Common parallel applications (1)

All common applications exhibit a very high fraction of potentially parallel operations (Amdahls Law: a very high, b very low)

Linear Algebra:

- Operations with vectors and matrices
- Systems of linear equations: $A \times x = b$
  - Solvers may work in a direct way, e.g. Gaussian-Elimination-Algorithm
  - Iterative solvers, e.g. Gauss-Seidel-Iteration, some very efficient solvers for sparse coefficient matrices A

Common parallel applications (2)

Solution of Differential Equations:

- Equations that contain x, a function $y(x)$ and deviations $y'(x)$.
- Numerical solution using discrete differences instead of symbolic differentiation
- Iterative algorithm on values x

Algorithmical basis of many 'simulations', in better words - numerical solution of PDE-problems

- Climate models and weather forecasting
- Deformation, explosion simulations

Common parallel applications (3)

Image processing:

- Local operators, e.g. spreading of spectrum, smoothing can be executed on different image parts in parallel
- Object matching, e.g. detection of geometric forms
- Finding of similar blocks in different images for detection of object movements
- (Soft) real-time multimedia

Example: Partial Differential Equation (1)

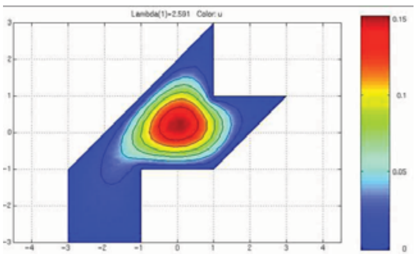
Laplace Partial Differetial Equation (Laplace PDE):

$$\triangle U(x,y) = \frac{\delta^2}{\delta x^2} U(x,y) + \frac{\delta^2}{\delta y^2} U(x,y) = 0$$

the values in $U(x,y)$ express for instance

- spacial distribution of electrical potential fields
- temperature on a surface
- level of ground water (e.g. for planning of building constructions)

Boundary values must be known for a solution of $\triangle U(x,y) = 0$



Example: Partial Differential Equation (2)

Discretization: express $U(x, y)$ by a two-dimensional array of values at discrete grid points

$$U(x, y) : U(i, j) \quad with \quad x_i = i \cdot h, \quad y_j = j \cdot h$$

with h as the distance of neighbor points in x, and in y direction.

Discrete approximation of differential operator:
Common practice is a substitution for the first order deviation, according to:

$$\frac{d}{dx} f(x) = f'(x) = lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx} f(x) = f'(x) = \frac{f(x+h) - f(x)}{h} + O(h)$$

...we need a discretization of $\frac{d^2}{dx^2} f(x) = f''(x)$

Example: Partial Differential Equation (3)

Using Taylor series:

$$\begin{aligned} f(x+h) &= f(x) + f'(x)h + \frac{1}{2}f''(x)h^2 + \frac{1}{6}f'''(x)h^3 + \dots \\ f(x-h) &= f(x) - f'(x)h + \frac{1}{2}f''(x)h^2 - \frac{1}{6}f'''(x)h^3 + \dots \\ f(x+h) + f(x-h) &= 2f(x) + f''(x)h^2 + \frac{1}{12}f''''(x)h^4 + O(h^6) \end{aligned}$$

This can be written ...

$$f''(x) = \frac{f(x+h) + f(x-h) - 2f(x)}{h^2} + O(h^2)$$

$$O(h^2) = -\frac{h^2}{12}f''''(x) + \dots$$

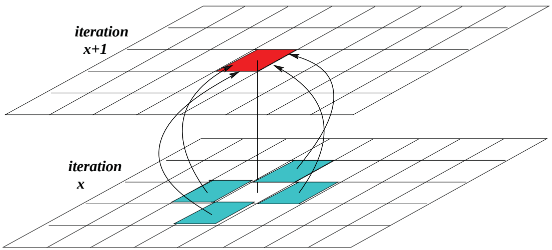
Example: Partial Differential Equation (4)

The differential operators can be written as differences:

$$\frac{U(i+1,j)+U(i-1,j)-2U(i,j)}{h^2}+\frac{U(i,j+1)+U(i,j-1)-2U(i,j)}{h^2}=0$$

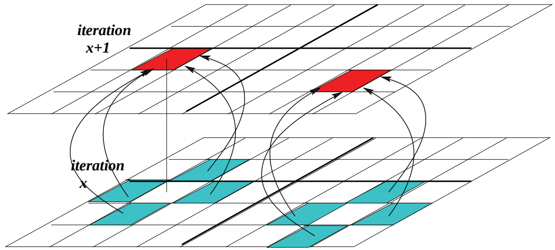
Finally, an iterative formula for U(i,j) is obtained:

$$U(i,j)=\frac{1}{4}(U(i+1,j)+U(i-1,j)+U(i,j+1)+U(i,j-1))$$



Example: Partial Differential Equation (5)

Can be transformed into a parallel iteration on separated areas for each processor



Access to neighbor areas:

- multiprocessor: via access to shared memory and synchronization
- multicomputer: by exchanging U(i,j) values that lay on the boundaries of the locally processed area (using messages).