University of Applied Sciences Dresden/Germany
Prof. Dr. Ludwig Paditz

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Exercise on Statistics with ClassPad300

A survey on the running costs during a month was carried out between 100 owners of the car type A and 100 owners of the car type B.
The samples give the following arithmetic means and variances:

$$\bar{x}_A = \frac{1}{100}\sum_{i=1}^{100} x_i = 291\ [€]\ ,\qquad \bar{y}_B = \frac{1}{100}\sum_{i=1}^{100} y_i = 302\ [€]\qquad \text{and}$$

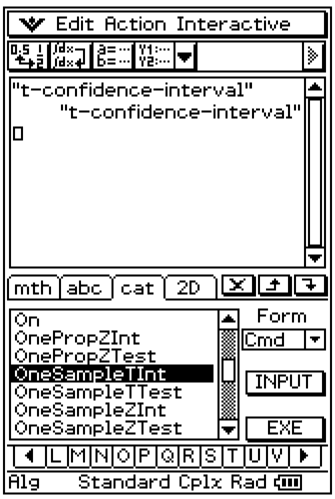
$$s_A^2 = \frac{1}{99}\sum_{i=1}^{100}(x_i - \bar{x}_A)^2 = 30\ [€^2]\ ,\qquad s_B^2 = \frac{1}{99}\sum_{i=1}^{100}(y_i - \bar{y}_A)^2 = 28\ [€^2]\qquad \text{respectively.}$$

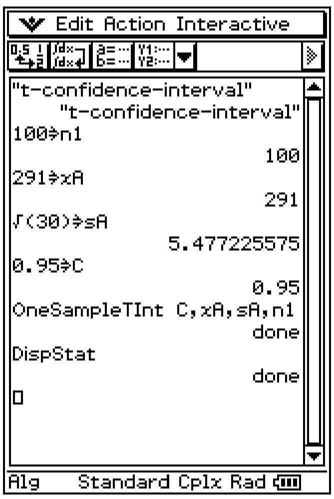
Suppose that the running costs during a month are $N(\mu_A,\sigma_A^2)-$ and $N(\mu_B,\sigma_B^2)-$distributed respectively.

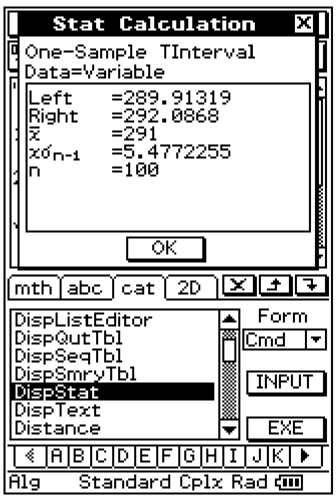
a) Compute a two-tailed confidence-interval for μ_A (C-level let be 95%).
b) Compute a two-tailed confidence-interval for σ_B (C-level let be 95%).
c) Test with a significance of $\alpha = 5\%$ the hypothesis $H_0:\ \sigma_A = \sigma_B$ against $H_A:\ \sigma_A \neq \sigma_B$
d) Assume that $\sigma_A = \sigma_B$. Test with a significance of $\alpha = 5\%$ the hypothesis $H_0:\ \mu_A = \mu_B$ against $H_A:\ \mu_A < \mu_B$.

Now we find the solutions by using the ClassPad300.

a) We get $289.91 \leq \mu_A \leq 292.09$ using the "OneSampleTint"-command (see the syntax of the command!)



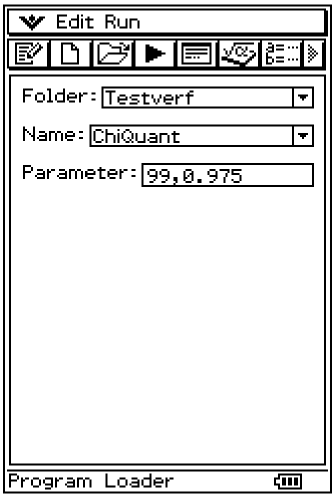


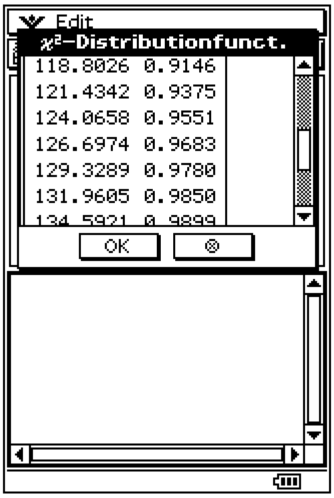


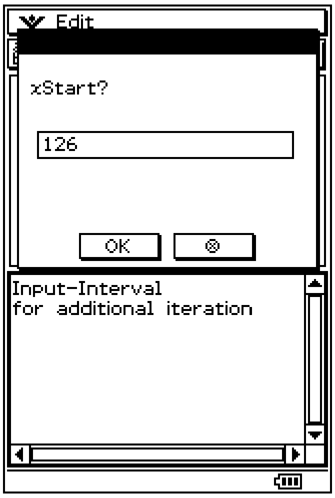
b) We use the well-known formula $\frac{(n-1)\cdot s_B^2}{\chi^2_{n-1,1-\alpha/2}} \le \sigma_B^2 \le \frac{(n-1)\cdot s_B^2}{\chi^2_{n-1,\alpha/2}}$ and need the χ^2_{n-1} – quantiles of the order 1-α/2 and α /2 respectively.

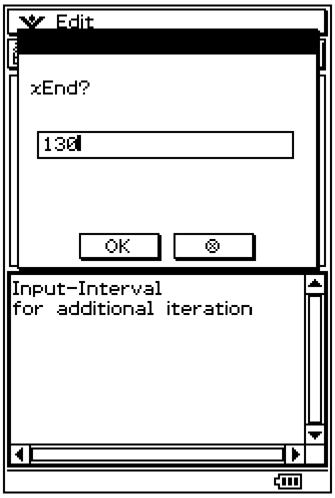
By the help of a small program we generate a table of values of the χ^2_{n-1} – distribution function, to find the needed quantiles $\chi^2_{99,0.975} = 128.42$ and $\chi^2_{99,0.025} = 73.36$.

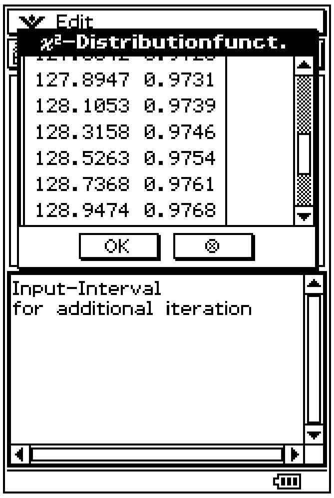
We draw the generated tables in form of a statistic graphic (xy-line) and in form of a CubicReg-function y = y1(x). Finally we solve the equation y1(x) = y2(x) with y2(x) =γ= 1 - α/2 and y2(x) =γ= α /2 respectively to get the wished quantiles.

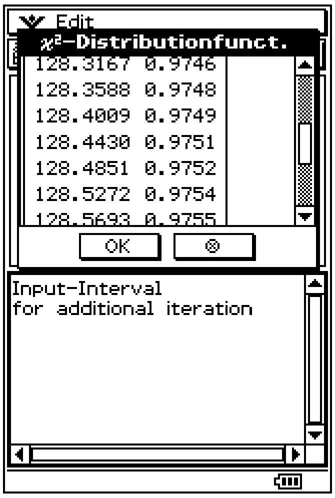


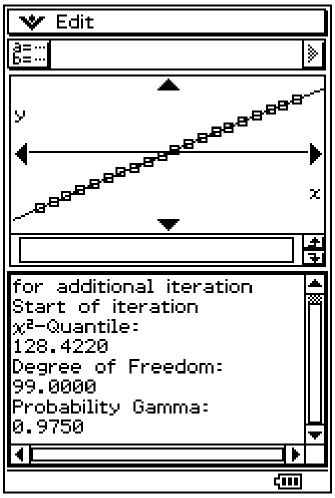


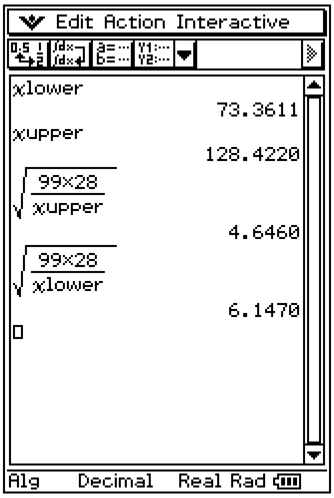






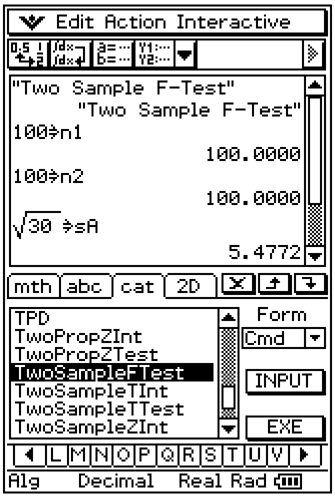


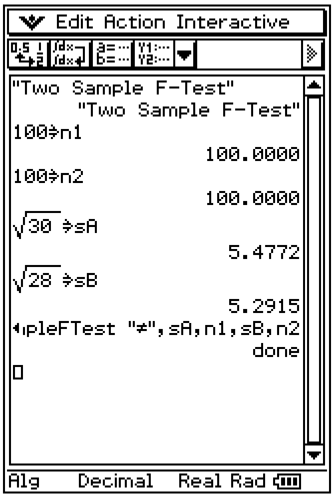


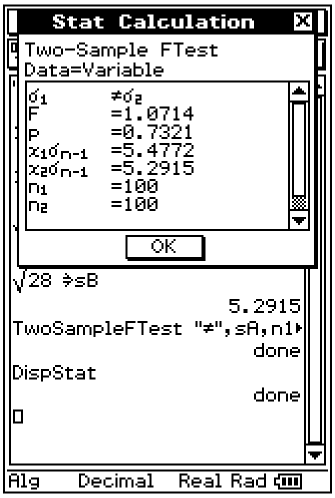


Thus we get the wished interval $\sqrt{\frac{(n-1)\cdot s_B^2}{\chi^2_{n-1,1-\alpha/2}}} = 4.646 \le \sigma_B \le \sqrt{\frac{(n-1)\cdot s_B^2}{\chi^2_{n-1,\alpha/2}}} = 6.147$.

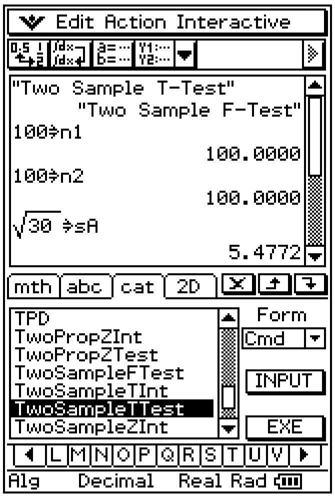
c) We get the p-value $p = 0.732 > \alpha = 0.05$ using the “TwoSampleFTest”-command (see the syntax of the command!), i.e. we have nothing against hypothesis H_0 .

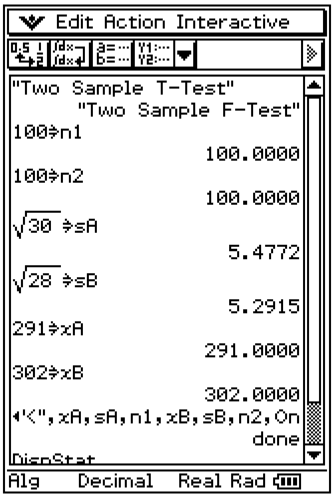


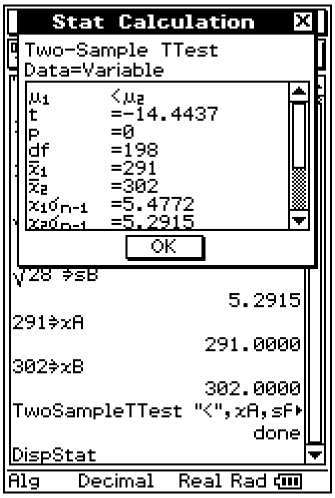




d) Because of the result in c) we assume $\sigma_A = \sigma_B$ (pooled variances) and use the “TwoSampleTTest”-command (see the syntax of the command!) and get the p-value $p = 0 < \alpha = 0.05$, i.e. we are against hypothesis H_0 .







Appendix (programs)

```
ClrText
Local xStart,xEnd,NumVal
ue,xxStart,tt,k,h,a,b,aa
,Chidf,Q,gamma
max(0,Chidf-50×(1-gamma
))⇒xStart
Chidf+50×gamma⇒xEnd
20⇒NumValue

0⇒aa
Goto b

Lbl a
aa+1⇒aa
If aa=2
Then
max(0,Q-(xEnd-xStart)/1
0)⇒xxStart
Q+(xEnd-xStart)/10⇒xEnd
```

```
Q+(xEnd-xStart)/10⇒xEnd
xxStart⇒xStart
IfEnd

Lbl b
seq(0,z,1,NumValue)⇒p
p⇒z
(xEnd-xStart)/(NumValue
-1)⇒h
For 1⇒k To NumValue Ste
p 1:ChiCD 0,xStart+(k-1)
×h,Chidf:prob⇒p[k]:xStart
+(k-1)×h⇒z[k]:Next

listToMat(p)⇒pp
listToMat(z)⇒xx
PrintNatural augment(xx,
pp),"χ²-Distributionfunct."
```

```
pp),"χ²-Distributionfunct."

If aa=0
Then
Print "Input-Interval"
Print "for additional iterat
ion"
Input xStart
Input xEnd
Goto a
IfEnd

DelVar x,y1,y2,tt,Q
CubicReg z,p,1,y1
Define y2(x)=gamma
solve(y1(x)-gamma=0,x,(
xStart+xEnd)/2,xStart,xE
nd)⇒tt
getRight(tt[1])⇒Q
```

```
If Q<0
Then
0⇒Q
IfEnd
PrintNatural Q,"χ²-Quantil
e-Estimation"
If aa=1
Then
Goto a
IfEnd

StatGraphSel Off
StatGraph 1,On,xyLine,z,
p,1,Square
SetStatWinAuto On

GTSelOn 1
GTSelOn 2
DrawStat
```

```
DrawStat
Pause

Print "Start of iteration"
If aa<2
Then
Goto a
IfEnd

Print "χ²-Quantile:"
Print Q
Print "Degree of Freedom:
"
Print Chidf
Print "Probability Gamma:"
Print gamma
If gamma≤0.5
Then
Q⇒χlower
```

```
Goto a
IfEnd

Print "χ²-Quantile:"
Print Q
Print "Degree of Freedom:
"
Print Chidf
Print "Probability Gamma:"
Print gamma
If gamma≤0.5
Then
Q⇒χlower
Else
Q⇒χupper
IfEnd

Stop
```