TI-80

TI-82

TI-83

TI-85

TI-86

TI-92

Modeling Trigonometric Data

By Nils Ahbel

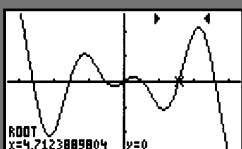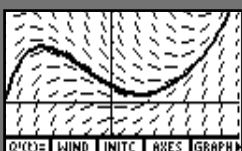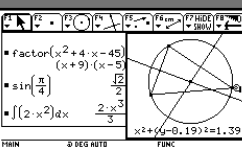**M**odeling data has become an important part of the secondary mathematics curriculum. Many Algebra and Precalculus courses take advantage of the regression packages in the TI-82, which include: linear, quadratic, cubic, quartic, natural log, exponential, and power. The TI-83 includes a trigonometric regression tool, but unfortunately, the TI-82 does not. This article explains how two programs can be used to aid in modeling trigonometric data. The first program, TRIGMDLS*, aids students in finding the model parameters through graphical analysis. The second, SINREG*, is a regression tool program similar to the built-in TI-83 sine regression feature.

Part I - Finding Model Parameters Through Graphical Analysis

Every sine wave can be expressed as a scale change and translation of $y = sin(x)$. Applying the linear change transformation $(x , y) \rightarrow (ax+h , by+k)$ to $y = sin(x)$ results in $\frac{y-k}{b} = sin\,\frac{x-h}{a}$, where the period $= 2\pi\,|a|$, amplitude $= |b|$, phase shift $= h$, and vertical shift $= k$ (1). From a data set a scatter plot can be drawn and a sine wave can be fitted by eye to the scatter plot. From this graph the period, amplitude, phase shift, and vertical shift can be estimated and values for the parameters a, b, h, and k as well. The program TRIGMDLS can assist in creating a scatter plot and finding appropriate values for the parameters. The program includes sample data of the average monthly temperature in Chicago for each month (2). The month (1 for January, 2 for February, etc.) are

IN THIS ISSUE:

LETTERS TO THE EDITOR

I just received my new "Eightysomething!." I was very concerned about something I saw (or didn't see) on the cover.

I phoned 1-800-TI-CARES to find out some answers and the operator suggested I e-mail the editors with my concerns.

I noticed that TI-81 did not make the cover; its icon was now removed from the left-hand column. My first reaction, what a tragedy!

The operator informed me that the TI-81 is being discontinued. That's fine. She also told me that many schools are "upgrading" their calculators to bigger and better models. That's fine, too.

The discontinuation of support for teachers still using the TI-81 in this publication is sad nonetheless.

I am a middle school teacher and we did a lot of fundraising to purchase the two class sets of TI-81s we have. We have no plans to "upgrade" and our students seem to enjoy the many experiences we provide them with the "discontinued" calculators we have. In fact, many are writing their own programs, hoping one day to have them published (I share "Eightysomething!" with them).

The message I receive from this publication is that the activities and experiences that middle school students may receive from the graphic calculators are no longer important—at least not important enough to dedicate at least a segment of your publication to it—not even important enough to at least recognize that many teachers, many students, many schools still get a lot of use out of these calculators.

My first reaction to the cover was this: Texas Instruments must be trying a new sales pitch to get schools to ditch their TI-81s and buy shiny new, technologically updated, fancier calculators. The reality, however, is that most public schools cannot afford new calculators. And why should we have to purchase something new when what we have seems to be suiting our needs—something we have worked hard to obtain in the first place.

I hope you will consider reinstating the TI-81 in your publication. After all, when you make a commitment to your products, you should maintain that support. I feel abandoned by this move and have a diminished level of respect for Texas Instruments and their products.

Please send me a written response. My students and I will be interested in hearing what you have to say.

Respectfully,
Steven J. Koponen
O. E. Dunckel Middle School
Farmington Hills, Michigan
Seventh grade math teacher, yearbook adviser

Steven,

The decision to delete the TI-81 screen from the masthead was based both on layout considerations (space problems when adding our newer products) and the fact that classroom use of the TI-81 has declined over time due to the increasing popularity of the TI-82 and more recently the TI-83. The introduction of these two calculators was in direct response to teachers requesting additional features and improvements required by an evolving curricula.

Although the TI-81 will be discontinued this year, we will continue to support your continued use of it. The following reflects our continued support for the TI-81:

Eightysomething! *- While fewer activities specific to the TI-81 are being submitted by users, the editors are certainly interested in publishing activities that are appropriate for the middle school curricula and the products being used there. Submit any articles to one of the editors listed on this page.*

Reference materials *- We have a list of publications, activity books, programming guides and other items related to the use of the TI-81. You can get a free copy of the list by simply visiting our web site at* ***www.ti.com/calc/docs/reference.htm***, *or calling* ***1-800-TI-CARES***. *These materials are available for order from the publishers referenced.*

Calc-TI *- This e-mail interest group supports the programming hobbyist with more technical information and discussions on our graphing calculators, including the TI-81.*

Graph-TI *- This e-mail interest group allows users of all TI graphing calculators to share ideas and solve problems relating to education.*

800-TI-CARES, *technical support - We will continue to answer questions regarding the TI-81 though our customer support lines.*

Repair *- any broken TI-81 will be repaired or replaced with a comparable unit for a period of five (5) years after we cease production.*

Retail availability *- at present, the TI-81 is carried by a number of national retailers and our dealers. This enables teachers or students currently using the TI-81 in class to easily obtain additional or replacement units.*

Volume Purchase Program *- Purchases of the TI-81 count toward obtaining free equipment. While TI-81 calculators and ViewScreens can be requested only as supplies last, purchases of the TI-81 do count toward obtaining other TI graphing models and classroom accessories. Further information is available on our web site at* ***www.ti.com/calc/docs/vpp.htm***

We hope this gives you a sense of how we value your adoption of the TI-81 in your classroom and will continue to support your use of TI graphing products.

Sincerely,
Texas Instruments
and the editors of **Eightysomething!**

The Calculator-Based Ranger™

By Bill Bower and Sam Gough

Math/Science Teachers, The Kiski School, Saltsburg, PA

The new Calculator-Based Ranger™ (CBR™) is the most recent example of how TI and teachers working together can develop the most effective solutions to specific educational needs. With the growing emphasis on connections between classroom theory and application, teachers have been looking for ways to collect and use real-world data. Unfortunately, many have found existing technologies much too difficult to learn and use. The CBR makes real-world data collection and analysis simple yet appropriate for all levels of mathematics, as well as physical science. Attractive, well-equipped, and modestly priced, the CBR is a self-contained motion data system that nicely complements the TI-82, TI-83, TI-85, TI-92, and the new TI-86.

Many educators have recognized the educational potential of data collection technologies, but questioned whether the time, effort, and financial investment that such systems demanded was justified. Teachers asked for a friendly plug-and-go device that would enable them and their students to simply open the box, collect data, and put it to effective classroom use, with little or no previous instruction. Teachers now have such a tool with the CBR.

We have been piloting the CBR with students in grades five through twelve math and science, and have watched it breathe new life into the classrooms we have worked with. High school students, always anxious to jump right into hands-on activities without being slowed by complex setup procedures, have seen their wishes come true with the CBR. The plug-and-go ease of use allows students to begin working immediately, assisted by simple setup prompts. The excitement of testing an idea has no time to fade as the students can immediately begin their activity and collect the data.

No programming knowledge or special hardware is needed with the CBR™. Just download the menu-driven multipurpose program from the CBR directly to the TI graphing calculator of your choice. The program allows you to instantly run a no-hassle sample using common "default" settings, or to customize the setup to meet the specific needs of you and your students.

Figure 1

The CBR collects distance, velocity, and acceleration data. Results can be plotted "real-time" (as it occurs), or stored to a list for subsequent graphical and statistical analysis. Plotting factors such as windows, labels, and scaling are automatically selected for hassle-free analysis. Available plot features include the ability to "smooth" the data so it appears less "noisy," and to select a subset of your collected data for closer analysis. Another selection from the easy-to-use "menu" of options is an interactive matching activity, in which students are shown a randomly-generated plot (Distance-Time or Velocity-Time) which they must reproduce with actual motion.
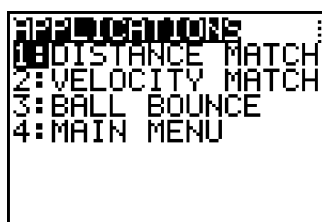

Figure 2

This feature provides instant feedback to the students as they explore graphing principles such as slopes and intercepts of lines. Another built-in feature simplifies the ball bounce activity, a simple yet powerful means of exploring parabolic and exponential relationships.
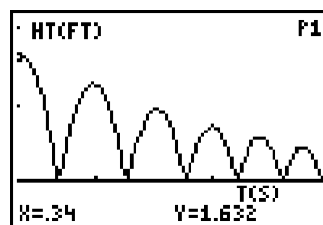

Figure 3

The CBR attaches to the calculator via a cord (included) much like an extended-length calculator unit-to-unit cord. This allows the calculator to "talk" to the CBR, telling it how and when to collect data. One setup option will let the user disconnect the CBR from the calculator and begin collecting data by pressing the TRIGGER button on the CBR. This provides a level of portability unmatched by any other motion probe. After the sample is finished, the CBR and calculator can be reconnected and the data downloaded to the calculator for analysis. The program also allows for time-delay triggering, in which sampling will begin ten seconds after the TRIGGER button or ENTER key is pressed. Watch the green light or listen to the "chirp" as they countdown to the start of sampling. This is very handy when one person is conducting an activity that needs a third hand!

Convenient mounting options further contribute to the CBR's ease of use. The sensor "flips" up and pivots through a range of 0 to 180 degrees, stopping at any desired angle. This allows you to point the sensor directly at the object, while the CBR is resting in a stable position. For example, the CBR can rest flat on a table while the sensor is precisely aimed at a dynamics cart traveling down an incline. A tripod socket is also located on the CBR, giving the user a wide range of options in setting up the motion sensor. A removable clamp is supplied that allows the CBR to be mounted to a table or door edge, a circular ring stand, or other convenient object.

The CBR has been an immediate hit in middle school math/science. For teachers it represents an easy way to bring data collection into the classroom and share it with students, even those not fully competent on the graphing calculator. For students it represents a fun way to learn.

Fifth and six graders who have little to no concept of algebra and higher mathematical concepts find the ease of data collection and display makes a study of graphing fundamentals accessible to them. They enjoy seeing results and discovering relationships, and quickly learn the effect of making small adjustments to the sample setup in order to best suit the activity.

Older middle school students, who are being exposed to pre-algebra and above, are fascinated to discover that there are real world applications to the concepts they are learning. This generates a great deal of enthusiasm and a never-ending stream of "what ifs?" The teacher's question, "How can we get them excited about math?" has been replaced by, "The bell already rang, how can I get them out of the room?" They have such fun that only later do they realize they have gained an understanding of graphing concepts. The CBR is definitely a hero at the middle school level to both the student and the teacher.

An unexpected bonus of the CBR was the degree of spontaneity and increased level of creativity it brought to the classroom. This is largely due to the ease with which the CBR's program accommodates various activity situations. Students no longer need to set up a time-consuming manual experiment (remember using yardsticks and stopwatches?), or write a program for a calculator or computer in order to try something different. Instead, they simply modify the desired parameter — for instance, the rate at which samples are taken. The all important "what if" question can now be answered immediately by the student, who has virtually complete control of how the data is collected. This in turn encourages more active participation, and produces better results from the students. They are truly excited by the prospect of finding out the answer to an idea immediately.

While the advantages of CBR are most apparent when attached directly to the graphing calculator, CBR remains fully compatible with the Calculator Based Laboratory™ (CBL™) and most microcomputer-based lab (MBL) devices. The ease-of-use and mounting options of the CBR offer several advantages over conventional motion detectors. Already using a favorite CBL motion program? No problem — motion programs written for the CBL can be used with the CBR, either directly connected to the calculator or working through the CBL.

Another feature teachers will value is the included book, "Getting Started with CBR™." This friendly guide features a quick-start path for first-time users as well as ready-to-use instruction and recording sheets for several classroom activities which explore specific mathematical principles using data collected with the CBR. You'll be running educational activities from the first day you open the box.

The CBR system really is the easy-to-use, self-contained data collection solution you have been waiting for. Think of it as a box full of real-world applications for your math class.◆

stored in L_1 and average monthly temperature (in °F) is stored in L_2. If you want to use the program for your own data, store the values of the independent variable in L_1 and values of the dependent variable in L_2. The following input and output screens use the sample data.

When the scatter plot appears, trace is active, which aids in determining the values of the four parameters.

Figure 1

Figure 2

Estimates of the parameters are entered, predicted y-values are stored in L_3, and the scatter plot is drawn again with the first estimate of a model based on the inputs graphed.
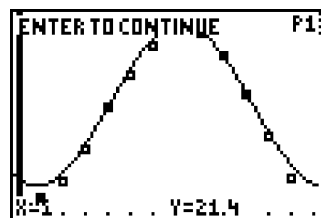
Figure 3

Figure 4

In this case it appears that the amplitude is too small and the vertical shift is too big. The program calculates the errors (actual - predicted y-values) and stores them in L_4; the squares of the errors are stored in L_5. The better the model, the smaller the sum of the squares of the errors. In this case (sum errors)$^2 \approx 59.9$. The next step is to choose to change a, b, h, and k, entering the same values for a and h and changing b and k.

Figure 5

Figure 6

Notice that the model reflected in the next two screens is more appropriate. It appears that the graph of the model lies closer to the data points. Also, the sum of the squares of the errors, ≈ 19, is much smaller than with the first set of para-

meters. This program can be used to estimate, check, and revise until the sum of the squares of the errors cannot be lowered appreciably.
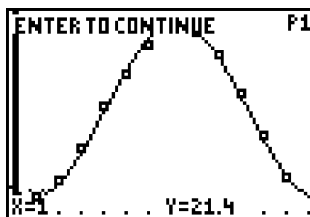

Figure 7

Figure 8

Of course students can model trigonometric data without the help of TRIGMDLS, but the program allows students to focus on the parameters without stumbling over keystrokes and misplaced and/or missing parentheses.

This program can be used in tandem with the type of program described in the article *Dealing with Data on the TI-82*, in **Eightysomething!**, Spring 1996, page 4 (download the article at http://www.ti.com/calc/docs/80xdown.htm#old). Teachers can write simple two-line programs which store the data into L_1 and L_2. Virtually no experience with programming is necessary to write these mini-programs.

Eventually students should learn how to solve the model for **y**, enter the function into "**Y=**", and choose the appropriate window. One logical first step is to focus on the parameters before the keystrokes. Note that you can download TRIGMDLS* which models using *y = sin(x)*, as well as TRIGMDLC* which models using *y = cos(x)*, from http://www.ti.com/calc/docs/80xthing.htm.

Part II - Finding Model Parameters With a Regression Program
*Program by John Kenelly, Department of Mathematical Sciences, Clemson University, and Charley Scarborough, Texas Instruments, Personal Productivity Products***

Program SINREG* is a trigonometric regression program which calculates the parameters $\frac{y-k}{b} = sin \frac{x-h}{a}$ based on four inputs. Two of the inputs are the location where the x-list and y-list are stored. If you ran program TRIGMDLS* and used the sample data (see Part I), sample data was stored in L_1 and L_2. The program needs an estimate of the period. Often the period is known, and if not, an estimate

of the period can be seen on the scatterplot. (The built-in trigonometric regression feature of the TI-83 does not require an estimate of the period.) The final input is the maximum number of iterations.

The program keeps searching for better values of the parameters until the maximum number of iterations have been run, or no appreciable improvement in the parameters occurs in successive iterations. In the input/output screens below, the same Chicago weather data was used as in Part I above. The data is monthly temperature data, so the period is 12. The first time the program was run, the maximum number of iterations was input as 1, allowing the program to run only one iteration. The results are shown below.

| Figure 9 | Figure 10 | Figure 11 |

The second time the program was run the maximum number of iterations was input as 10. With this data set, each iteration took about 15 seconds. Although the maximum number of iterations was 10, the program stopped after 6 iterations. The parameters after 6 iterations had a better fit. The sum of squares of the errors decreased from about 23.1 to 5.9. You have the option of storing the equation in Y_1. If you do, it will be in the form $y = b \cdot sin\left(\dfrac{x - h}{a}\right) + k$. The results of the second run are shown below.
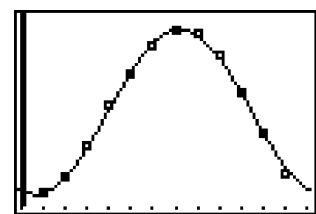
| Figure 12 | Figure 13 | Figure 14 |

The two programs described above can be used together to aid students in the study of trigonometry as it pertains to data. TRIGMDLS can be used as a tool to help students develop an understanding of the parameters of a trigonometric model. SINREG can be used the same way the built-in regression tools of the TI-82 are already used, to quickly find an appropriate model for data.◆

*All programs mentioned in this article are available at http://www.ti.com/calc/docs/80xthing.htm.

** *The program written by John and Charley calculated a, b, c, and d in the equation. The program was modified so that the output of both programs would be the same.*

(1) *Functions, Statistics, and Trigonometry*, The University of Chicago School Mathematics Project, Scott, Foresman, 1992, p. 353.

(2) Ibid., p. 362.

Meet the Editor

John F. Mahoney

John F. Mahoney has taught mathematics at Sidwell Friends School, an independent coeducational Quaker day school in Washington, DC, for the past 20 years. He previously taught at Moorestown Friends School (NJ) and Wilmington Friends School (DE). He taught at the Truro School in Cornwall, England in 1991 - 1992 as a participant in a teacher exchange.

John, a native of Westfield, New Jersey, was a member of the Slide Rule Club in high school. He did his undergraduate work in mathematics at Bucknell University and his graduate work at Temple University. He was awarded the Presidential Award for Excellence in Science and Mathematics Teaching in 1994.

In 1972, when he first started teaching high school, there was only one student in his school who was wealthy enough to afford a calculator which was sufficiently powerful to compute square roots. Whenever John had to calculate a lot of square roots, he tracked down this student to borrow his calculator. Eventually prices came down and John bought a Texas Instruments SR-10 calculator. In 1977, John bought a Texas Instruments TI-58. This programmable calculator had a single line display and came with interchangeable plug-in memory modules. Both of these calculators came with black, plastic belt pouches.

In 1979 John and a colleague bought Sidwell Friends' first micro computer — an Apple II+. John received a grant from the Apple Education Foundation to write and market graphical calculus programs for the Apple in 1980. Since then John has led workshops on using graphing calculators and computers in mathematics teaching. He has been a Reader and a Table Leader for the College Board's Advanced Placement Mathematics program and is currently a Consultant for the College Board.

John is the clerk (president) of the Board of Trustees of Sandy Spring Friends School (MD). He has four children whose ages are 14 and 17. None of them is a twin. Each of them uses graphing calculators extensively. John and his family live in an 1844 farm house in Bethesda, MD. John's other interests are carpentry, opera, cycling, and watching "Seinfeld."

As an editor of **Eightysomething!**, John's focus is to write and edit articles about innovative and straightforward ways of using calculators in the classroom.

Continued Fractions on Graphing Calculators

By John F. Mahoney

Middle schoolers are frequently intrigued by continued fractions. For example, ask your students to contemplate finding the value of:

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+...}}}}}}}}}$$

Some students will ask whether the expression has a value. That is actually a very insightful question.

Here's a way, using a graphing calculator, to determine whether the expression has a value.

First, press 1 and then ENTER. Then type the expression: 1 + 1/Ans ▶ Frac and press ENTER again. The calculator will respond with 2. Keep pressing ENTER and the calculator will respond with a series of fractions.
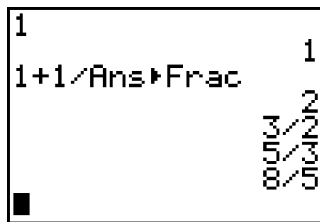
```
1                      1
1+1/Ans►Frac
                       2
                     3/2
                     5/3
                     8/5
■
```
Figure 1

Eventually the fractions will turn into decimals.

```
            2584/1597
            4181/2584
            6765/4181
           10946/6765
           1.618033985
           1.61803399
           1.618033988
■
```
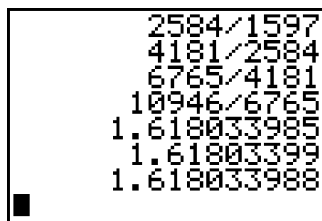*Figure 2*

If students continue to press ENTER the values of the decimal will become constant at 1.618033989, which indicates that this process converges.

Analytically, algebra students can come up with the same result. For this process to converge to a value, x, then x must have the property that:

$$1+\frac{1}{x}=x, \quad \text{or} \quad x+1=x^2, \text{ so}$$

$$x^2 - x - 1 = 0$$

which can be solved by quadratic formula to get

$$x = \frac{1 \pm \sqrt{5}}{2}.$$ The positive value of x is

$$\frac{1+\sqrt{5}}{2} \approx 1.618033989$$

Further, some students will recognize that the numbers in the fractions are terms of the Fibonacci sequence {1, 1, 2, 3, 5, 8, 13, 21, ... , 1597, 2584, ...} which has the property that every term is the sum of the two preceding terms. Students thus, in completing this example, are actually verifying that the ratio of successive terms of a Fibonacci sequence converges to $\frac{1+\sqrt{5}}{2}$.

Notes:

1. Encourage your students to experiment with other continued fractions with different initial values and patterns. Specifically, replace the 1's in the example with 2's.

2. You may have a student who asks about the other root ($\frac{1-\sqrt{5}}{2}$) to the quadratic equation above. Suggest that they experiment with starting with that number -0.618033989 as their initial value on their calculator. Another alternative is to use the expression 1-1/Ans ▶ Frac with an initial value different from 1. This leads to a sequence of numbers which, while not converging, does have a cycle of period 3. If your students have access to a TI-82 or a TI-83, they can explore CobWeb diagrams using their manuals.

3. Users of the TI-80 should use the expression: 1 + 1/Ans ▶ b/c instead of 1 + 1/Ans ▶ Frac. Since the TI-81 doesn't have fraction capability, users should use the expression 1 + 1/Ans to get decimal values. (A program that converts decimals to fractions can be found in the TI program archive at http://www.ti.com/calc/docs/arch.htm.) ◆

Does a TI-8x Cast a Fair Die?

By Constance C. Edwards

In Chapter 6 of DATA ANALYSIS AND STATISTICS from the NCTM CURRICULUM AND EVALUATION STANDARDS FOR SCHOOL MATHEMATICS (1992), there is a cute little discourse on chi square. It starts with a die which has been rolled 60 times. Each face does not appear an equal number of times. Is there something wrong with the die? Chi square is computed and found to be 7.0. If chi square were near zero, we could be confident that the die was "fair." But how close to zero must it be? The article then states, "One way to answer this question is to look at chi squares generated from purely random events...," and suggests that one use a calculator or computer program to do this. We will show how a TI-8x can be used to decide the fate of this die. And, along the way, we will answer the question, "Does a TI-8x cast a fair die?"

Okay, so you already know that the TI-8x casts a fair die. After all, all we have to do is use its random number generator to emulate the throw of a die, and we'll have a fair die. Right? Well, maybe we'd better check this out. But before doing this, maybe we should explain how chi square plays a role in answering this question.

Chi square is a statistical measure of the difference between the **expected outcome** and the **actual outcome**. If a fair die is rolled N times, probability theory tells us that we should **expect** each face of the die to appear $\frac{1}{6} N$ times. But in **actuality**, this usually does not happen. If E denotes the **expected** number of times which probability theory tells us the event should occur, and if A denotes the **actual** number of times that the event did occur, then (E-A) gives a measure of the difference between these events. To compensate for the problem of dealing with negative versus positive differences, it is easier to look at (E-A)^2. And to turn this into a ratio which compares this unsigned result to the **expected** outcome, one looks at (E-A)^2/E. The sum of these values is called chi square. That is, $chi\ square = \sum \frac{(expected - actual)^2}{expected}$.

From this definition, it is easy to see that a chi square value which is close to zero tells you that you have, for example, a fair die. But what does a chi square of 7.0 tell you?

To help answer the fair die questions, here's a program, ROLLDIE, which gets a TI-80, 82, or 83 to roll a die N times and compute chi square .

```
: ClrHome
: Input "NUM ROLLS", N
: ClrList L₁, L₂
: For(I, 1, 6, 1)
: I→L₁(I): O→L₂(I)
: End
```

```
: For(I, 1, N, 1)
: int (6rand+1)→D
: L₂(D)+1→L₂(D)
: End : N/6→E
: sum ((L₂-E)²/E)→C
: Disp "CHI = ", C
```

The program will place the face number of the die in list **L₁**, and their corresponding frequencies in **L₂**. The value of chi square will be housed in the variable **C**. If you are having trouble programming your calculator, the appendix of the TI-GUIDEBOOK is a great resource.

The **int(6rand+1)** command emulates the roll of our TI-Die. (On a TI-80 or 83, **RandInt (1,6)** can be used in place of **int (6rand+1)**. **RandInt** is not available on the TI-82.) **Rand** generates random numbers
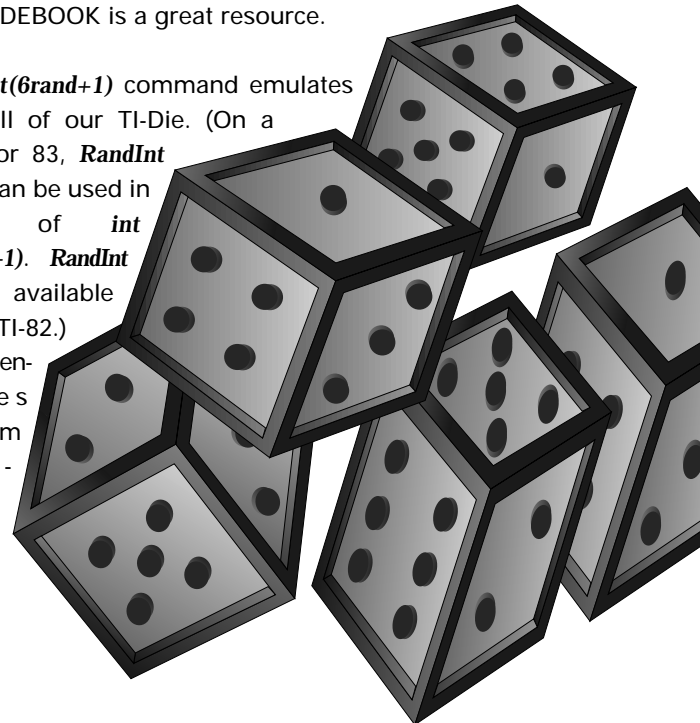
strictly between *0* and *1*. So *6rand + 1* generates numbers strictly between *1* and *7*. And *int(6rand+1)* gives the integer part of these numbers. Since this is based on a valid random number generator, this TI-Die is a fair die. But, as the figures below will show, this TI-Die does not always appear to be fair.

Figure 1

Figure 2

We're taught that if a fair die is rolled a sufficient number of times, each face will appear an equal number of times. Many people would think that a TI-8x would be capable of showing such equality of faces after about a million rolls. We made only one attempt at proving this with a TI-80. After 30 hours the experiment was aborted because the batteries were showing signs of dying. The results appear in Figure 1. The program was terminated after 911,073 rolls with chi square ≈11.5. Does that value of chi square make you feel that the TI-80 casts a fair die?

In Figure 2, L_2 shows the result of rolling the TI-82 60 times with chi square ≈ 3.8. That sounds a bit fairer than a chi square of 11.5, but how good is it? L_3 shows the result of replacing the *int(6rand+1)* algorithm in the above program with another algorithm which will be discussed later. We will call this new die the L_3-Die. It was also rolled 60 times with chi square ≈1.4. Is the L_3-Die better than the TI-Die?

The answers to the questions about the fairness of the TI-Die, the L_3-Die, and the die mentioned at the beginning of this article, all boil down to answering one question: What is the significance of chi square? How close to zero must it be before we can conclude that we have a fair die?

The NCTM article cited above suggests that we compare our results with "chi squares generated from purely random events." The random number generator possessed by the TI-8x, which is based on proven mathematical algorithms, is capable of doing this.

So to check out the die mentioned at the beginning of this article (chi square = 7) and the L_3-Die (chi square = 1.4), we need to compare these results to the chi squares of a TI-Die which is rolled 60 times, with the experiment being repeated enough times to generate a large distribution of chi squares. Each student in a class could run the program once to collect data on chi square values.
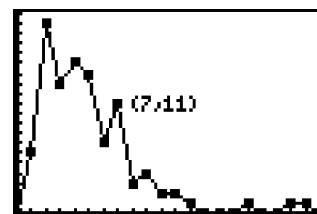

Figure 3

Figure 4

Figures 3 and 4 show the results of running a program, NCHIDIE, which was written to perform this experiment 100 times. The integer part of possible chi squares is listed in L_1 with L_2 containing their frequencies. Our largest chi square was 20. This table shows that a chi square of 6 or less occurred 74 out of 100 times. Thus, the probability of a chi square of 7 or more is 26%. So with a fair die, a chi square of 7 or more will happen about 1 out of 4 times. The NCTM article concludes that this is not a very unusual outcome, so the die is most likely a fair die.

Couldn't the same be said of the L_3-Die? After all, this die had a chi square of 1.4. Figure 3 suggests that this die, when compared to the TI-Die, has a 94% probability of being a "fair die."

But the L_3-Die is not a fair die. It used the algorithm *int(11rand)→R :R - 6int(R/6) + 1 →D* to emulate the roll of a die. This algorithm generates numbers between and including 0 and 10 and adds 1 to the remainder when that number is divided by 6. So the probability that a 6 is rolled is only 1/11, while the probability of the other faces is 2/11. This is definitely an unfair die. What does this say about the NCTM conclusion mentioned above?

It is hoped that this article has raised more questions than it has answered, questions which can be discussed in a classroom situation. The TI-Die is a fair die, even when it gives a chi square of 11.5. And an unfair die can give misleadingly positive chi squares of 1.4. So is it correct for the NCTM article to conclude that its chi square = 7 die is "most likely a fair die?" ◆
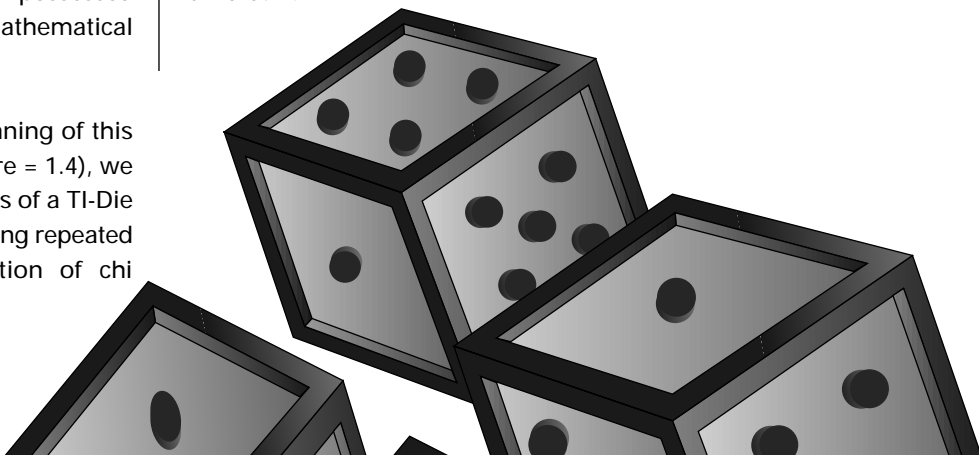
Exact Radical Solutions with TI-8X

By Ames Brown, student, Kent, CT

Ames is a Junior at Kent School in Kent, CT. He is an excellent math student, a talented computer scientist, and captain-elect of the Varsity Cross Country Team. You can e-mail him at browna@kent.pvt.k12.ct.us or visit his home page at http://www.kent.pvt.k12.ct.us/students/browna/ameshome.htm.

One major limitation of the TI-8X series of calculators is its inability to display solutions as radicals; however, a simple scanning program can emulate this feature of more sophisticated calculators.

The EXACTOR program (available at http://www.ti.com/calc/docs/80xthing.htm) can convert a decimal of six or more digits into the form $\pm \frac{\sqrt{W}+T}{U}$, when possible. The 4th, 6th and 8th lines of the program allow you to change the maximum searchable values of T, W, and U respectively. (The 3rd, 5th, and 7th lines of the program allow you to change the minimum searchable values of T, W, and U respectively.) The EXACTOR program accepts input from Ans. Here are some sample input values and the EXACTOR program's output.

When the following expression is evaluated:	and "Ans" is:	Exactor's output is:
$1/\sqrt{2}$	.7071067812	$\sqrt{2}/2$
$\sqrt{8}$	2.828427125	$2\sqrt{2}$
-2.724744871	-2.724744871	$-\sqrt{6}+3/2$
cos(150°)	-.8660254038	$-\sqrt{3}/2$
tan(-π/6ʳ)	-.5773502692	$-\sqrt{3}/3$

An example of a calculation and the actual output from EXACTOR is shown below.
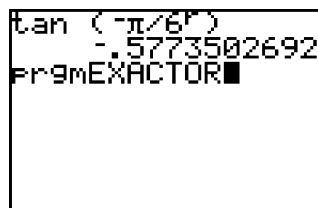
Figure 1
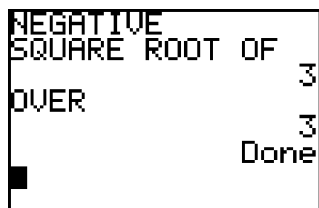
Figure 2

EXACTOR is a simple scanning program that attempts to match thousands of values with the input value in order to determine the exact value it represents. It begins by taking the square root of every positive whole number from one to six and comparing its decimal equivalent to the input value. If there is no match, the program multiplies each of these six values by every positive whole number from one to six. If there still is no match, the program divides the 36 numbers from the first two scans by every whole number from one to six. If there is no match in 1,296 values, the input is not within the domain of the program. The program's domain can be easily expanded or restricted by changing the first digit of the second line of the program to any positive whole number.

In order to expedite the program's scanning, the program scans for negative matches as it scans for positive matches. It does this by taking the absolute value of the input for each match attempt. If a match is found, EXACTOR then checks whether the match is positive or negative. If the program scanned each match individually for a negative match, the scan would take twice as long to complete.

A complete scan of all 1,296 solutions takes just over 20 seconds. Finding a simple radical like the first example takes less than 2 seconds, but more complicated solutions such as the third example can take about 13 seconds to find. If the range is increased, the waiting time increases exponentially because the calculator is solving for more matches. If X is the limit of the scan, EXACTOR searches for X^4 matches.◆

Modeling Titration Curves

A Point of Integration with Chemistry and

By Kent J. Crippen, Robert Mann, and David W. Brooks, Lincoln, Nebraska

Titration is a procedure used by chemists to determine the concentrations of dissolved chemical species. Chemistry teachers often use titrations to teach about acid/base behavior. The chemistry teacher uses the shapes of titration curves when explaining a variety of chemical behaviors. What a chemistry teacher describes as the buffering zone and the equivalence point of a titration curve, the mathematics teacher describes as inflection points of a function. The titration curve represents a possible way to connect these two disciplines.

We have developed a simple calculator program that generates a titration curve for any reacting acid species with a strong base (such as sodium hydroxide). The user enters these quantities: molarity of acid; dissociation constant for the acid (Ka); dissociation constant for water (Kw); volume of acid to be titrated; and molarity of titrant (such as NaOH). The calculator then generates a titration curve plotting pH vs. volume titrant added (Figure 1).
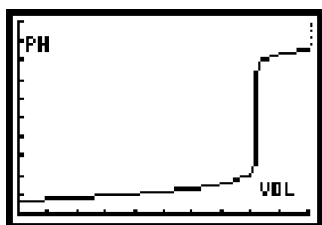
Kw = 1.0×10^{-14}	
Ma = 0.1 M	
Mb = 0.1 M	
Va = 100 mL	
Ka = 100	

Figure 1

The program allows students to generate multiple titration curves without performing the actual titration. The chemistry instructor can then focus instruction on the differences between curves as seen in Figures 2-4. These are interpreted in terms of the chemical behavior of the reacting acid.
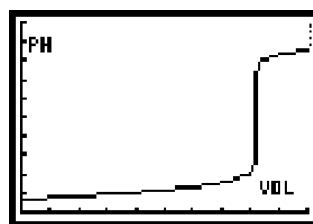
Ka = 100

Figure 2
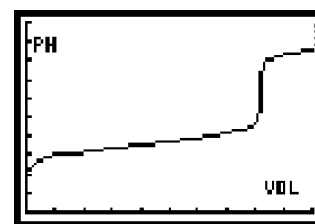
Ka = 1×10^{-5}

Figure 3
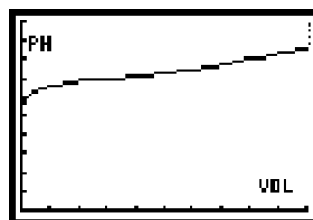
Ka = 1×10^{-10}

Figure 4

Kw = 1.0×10^{-14}
Ma = 0.1 M
Mb = 0.1 M
Va = 100 mL

Using the Titrate program*, the math teacher can generate multiple samples for analysis of various slopes and critical values (Figure 5).

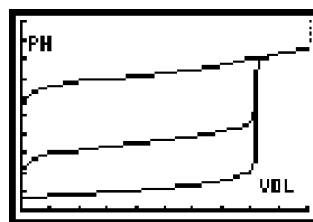| |
|---|
| Ka1 = 1×10^{-10} |
| Ka2 = 1×10^{-5} |
| Ka3 = 100 |

Kw = 1.0×10^{-14}
Ma = 0.1 M
Mb = 0.1 M
Va = 100 mL

Figure 5

In pre-calculus, an exciting extension using this program would be a "find the function" activity for students. Since the titration curves are generated by a program using data manipulation and not an equation, the students are given a graph, but no function. A terrific problem-solving opportunity

has emerged. Instruct the students to find a function which will generate a curve which best matches the titration model provided. Have them then use this curve to predict pH values for unknown volumes and volumes for currently unknown pH values. They can then check their predictions by slightly altering the range and/or scale of the original program. As the students try to create a function with a matching graph, they should gain valuable knowledge about function properties, transformations, and piece-wise functions. They should also learn how to better manipulate their calculators and how to interpret function inputs and outputs. As the students explore this open-ended question, you may give as little or as much guidance as you like. You may, for instance, suggest that the students try piece-wise and radical functions.   The following function modeled one titration curve very well (Figure 6):

$$y = 5(x-8)^{\wedge}(1/7) + 8$$
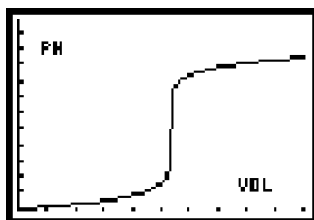
Figure 6

Follow-up discussions should include basic function concepts as well as the effects of stretches and shifts upon the original curve.

Another extension would require the students to adapt the program (or the current lists) to find the derivative of these titration curves. This would allow the students to talk about the inflection points of the original curves and how they relate back to the scientific experiment that developed them. The point of maximum slope is of particular importance since this is the equivalence point—the point where the reactants are equal in quantity. By analyzing a variety of curves and determining the point of maximum slope (the derivative of the function), students discover that equivalence points are not the same for all titrations.   More importantly, that chemical behavior, specifically the ionic character (or degree of dissociation into ions) of the reacting specie dictates the point of equivalence.

This program is an excellent tool for teachers integrating Chemistry and Mathematics. Students can use the program to investigate the functions and critical values inherent to a titration curve, or to explain those functions and values based upon the nature of reacting compounds. In either case, the mathematics and chemistry are obviously intertwined. Instruction that emphasizes both the Chemistry and Mathematics will help the student develop a more thorough understanding of this phenomenon.◆

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*The 82/83 program can be downloaded from
ftp://archive.ppp.ti.com/pub/graph-ti/calc-apps/
http:///www.ti.com/calc/docs/80xthing.htm,
or **http://www.cci.unl.edu/calculators/titrate.html.**

Correcting a Calculator "Lock-Up"

by Terry Walsh

Recently, my TI-92 locked up and I couldn't get it to operate, even after I put in brand new batteries. Contacting TI, I received the following suggestions to try:

1. Pull out one of the batteries.

2. Hold down the ON key for at least 10 seconds.

3. Replace the battery.

4. Turn the calculator on and adjust the contrast ("diamond" and "+" keys on the TI-92 or 2nd ↑ for the TI-8X series).

5. If these four steps don't work, on the TI-92, try the reset sequence of lock, diamond and on.

The first four steps can be tried on any of the TI graphing calculators. If your calculators ever freeze, try these steps if nothing else works. If it is still frozen, call 1-800-TI-CARES, because they do and they'll help you troubleshoot the difficulty.

Have Your Students Plot a Logistic Curve

By John Mahoney

Populations which have a limit to their growth can frequently be modeled by a logistic curve in the form:

$$f(x) = \frac{c}{1 + ae^{-bx}}$$

where a, b, and c are constants. Your students can use their calculators to explore the significance of changing the values of a, b, and c. Better yet, you can use a calculator in the class to develop a specific example of a logistic curve.

Probably the easiest example of a logistic curve for students to understand is the spread of a rumor in a school. Initially only one or two students know of the rumor. As it spreads, it starts to spread more and more quickly. After awhile, when almost everyone knows of the rumor, the rate of increase decreases. After a long time the last person in the school (a teacher? a principal?) finally hears of the rumor.

Give each of your students a number from 1, to say 30. If you are fortunate to have a small class, assign numbers to each of the students' hands. In this article, I'll assume there are 30 students. Have the calculator produce a random integer between 1 and 30 by typing in the expression: **iPart(30rand+1)**. [The sequence of keys on the TI-82 to accomplish this is: MATH, ⇒, ⇓ , ENTER, (, 3, 0, MATH, ⇒, ⇒,⇒, ENTER, +, 1,)]

Once that expression is on the screen, press ENTER and the calculator will return an integer from 1 to 30. My calculator returned a 4. Yours will probably be different. In my case, this means that student #4 originates the rumor. Since just one person knows the rumor, press ENTER once again. My result was 14. Now two students know the rumor, so we'll press ENTER twice more to find out who they "spread" it to. My result was 30 and 21. Start making a table of how many students know the rumor during each time period.

You should ask the students to raise their hands at the end of each time period if they have "heard" the rumor. At the end of the third time period four students have heard the rumor (#4, 14, 21, and 30) and so I press the ENTER key four times and get #17, 20, 21, and 28. 17 is a repeat and thus after four time periods only 7 (of the 30) students have heard the rumor. Continue this process. To make it easier, clear the calculator screen at the beginning of each time period by pressing CLEAR. The TI-82 and TI-83 can display 7 lines of data in each screen, so you can press ENTER seven times in succession.

Continue this process until all of the students have "heard the rumor." In my situation, this took 12 time periods as shown below.

Rumor Spreading

Time Period	Number of Students Who've heard the rumor
1	1
2	2
3	4
4	7
5	12
6	18
7	24
8	26
9	28
10	28
11	29
12	30

Table 1

Now store the data, above, in lists, on the TI-82 or TI-83. Plot the graph of the data using STAT PLOT. The TI-83 has a Logistic regression procedure and you can use it to compute an equation of a logistic curve which best fits the data. You can plot the graph of this equation easily by going to Y=, choosing the VARS menu, Statistics, EQ, RegEQ. This process places the most recently computed regression equation directly into Y1. Users of other calculators can experiment with different values of the parameters a, b, and c in the logistic equation to come up with a function that best fits the data. As a hint, c should be close to the number of students in the class.◆

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

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