Does a TI-8x Cast a Fair Die?

By Constance C. Edwards

In Chapter 6 of DATA ANALYSIS AND STATISTICS from the NCTM CURRICULUM AND EVALUATION STANDARDS FOR SCHOOL MATHEMATICS (1992), there is a cute little discourse on chi square. It starts with a die which has been rolled 60 times. Each face does not appear an equal number of times. Is there something wrong with the die? Chi square is computed and found to be 7.0. If chi square were near zero, we could be confident that the die was "fair." But how close to zero must it be? The article then states, "One way to answer this question is to look at chi squares generated from purely random events...," and suggests that one use a calculator or computer program to do this. We will show how a TI-8x can be used to decide the fate of this die. And, along the way, we will answer the question, "Does a TI-8x cast a fair die?"

Okay, so you already know that the TI-8x casts a fair die. After all, all we have to do is use its random number generator to emulate the throw of a die, and we'll have a fair die. Right? Well, maybe we'd better check this out. But before doing this, maybe we should explain how chi square plays a role in answering this question.

Chi square is a statistical measure of the difference between the *expected outcome* and the *actual outcome*. If a fair die is rolled N times, probability theory tells us that we should *expect* each face of the die to appear $\frac{1}{6} N$ times. But in *actuality,* this usually does not happen. If E denotes the *expected* number of times which probability theory tells us the event should occur, and if A denotes the *actual* number of times that the event did occur, then $(E-A)$ gives a measure of the difference between these events. To compensate for the problem of dealing with negative versus positive differences, it is easier to look at $(E-A)^2$. And to turn this into a ratio which compares this unsigned result to the *expected* outcome, one looks at $(E-A)^2/E$. The sum of these values is called chi square. That is, $\text{chi square} = \sum \frac{(\text{expected - actual})^2}{\text{expected}}$.

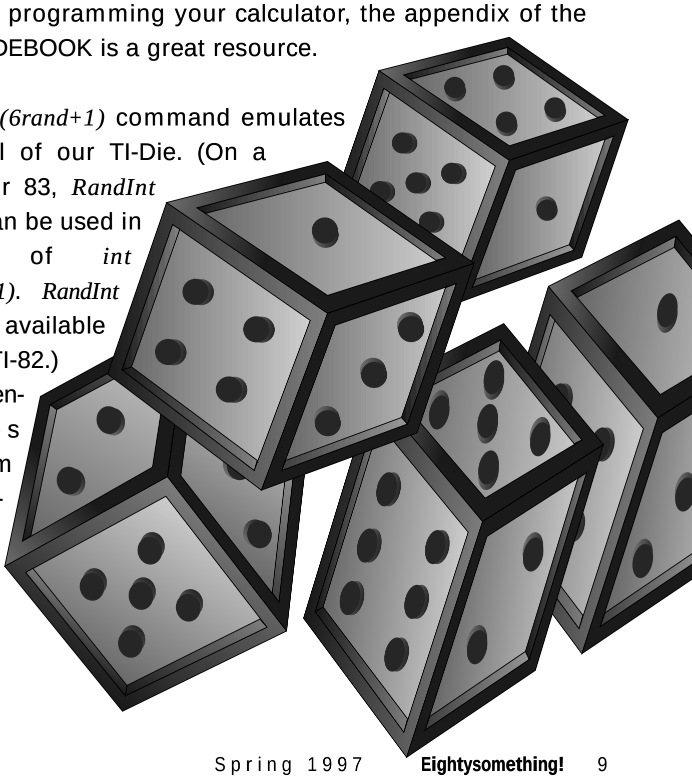
From this definition, it is easy to see that a chi square value which is close to zero tells you that you have, for example, a fair die. But what does a chi square of 7.0 tell you?

To help answer the fair die questions, here's a program, ROLLDIE, which gets a TI-80, 82, or 83 to roll a die N times and compute chi square .

```
:ClrHome
:Input "NUM ROLLS",N
:ClrList L1,L2
:For(I,1,6,1)
:I→L1(I):0→L2(I)
:End
```

```
:For(I,1,N,1)
:int (6rand+1)→D
:L2(D)+1→L2(D)
:End :N/6→E
:sum ((L2-E)²/E)→C
:Disp "CHI = ",C
```

The program will place the face number of the die in list L_1, and their corresponding frequencies in L_2. The value of chi square will be housed in the variable C. If you are having trouble programming your calculator, the appendix of the TI-GUIDEBOOK is a great resource.



The *int(6rand+1)* command emulates the roll of our TI-Die. (On a TI-80 or 83, *RandInt (1,6)* can be used in place of *int (6rand+1)*. *RandInt* is not available on the TI-82.) *Rand* generates random numbers strictly between *0* and *1*. So *6rand + 1* generates numbers strictly between *1* and *7*. And *int(6rand+1)* gives the integer part of these numbers. Since this is based on a valid random number generator, this TI-Die is a fair die. But, as the figures below will show, this TI-Die does not always appear to be fair.

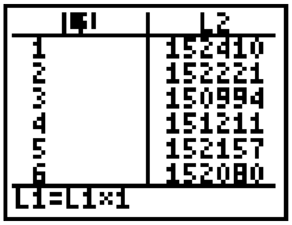


Figure 1

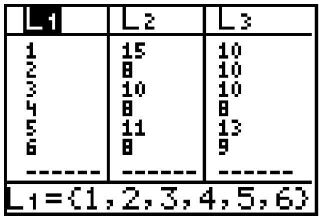


Figure 2

We're taught that if a fair die is rolled a sufficient number of times, each face will appear an equal number of times. Many people would think that a TI-8x would be capable of showing such equality of faces after about a million rolls. We made only one attempt at proving this with a TI-80. After 30 hours the experiment was aborted because the batteries were showing signs of dying. The results appear in Figure 1. The program was terminated after 911,073 rolls with chi square ≈11.5. Does that value of chi square make you feel that the TI-80 casts a fair die?

In Figure 2, L_2 shows the result of rolling the TI-82 60 times with chi square ≈ 3.8. That sounds a bit fairer than a chi square of 11.5, but how good is it? L_3 shows the result of replacing the *int(6rand+1)* algorithm in the above program with another algorithm which will be discussed later. We will call this new die the L_3-Die. It was also rolled 60 times with chi square ≈1.4. Is the L_3-Die better than the TI-Die?

The answers to the questions about the fairness of the TI-Die, the L_3-Die, and the die mentioned at the beginning of this article, all boil down to answering one question: What is the significance of chi square? How close to zero must it be before we can conclude that we have a fair die?

The NCTM article cited above suggests that we compare our results with "chi squares generated from purely random events." The random number generator possessed by the TI-8x, which is based on proven mathematical algorithms, is capable of doing this.

So to check out the die mentioned at the beginning of this article (chi square = 7) and the L_3-Die (chi square = 1.4), we need to compare these results to the chi squares of a TI-Die which is rolled 60 times, with the experiment being repeated enough times to generate a large distribution of chi squares. Each student in a class could run the program once to collect data on chi square values.

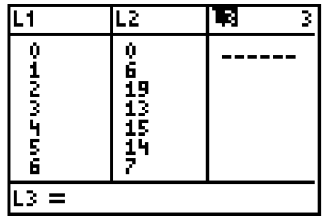


Figure 3

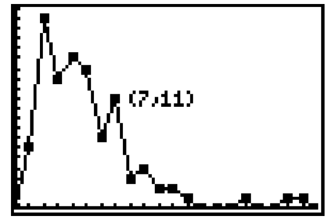


Figure 4

Figures 3 and 4 show the results of running a program, NCHIDIE, which was written to perform this experiment 100 times. The integer part of possible chi squares is listed in L1 with L2 containing their frequencies. Our largest chi square was 20. This table shows that a chi square of 6 or less occurred 74 out of 100 times. Thus, the probability of a chi square of 7 or more is 26%. So with a fair die, a chi square of 7 or more will happen about 1 out of 4 times. The NCTM article concludes that this is not a very unusual outcome, so the die is most likely a fair die.

Couldn't the same be said of the L_3-Die? After all, this die had a chi square of 1.4. Figure 3 suggests that this die, when compared to the TI-Die, has a 94% probability of being a "fair die."

But the L_3-Die is not a fair die. It used the algorithm *int(11rand)→R :R - 6int(R/6) + 1 →D* to emulate the roll of a die. This algorithm generates numbers between and including 0 and 10 and adds 1 to the remainder when that number is divided by 6. So the probability that a 6 is rolled is only 1/11, while the probability of the other faces is 2/11. This is definitely an unfair die. What does this say about the NCTM conclusion mentioned above?

It is hoped that this article has raised more questions than it has answered, questions which can be discussed in a classroom situation. The TI-Die is a fair die, even when it gives a chi square of 11.5. And an unfair die can give misleadingly positive chi squares of 1.4. So is it correct for the NCTM article to conclude that its chi square = 7 die is "most likely a fair die?"◆