Discovery of the power rules
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Definition

Remark 1

Remark 2

In general it holds:

a^2 = a^2 = a•a

b^3 = b^3 = b•b•b

Thus what is:

$a^2 \cdot a^3$?

Exercise 1

Remark 3

Thus, what is:

$b^4 \cdot b^4$?

Exercise 2

Remark 4

Thus to multiply powers
it holds the following rule:

Rule 1

Task 1

Our next question is:
How to proof the equation

$a^2 \cdot b^2 = (a \cdot b)^2$?

Exercise 3

Remark 5

Rule 2

Task 2

The last question is:
How to proof the rule

$\left(a^s\right)^r = a^{(s \cdot r)} = a^{s \cdot r}$?

Remark 6

Rule 3

Task 3

By the help of the rules
1 and 2 you get the rule
for the quotients:

$$\frac{a^r}{a^s} = a^{(r-s)} = a^{r-s}$$

and

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

Reconstructing of a procedure:

Heron of Alexandria

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Remark

Application

Find the following number: $\sqrt{a}$

Factoring the number a:

$a = x_1 \cdot y_1$

Choose an arbitrary but appropriate first value,
the initial value x_1, and
compute the other factor y_1:

$$y_1 = \frac{a}{x_1}$$

Remark 1

Working 1

Remark 2

Remark 3

Working 2

Formation of the average value:

$$x_2 = \frac{x_1+y_1}{2} = \frac{1}{2}(x_1+y_1)$$

Replace y_1 by $\frac{a}{x_1}$

(2^{nd} approximate value):

$$x_2 = \frac{1}{2}\left(x_1+\frac{a}{x_1}\right)$$

It must apply:

$$y_2 \cdot x_2 = a$$

Computation of y_2:

Working 3

Remark 4

In general it holds:

(n+1)th approximate value:

$$x_{n+1}=\frac{1}{2}\left(x_{n+1}+\frac{a}{x_{n+1}}\right)$$

Working 4

Remark 5

If both rectangle sides are equivalent long, it concerns a square:
This represents the solution of our problem.

Theorem

Task 1

Working 5

Exercise with solutions
to the addition procedure
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1. I: $x+5y = -14.5$
$\wedge$ II: $-x-y = 2.5$

Step 1

$(x+5y=-14.5)+(-x-y=2.5)$

$4\cdot y=-12$

Step 2

solve$(4\cdot y=-12,y)$

$\{y=-3\}$

Step 3

solve$(-x-y=2.5|y=-3,x)$

$\left\{x=\frac{1}{2}\right\}$

solution set: **L =** $\left\{\left(\frac{1}{2}|-3\right)\right\}$

2. I: $2x+y = 10.3$
$\wedge$ II: $3x-y = 1.2$

Step 1

$(2x+y=10.3)+(3x-y=1.2)$

$$5\cdot x=\frac{23}{2}$$

Step 2

$\text{solve}(5\cdot x=\frac{23}{2}, x)$

$$\left\{x=\frac{23}{10}\right\}$$

Step 3

$\text{solve}(2x+y=10.3 \mid x=\frac{23}{10}, y)$

$$\left\{y=\frac{57}{10}\right\}$$

solution set: L= {(2.3|5.7)}

3. I: $x-5y = 18.2$

$\wedge$ II: $x-2y = 6.2$

Working with 3.

4. I: $2x-3y = -39$

$\wedge$ II: $-2x+4y = 51$

Working with 4.

Exercise with a test character to the addition procedure

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1. $\quad$ I: $2x-3y = 10$

$\wedge$ II: $-x-y = -4.5$

Step 1

$(-x-y=-4.5)\cdot(-3)$

$$3\cdot(x+y)=\frac{27}{2}$$

Step 2

$\left(3\cdot(x+y)=\frac{27}{2}\right)+(2x-3y=10)$

$$3\cdot(x+y)+2\cdot x-3\cdot y=\frac{47}{2}$$

$\text{simplify}\left(3\cdot(x+y)+2\cdot x-3\cdot y=\frac{47}{2}\right)$

$$5\cdot x=\frac{47}{2}$$

Step 3

$\text{solve}\left(5\cdot x=\frac{47}{2},x\right)$

$$\left\{x=\frac{47}{10}\right\}$$

Step 4

$\text{solve}\left(2x-3y=10 \mid x=\frac{47}{10}, y\right)$

$$\left\{y=-\frac{1}{5}\right\}$$

solution set:

$L = \left\{\left(\frac{47}{10} \middle| -\frac{1}{5}\right)\right\}$

$= \{(4.7|-0.2)\}$

Remark

Working

2. **I: $3y-4x = 13.5$**

∧ II: $-x-y = -4.5$

Working with 2.

3. **I: $y-0.7x = 0.4$**

∧ II: $y+7x = -15$

Working with 3.

4. **I: $3x-4y = 1.2$**

∧ II: $4y+1.2 = -3x$

Working with 4.

5. **I: $-x+7y = -12$**

∧ II: $2x-y = 11$

Working with 5.

Addition procedure

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Remark 1

Remark 2

1. Example:

I: $4x-3y = -5$

$\wedge$ II: $3x+y = 6$

Step 1

(3x+y=6)•3

3•(3•x+y)=18

simplify(3•(3•x+y)=18)

9•x+3•y=18

Step 2

(4x−3y=−5)+(9x+3y=18)

13•x=13

Step 3

solve(13•x=13,x)

{x=1}

Step 4

solve(4x−3y=−5|x=1,y)

{y=3}

⇒ L = {(1|3)}

Remark 3

Working 1

Working 2

Linear System of equations
in tasks with text
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Task 1

Step 1

original length x
original width y

Step 2

Step 3

Remark

$A_1 = x \cdot y$
$A_2 = (x+5) \cdot (y+2)$
$A_2 = A_1 + 40 [cm^2]$
$A_3 = (x+1) \cdot (y-3)$
$A_3 = A_1 - 14 [cm^2]$

Step 4

Working 1

$$\text{I:}\quad x\cdot y+2\cdot x+5\cdot y+10=x\cdot y+40$$
$$\wedge\ \text{II:}\quad x\cdot y-3\cdot x+y-3=x\cdot y-14$$

Step 5

Working 2

$$\text{I:}\quad 2\cdot x+5\cdot y-30=0$$
$$\wedge\ \text{II:}\quad -3\cdot x+y+11=0$$

Step 6

solve(2•x+5•y−30=0, y)

$$\left\{y=\frac{-2\cdot x}{5}+6\right\}$$

solve(−3•x+y+11=0, y)

$$\{y=3\cdot x-11\}$$

Step 7

$$\text{I:}\quad y=\frac{-2\cdot x}{5}+6$$
$$\wedge\ \text{II:}\quad y=3\cdot x-11$$

Step 8

$$\text{solve}\left(\frac{-2\cdot x}{5}+6=3\cdot x-11,x\right)$$

{x=5}

Step 9

solve(2·x+5·y−30=0|x=5,y)

{y=4}

Step 10

Working 3

Task 2

Protocol by

Markus Mustermann

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Remark 1

Task

Step 1

(3x+y=6)•3

3•(3•x+y)=18

Step 2

(3•(3•x+y)=18)+(4x−3y=−5)

3•(3•x+y)+4•x−3•y=13

simplify(3•(3•x+y)+4•x−3•y=13)

13•x=13

Step 3

solve(13•x=13,x)

{x=1}

Step 4

solve(3x+y=6|x=1,y)

{y=3}

⇒ L = {(1|3)}

Remark 2

Working

In order to control the result, one uses best the "judge"-instruction.

judge(3x+y=6|x=1|y=3)

TRUE

judge(4x−3y=−5|x=1|y=3)

TRUE

Nachvollziehen eines Verfahrens:

Keplersche Fassregel

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Einsatz

Ziel: Berechnung des Integrals $\int_0^{\pi} x \cdot \sin(x)+1 dx$.

Vorgehen: Zunächst werden die Flächen von Näherungsfiguren bestimmt. Danach wird ein Mittelwert aus diesen beiden Flächen bestimmt. Dieser Mittelwert stellt die Näherungslösung dar.

Wir definieren:

Define f(x)=x•sin(x)+1

done

Bemerkung 1

Schritt 1

Bemerkung 2

Sehnentrapeze

Bemerkung 3

Die Sehnentrapeze haben also die Flächen

$$A1 = \frac{f(0)+f\left(\frac{\pi}{2}\right)}{2}\cdot\frac{\pi}{2} \text{ und}$$

$$A2 = \frac{f\left(\frac{\pi}{2}\right)+f(\pi)}{2}\cdot\frac{\pi}{2}$$

$$\frac{f(0)+f\left(\frac{\pi}{2}\right)}{2}\cdot\frac{\pi}{2}$$

2.804496877

$$\frac{f\left(\frac{\pi}{2}\right)+f(\pi)}{2}\cdot\frac{\pi}{2}$$

2.804496877

□

Schritt 2

Bemerkung 4

Tangententrapez

Die Fläche des Tangententrapezes berechnet man mit der Formel

$$A = f\left(\frac{\pi}{2}\right)\cdot\pi$$

$$f\left(\frac{\pi}{2}\right)\cdot\pi$$

8.076394854

Bemerkung 5

Schritt 3

Die Näherungsfläche wird also durch die Formel

$$A = \frac{1}{3}\cdot\left(2\cdot\left(\frac{f(0)+f\left(\frac{\pi}{2}\right)}{2}\cdot\frac{\pi}{2}+\frac{f\left(\frac{\pi}{2}\right)+f(\pi)}{2}\cdot\frac{\pi}{2}\right)+f\left(\frac{\pi}{2}\right)\cdot\pi\right)$$

berechnet.

$$\frac{1}{3}\cdot\left(2\cdot\left(\frac{f(0)+f\left(\frac{\pi}{2}\right)}{2}\cdot\frac{\pi}{2}+\frac{f\left(\frac{\pi}{2}\right)+f(\pi)}{2}\cdot\frac{\pi}{2}\right)+f\left(\frac{\pi}{2}\right)\cdot\pi\right)$$

6.431460787

Wertet man das Integral direkt aus, erhält man:

$$\int_0^{\pi} x\cdot\sin(x)+1dx$$

6.283185307

2π

6.283185307

Trotz der nur groben Annäherung erhält man ein relativ gutes Ergebnis.

Bemerkung 6

$$\int_a^b g(x)\,dx \approx$$

$$\frac{1}{3}\cdot\left(2\cdot\left(\frac{g(a)+g\left(\frac{a+b}{2}\right)}{2}\cdot\frac{b-a}{2}+\frac{g\left(\frac{a+b}{2}\right)+g(b)}{2}\cdot\frac{b-a}{2}\right)+g\left(\frac{a+b}{2}\right)\cdot(b-a)\right)$$

$$= \frac{1}{6}\cdot(b-a)\cdot\left(g(a)+4\cdot g\left(\frac{a+b}{2}\right)+g(b)\right)$$