The Modelling Process:

There are many ways to describe the mathematical modelling process, but a simplified approach is shown in figure 1. Each of the 7 stages will be explained in greater detail as I present a real life problem that students are required to model.

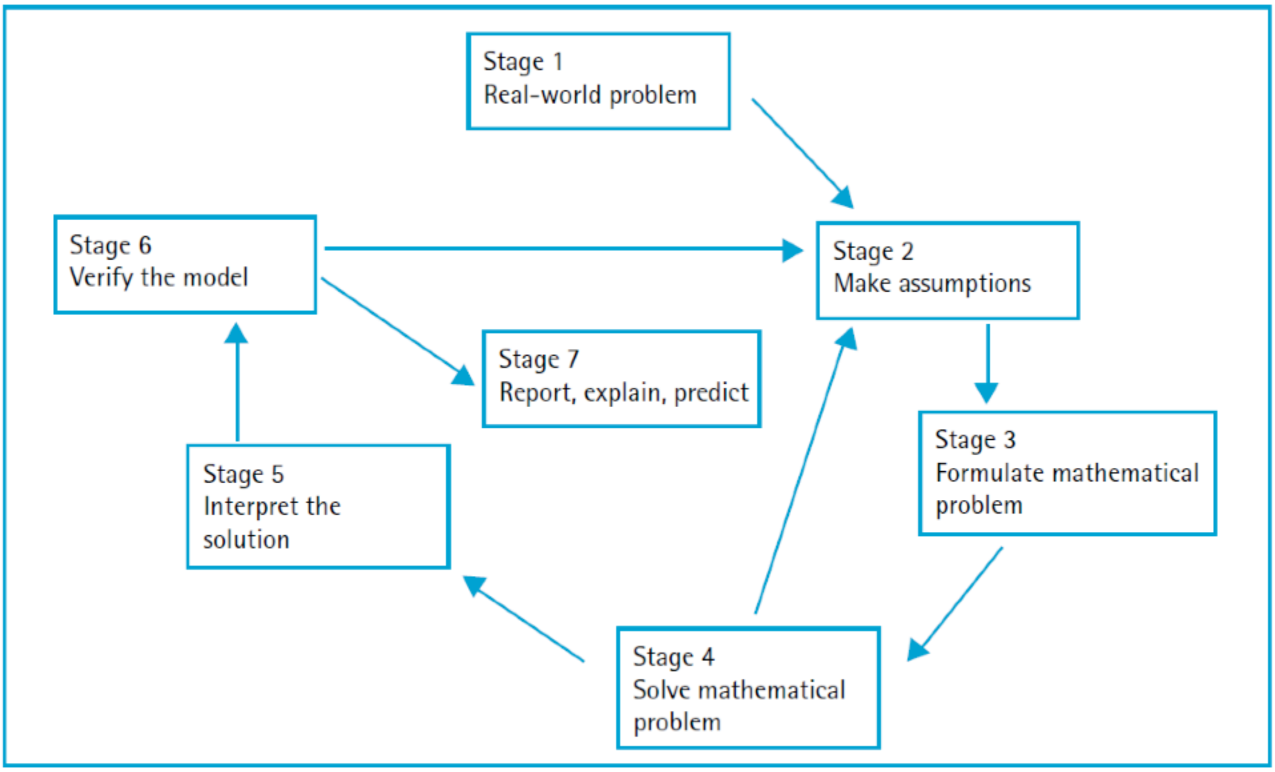

Figure 1. The modelling process

The Modelling Exercise:

Following an introduction to matrices, which also explores some applications of matrices, students are given 8 weeks to develop a dominance theory model for the national Rugby League competition. The task is given to the students a week prior to the commencement of the competition and provides an opportunity for individual creativity in developing a final model. It is made very clear to the students that the end result of picking 8 winners from 8 games is NOT the main objective, but rather students are to concentrate on documenting the stages of the development of their model, and to justify any changes that take place to produce the final working model.

Stage 1: A Real World Problem

The problem is presented simply, free from data. It is a topic (Rugby) that appeals to the cohort (boys). There is no one correct approach, however the first step for all is to collect the data. Dominance Matrix theory has been taught prior to the problem being presented and the students will have practiced simple examples.

Dominance/Supremacy Matrices:

The first 7 rounds of the 2011 NRL competition are listed on the following pages.

- You are to record the results of these matches, along with conditions under which each game was played.

Based on these results,

- Model these results using matrices and dominance procedures to rank the teams.
- Outline any enhancements you have made as you have refined your model as the season has progressed. List strengths and limitations there may be with your technique.
- State any assumptions you are making and any affect these have on the outcomes

(To better fine-tune your system you may like to use the results of the first 3 rounds to predict the winner of the 4th round, and then make any adjustments if necessary to predict the winner of the 5th round and so on. Document each model you develop together with your reasons for any changes you may make in developing subsequent models.)

- Make sure you address the criteria on the attached pages

- Finally,

Use the rankings you have established using your model at the end of round 7 to predict the winners of round 8 below.

Round 8:

Brisbane Broncos	vs.	Canterbury-Bankstown Bulldogs
North Queensland Cowboys	vs.	Manly-Warringah Sea Eagles
St George-Illawarra Dragons	vs.	Parramatta Eels
South Sydney Rabbitohs	vs.	Cronulla-Sutherland Sharks
Canberra Raiders	vs.	Wests Tigers
Melbourne Storm	vs.	Newcastle Knights
Gold Coast Titans	vs.	Sydney Roosters
Auckland Warriors	vs.	Penrith Panthers

Stage 2: Make Assumptions

Done properly, this is a time consuming part of the exercise. A list of all the variables should be made and these modified or simplified before proceeding further. In this example, variables such as whether the game is played in the day or the night, home or away, fine or wet weather, teams are full strength or with injury substitutes etc, could be taken into account. As with any modelling exercise, whenever the real world model is over simplified it is a very strong possibility that the model will lose accuracy when it is used for predicting future outcomes.

Stage 3: Formulate mathematical Problem

A standard Dominance Matrix has a 1 for a win, and a zero for a loss. Students are presented with the opportunity to vary these values. Would it be more appropriate to incorporate home and away wins? For example, 2 for an away win and 1 for a home win…..or 1.5 for an away win? Should a greater value be given for a win margin of more than 20 points? Students are given the freedom to use whatever values they can justify.

Stage 4: Solve the mathematical problem

Once it has been decided what values to use in the matrix, it is necessary to determine what weight needs to be applied to second order, possibly third order dominance to produce the working model. After 3 rounds the teams are then ranked and used to predict the 4th round of matches. From experience, very few students will pick 8 winners from 8 games, so its time to relook at Stages 2 and 3 and adjust the model. Students are encouraged to re-examine their assumptions and develop a new model to predict rounds 5, 6 and 7.

Stage 5: Interpret the solution

The model is further refined until there is some consistency in results. The students are encouraged to revisit the initial problem, and ensure that their model is working within the constraints set. One important aspect of this stage is that they soon realise that their solution is quite clearly governed by the constraints, and is not easily transferred to other situations.

Stage 6: Verify the model

This stage requires each student to look for strengths and limitations of their mathematical model. Reflecting on their model, and the success they have experienced in predicting winners for rounds 4,5,6 and 7, it is clear that their models have limitations caused mainly by a very simplistic approach to the many variables that effect the results of a football match. Just as it is important to identify the variables used it is also of value, and indeed probably more important, to identify the variables that are ignored. This is highlighted in the following Criteria for Modelling and Problem Solving

Criteria for Modelling and Problem Solving

Rating	Description
A	• shows initiative and insight in developing the matrix model • details the assumptions made in the selection and use of their matrix model • details all refinements that took place during the development of the model • explores the strengths and limitations of the matrix model • uses effective strategies to synthesise a matrix model
B	• shows some initiative in developing the model • details the assumptions made in the selection and use of their individualised model • details some refinements that took place during the development of the model • uses effective strategies to obtain a matrix model
C	• develops a basic model and uses some data • uses some effective strategies

Stage 7: Report, explain, predict

In the week leading up to Round 8, students submit their assignments, along with the teams their model predicts will win that round. Students are required to outline the development of their model, and explain all aspects of the process. At the conclusion of round 8, students are handed back their assignments (unmarked), with the results of the round. Under exam conditions students are given 45 minutes to appraise their model based on how well they were able to predict the winner of round 8. They are also encouraged to discuss what future enhancements could be made to their model for the remaining games.

Conclusion:

I have used this learning experience/assessment item on a number of occasions over the last 4 or 5 years. Without a doubt it has proved challenging, rewarding and enjoyable for my students. Other observations are as follows:

- Mathematical modelling is best taught by students "doing"
- It is open ended, which gives students the freedom to explore
- Emphasis is placed on the modelling process and its development rather than predicting 8 winners from 8 games
- It challenges students to critically evaluate, reflect and formulate – a difficult set of important skills to teach in mathematics.
- Above all else the students find it FUN to do.
- I have yet to see a student's model that consistently predicts winners, so I suppose winning those millions of dollars is still just around the corner.

Teaching and learning high school mathematics through an interdisciplinary approach

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Abstract

The aim of this paper is to share the results of an implemented action-research project in teaching high schools mathematics through an interdisciplinary approach.

The starting point of the action research project was the high school students' lack of motivation in studying mathematics which had an impact on the students' learning outcomes. Few students learn math simply because they like it. Most students learn math if they understand why they have to learn it, if they have the possibility to apply mathematical concepts and algorithms in real life or within other subjects or various contexts.

The Romanian mathematics textbooks do not support teachers in motivating students' learning. In these textbooks they can find exercises like: "solve the equation …." or "determine the derivative of the function …".

The action research project answers the following questions:

- what are the effects of the interdisciplinary approach on the students' learning in mathematics?
- what are the effects of the interdisciplinary approach on reaching both the specific mathematical aims and mathematical literacy & competence in science and technology - as a key competence domain?

The action research project is documented with results of assessment and evaluation of the mathematical learning. Students' reflections on their interdisciplinary experience, on their learning mathematics through different subjects (physics, chemistry, music, etc.) are included in the paper.

The paper concludes that the interdisciplinary approach enhances both, the students' mathematical learning and their mathematics, science & technology competence.

Introduction

Students' motivation in learning mathematics is decreasing year by year. If students enjoy learning mathematics in the primary school – as during this stage it is more closely related to operations with natural numbers, their enjoyment in learning math is getting lower and lower as the level of abstraction is increasing. Most high-school students learn math only because they have to take the final examination at the end of the 12th grade. As a mathematics teacher, I have often heard the same question asked by different high-school students: "Why do we have to learn …. ?" Lack of intrinsic motivation in learning mathematics has an impact on the students' learning outcomes. Even if the Romanian mathematics curriculum is quite generous, as it states that *the main aims of studying mathematics in high-school are to develop students' skills to reflect upon the world and provide them with knowledge to act upon the world; to formulate and solve problems by using knowledge across different domains*, mathematics textbooks do not support teachers and learners in reaching these aims. Exercises and problems in the math textbooks have a high level of abstraction and they allow for the use of mathematical concepts and algorithms only in mathematical contexts.

The list of key competences for lifelong learning (European Commission, 2007) which *are particularly necessary for personal fulfillment and development, for social inclusion, active citizenship and employment*, includes mathematical competence and basic competences in sciences and technology. Mathematical competence is defined as *the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations, with the emphasis being placed on process, activity and knowledge,* while basic competences in sciences and technology refer to the use and application of knowledge and methodologies that explain the natural world. This key competence includes understanding of the changes caused by humans and each individual's responsibility as a citizen. Moreover, this key competence is one of the eight key competences which are all interdependent.

Mathematical thinking is not easy to define. Based on the Principles and Standards for School Mathematics published by the National Council of Teachers of Mathematics (NCTM), mathematical thinking is related to reasoning, problem solving, communicating and connections; it is a thinking process for building mathematical understanding. Needless to say, developing students' ability to apply mathematical thinking to solve everyday life problems is a long process which is based on modeling the real world and developing problem solving skills. During the learning process, students have to learn how to apply mathematical thinking in different contexts.

In autumn 2010, I started teaching mathematics to a class of 31 ninth graders, who focus on studying sciences and English language (bilingual track). At the beginning of the school year, asked why they decided to take the sciences path, the students answered that:

- they wanted to study medicine (45%);
- they wanted to become chemistry researchers (10%);
- they wanted to study some mathematics during high-school, but not as much as those students whose curriculum focuses on mathematics and computer sciences and not as little as those students who focus on social sciences (45%).

I asked the students *what* they would like to learn in their high-school mathematics class; they answered that they would like to learn:

- things that would be useful in their personal and professional life (80%);
- concepts and algorithms that they needed to know to get a good score in the final exam (71%);
- how to reason (26%).

Being asked how they would like to learn mathematics, the students answered that they would like to learn mathematics:

- in as enjoyable a manner as possible;
- without being stressed;
- by interaction;
- with examples of practical/ everyday life problems which stimulate thinking/ reasoning;
- in English.

I carried out the initial/ diagnostic assessment in order to identify the level of the students' mathematical skills and their ability to use mathematical concepts and

algorithms they would have to use during the high-school math. The average score of the initial assessment test was 56.60 out of 100.
Considering all the above mentioned issues, I designed my action-research project in teaching high-school mathematics to this group of students through an interdisciplinary approach. I was, inspired by the results of the ScienceMath Project and the conceptual framework for cross-curricular teaching (Beckmann, 2009) about which I had learnt in the European Science Math teacher training, as well as by the philosophy of inquiry-based learning (Dewey, 1916).

Teaching high-school mathematics through an interdisciplinary approach

As I had to follow the national mathematics curriculum which limits my academic freedom, I integrated the interdisciplinary approach in teaching high-school mathematics in both my lessons and on the e-learning platform I developed for the ninth graders. The e-learning platform is a "moodle" which contains specific tasks for students. The forum on the platform as well as individual e-messaging and the feed-backing option when assessing students' work allow individual support of learners. Currently, the e-platform contains each student's e-portfolio.
During the first year of my action research project, the interdisciplinary approach took shape in the following learning activities:

Topic	Involved subject(s)	Students' tasks	Specific methodology/ strategies used	Mathematic learning objectives
Real numbers	Music	- students compose and play a piano/ guitar piece to represent different rational and irrational numbers.	Music specific methodology	Students distinguish and explain differences between rational and irrational numbers
Number systems	Literature	- students read a passage from Ian Stewart's book *Nature's Numbers* using text coding. - students write an essay on Leopold Kronecker's quote "The dear God has made the whole numbers, all the rest is man's work." - students watch Marcus du Sautoy's videos *From Zero to Infinity, Safety in numbers* and the video *The Teaching Challenge – Simon Singh* (the videos were available on the teachers.tv website) and answer questions (conceptual clarification, probing reasons, perspectives, probing	Reading and writing strategies for understanding texts (reading with text coding, writing essays, Socratic questioning)	Students measure cardinality of number sets, explain the development of the number systems, explain the use of different numbers in everyday life situations and technology, appreciate contribution of different cultures to developments in mathematics.

Topic	Involved subject(s)	Students' tasks	Specific methodology/ strategies used	Mathematic learning objectives
		implications and consequences) which addresses different thinking levels.		
Numerical functions	Sciences (chemistry, biology)	- students collect water samples, measure dissolved oxygen, turbidity, temperature, pH; - students specify relationships among water quality indicators collected data; - students define and graph the numerical functions which described the established relationships.	Inquiry and project based learning Chemistry experiments	Students use different ways of describing a numerical function, analyze practical situations and describe them by using numerical functions.
Quadratic functions	physics	- students watch the video of the freely falling ball experiment and use the freely falling ball graph (position depending on time); - by reading the graph, students identify the position of the ball at different moments, the interval of time between two different positions of the ball, the domain and codomain of the function, axes intercepts and their meaning related to the experiment, coordinates of the vertex point, monotonicity of the function, the image of the function; - students calculate the 2nd degree polynomial which defines the graphed function.	Inquiry based learning	Students interpret functions from graphs, analyze quadratic functions to describe the motion of an object, identify applicability of quadratic functions.
Vectors	Physics, English language	Students explain in writing, in Romanian language, the solution of the two vectors problems (use of addition vectors tools to solve relative velocity); both problems and solutions are provided in	Reading and writing proofs for understanding math concepts and processes	Students use addition of vectors to solve problems in non-mathematical context, analyze and evaluate

Topic	Involved subject(s)	Students' tasks	Specific methodology/ strategies used	Mathematic learning objectives
		English language (the problems were available on Homework helper - Physics and Mathematics website www.jfinternational.com).		written proofs, read scientific text in English language, identify applicability of vectors.

Results of the action research project in teaching high-school mathematics through an interdisciplinary approach

I analyzed the results of the so far implemented action research project by answering my lead questions:

- What are the effects of the interdisciplinary approach on the students' learning in mathematics?
- What are the effects of the interdisciplinary approach on reaching both the specific mathematical aims and mathematical literacy & competence in science and technology – as key competence domain?

Interdisciplinary approach better addresses students' learning styles. For example, two students who are excellent guitar players were very proud about their performance when they represented real numbers through music. When they started the learning activity, they did not remember anything about rational and irrational numbers, even if they had learnt about them in the seventh grade. After six months, they still remembered the learning activity and they easily explained differences between rational and irrational numbers and gave examples of rational and irrational numbers. Learning was a process of "personal discovery of meaning" (Fatzer 1998, p.66) as these students understood real numbers very well; playing the guitar to demonstrate their learning was an important emotionally charged moment for them.

Twelve students (39%) who have predominant verbal/ linguistic intelligence were enthusiastically engaged in learning about the number systems by reading literature and writing essays, and they also obtained excellent results.

The interdisciplinary approach has improved students' motivation to learn mathematics. Their reflections on their interdisciplinary experience are relevant:

"My best work in the grading period is the essay on Leopold Kronecker's quote *The dear God has made the whole numbers, all the rest is man's work*. This is my best work because I took pleasure at writing it. Mathematics does not provide too many opportunities for free-writes. This assignment gave me the freedom to write my own thinking without being afraid that I'm wrong. I enjoyed working with mathematical concepts." (from the semestrial self-assessment report of Adela F., student)

"In the beginning, I didn't enjoy learning the Vectors unit. I thought this unit was useless; it was so different from what we'd learnt in geometry until then. Later on, I found that vectors are useful for solving some geometrical problems – and the proofs were short, and elegant, and that vectors are tools for physics in working with motion and forces. By studying the Vectors unit, I started using vectors to solve geometry problems (which I find difficult) and it has been really easy to solve mechanics problems in physics now I understand the operations with vectors. These motivated me for learning." (from the Learning Journal of Ana A., student)

"I don't like mathematics very much – I'm good at physics, but I have to say that after understanding the vectors applicability in physics I started to think that learning math supports me in better understanding physics. Understanding the concept and the use of operation with vectors helped me to better understand velocities and forces." (from the Learning Journal of Alex K., student)

"From my point of view the most important thing was that we learnt to work in a team and how to cooperate with our classmates. We had a lot of fun while collecting water samples and we wanted to act together to improve water quality. Defining the functions was a bit difficult but we managed to do it together." (from the Feedback sheet – Water Monitoring Activity of Alina S., student)

During the implementation of the action research project 10 students (32%) improved their reasoning and logical thinking, 16 students (51%) used tools, physical models or technology appropriately, and all students were able to make at least one connection to topics outside mathematics.

The average score of the students' assignments is 75.71 out of 100, which shows a relevant improvement in students' learning outcomes (+19.11 as compared with the initial assessment).

Closing remarks

The theme of the South African Conference is *Turning Dreams into Reality: Transformations and Paradigm Shifts in Mathematics Education* has encouraged me "to share innovative and creative ideas for effecting reform and transformation in the area(s) of […] classroom practices". My ideas for transformation may be implemented with no or few adaptations to the learning environment and the specific group of students in mathematics. I do hope that my paper convinced you about the efficiency of the interdisciplinary approach in achieving improved mathematics learning. For me, this approach really is '*Turning Dreams into Reality*' as my dream is to teach mathematics to students that are eager to learn it because they understand it.

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A New Elementary Mathematics Curriculum: Practice, Learning and Assessment Some Classroom Episodes

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Abstract

The aim of this paper is to present the new and innovative Mathematics Curriculum for elementary levels that is being implemented in the Portuguese basic education system (students from 1st to 9th grade) through an overview of an ongoing study of implementation/experimentation of this curriculum. A specific mechanism was implemented in the field to provide scientific and pedagogical support to the development of the new elementary mathematics curriculum (NPMEB) implementation at all grade levels and all over the country. In particular, the NPMEB is being experimented by a set of teachers that teach in their own classes and that have been trained and accompanied along the experience by the different authors of the program. We will focus on some classroom practices, sharing innovative and creative ideas of teachers and students, grounded on some of the tasks used by the teachers. The preliminary results suggest that some improvements are already visible, namely regarding students' attitudes and mathematical competences and teachers' practice.

Introduction

In the current Portuguese education system there is a new and innovative elementary mathematics curriculum (students from 1st to 9th grade), the NPMEB (ME, 2007) that includes a series of changes of the government's responsibility to improve the conditions of the teaching and learning of that discipline. To this have contributed the discontent with the results obtained by students in national external (e.g. standardized tests, examinations) and international assessments (e.g. Program for International Student Assessment - PISA). This curriculum was designed to gather some disperse curricular documents and substitute the current syllabus/curriculum/program, published in the early 90s, but mainly to provide the sustained development of students' mathematical learning focused on the more recent recommendations of mathematics teaching and learning.

A specific mechanism was implemented in the field to provide scientific and pedagogical support to the development of the NPMEB implementation through all the grade levels and all over the country. This approach was not generalized but applied to a sample of classrooms/teachers. In particular, the NPMEB is being experimented since 2008 by a set of teachers that teach in their own classes and whom have been trained and accompanied along the experience by the different authors of the program. At the same time the Ministry of Education named a team of mathematics educators for an evaluation study of the process of implementation/experimentation of the NPMEM for three years. The study was designed in three phases with the following purpose: to describe, analyze and interpret teaching practices and assessment developed by teachers of the experimentation and/or teachers to teach in the process of generalization; to describe, analyze and interpret the involvement and participation of students in developing their learning in the context of the classroom; and to evaluate such practices and other curricular materials applied. It was expected to have three

multiple case studies (one for each cycle of basic education), an evaluative global summary and some recommendations.
We propose to present an overview of some of the tasks used by the teachers and the work of the students in accordance with NPMEB. as well as the strategies used by the teachers. Data has suggested that some improvements are already visible, namely regarding students' attitudes and competences and teachers' practice. Before presenting the method of evaluation of the study and some classroom episodes, we began to identify the main ideas of the NPMEB.

Main ideas of the NPMEB

The NPMEB is no more than a readjustment of the existing program, with nearly twenty years, for grades 1-4 (1st cycle of basic education), 5-6 (2nd cycle of basic education), and 7-9 (3rd cycle of basic education), (ME, 1990, 1991), which points to significant changes on mathematics teaching and learning, and to professional practices of teachers. In our opinion, the most innovative aspect of the NPMEB it was to replace the three existing programs by a single one that involves all grades from 1 to 9 with the same structure and the same mathematical themes. On the other hand transversal skills are addressed in the same way that mathematical topics, i.e. to which were suggested methodological guidelines, resources and examples of tasks.
The aim of the new curriculum (ME, 2007) is to promote, in the students, the acquisition of information, knowledge and experience in mathematics. On the other hand intends to develop the capacity of integration and mobilization in different contexts and also indicate the development of positive attitudes towards mathematics and the ability to appreciate this science of all students. Thus three indissociable basic aspects are pointed out for mathematics education - the acquisition of knowledge, ability to use it appropriately and develop general relationship with the discipline. At a subsequent stage of the program is organized in each cycle, around four major mathematical themes (Numbers and Operations, Geometry, Algebra and Organization and Data Analysis) and three transversal fundamental capacities (Problem Solving, Reasoning and Communication).
The NPMEB also presents several general methodological guidelines, with emphasis on the need for diversification of tasks and giving particular attention to its nature, mainly to the challenge they promote, the role of situations in context, the importance of mathematical representations and the connections in mathematics and with extra-mathematical aspects, the educational value of group work and moments of collective discussion in the classroom, the importance of appropriate use of technology and other materials. It is an opportunity to: value certain features of mathematics that were forgotten or worked in a decontextualized way (e.g. mental computation, number sense, demonstration, visualization, geometric transformations, patterns, algebra, statistics); introduce some topics earlier (e.g. rational numbers, algebra); value mathematical processes (problem solving, reasoning and communication); and value the mathematical tasks and the roles of the teachers and students.
The program outlines a set of general principles for the evaluation and especially emphasizes the importance of curriculum management held at the school level. Such program involves a process of curriculum change and demand for an exploratory teaching-learning with a new kind of classroom culture, where students must be much more active and be part of the construction of new knowledge, where teacher must offer appropriate tasks within a challenging element. The tasks have a crucial importance in this change. It is the teacher who can start by giving a task that uses students' knowledge, while allowing the development of new concepts or processes so that they effectively engage in work and interpret correctly the task proposed. This work can be done in different ways, discussing and arguing ideas, orchestrated by the teacher, in order to avoid repetition, and highlight what is mathematically essential. In this perspective the traditional classroom is replaced by

discovery and the development of higher-order capabilities such as testing, conjecturing, reasoning and proof, can be shared by students and teachers. To face these new challenges it is asked for a change of attitude towards mathematics and its teaching, for proper teacher training programs, appropriate educational materials, as well new organizations in schools.

NPMEB: The Evaluation Study

The Ministry of Education through the Department of Curriculum Development and Innovation (DGIDC) requested under the Process of Experimentation of the NMPEB a evaluation study, that it's ultimate purpose is to produce a set of evaluative synthesis and recommendations that could contribute to regulate and/or enhance the development of NPMEB. The DGIDC devised a plan to implement the new program in basic schools. This plan provided five actions such as: a) experimentation, during 2008/2009, the NPMEB in 40 pilot classes of three cycles of basic education; b) the beginning of widespread NPMEB in the academic year 2009/2010; c) the production and distribution of curriculum materials of different nature (e.g. thematic booklets, assignments for use in classrooms; lesson plans); d) a support structure for the beginning of widespread NPMEB in 2009/2010 (e.g. new program coordinators in each group; set of accompanying teachers); and e) teachers' training. In the training, all teachers experimenters participated in training throughout the school year (50 hours classroom and 50 hours of autonomous work) which, in essence, was of the responsibility of NPMEB' authors. Yet developed a monitoring process (e.g. visits to classrooms and meetings with teachers experimenters) through a coordinator group for each cycle that met on average once a month and all groups three times per year. All teachers had reductions in their school hours and shared their classes with a pedagogical pair. The process of experimentation began in forty pilot classes equally distributed by the 3, 5 and 7th grades.

The First Phase included the design of the conceptual framework of the process of the NPMEB' experimentation. So it was taken into account: 1) the NPMEB framework; 2) the design of the implementation plan; 3) the structure of support for plan implementation; 4) the plan to support the process of experimentation; 5) the management system of the process; and 6) the teachers experimenters. The data was obtained through: *interviews* – to all the 40 teachers experimenters (EP) and 42 others intervenient (from coordination group, consultative council, authors program, and professional and scientific associations, mathematicians and mathematics educators, teachers of the three cycles of basic education, administration) translated into about 110 hours of material recorded and transcribed in full; *document analysis*; *field notes* and a *questionnaire* – to the teachers experimenters.
We can summarize the main results of this first phase of the evaluative study, in the following ideas: 1) An innovative process of implementation 2) A hard beginning of the process of implementation 3) a demanding program (the NPMEB) 4) A well achieved teachers education training. However there were some problems with the educational materials available, the follow-up of the teachers experimenters had some virtues and difficulties, it is expected a difficult generalization so some care will be need.

The Second Phase. In this phase we were mainly concerned with classroom. Taking into account the objectives of the study we intended to describe, analyze and interpret the learning environments, students participation, teaching and assessment of the classrooms in the process of experimentation and generalization of NPMEB. The dynamics of the classroom and its complexity is always difficult to be categorized, because most times there are overlaps and interactions that can not be translated directly into an "instrument". However we designed a assessment matrix where considered three main objects of evaluation and fourteen dimensions, indicated in parentheses, to support the team in the data collection and systematization and also to organize and structure the first report: 1) *Teaching practices* (e.g. teaching planning, organization, resources, materials, tasks, classroom dynamics; role of teachers and students, time management); 2) *Practice assessment* (e.g. integrating, articulation with teaching, predominant assessment tasks; nature, frequency and distribution of feedback; dynamics of assessment; predominant role of teachers and students); and 3) *Participation of students* (dynamics, frequency, nature).

This evaluation study is descriptive, analytical and interpretive in nature and therefore it was decided that the treatment data should follow closely the recommendations of Wolcott (1994). We used for data collecting lessons' observations, semi-structured interviews with teachers and students, and field notes for information from informal conversations with teachers and students. Were also consulted and analyzed various kinds of documents (eg, legislation, guidelines produced by ME; specific bibliography). This study took place over about ten months where participated six teachers, two for each cycle of basic education, that were interviewed and observed their classes and interviewed thirty-eight students. We decide to focus on description, analysis and reflection of what it was listened from teachers and students and of what was observed in the classrooms of each cycle, producing just a narrative by cycle.

Two Classroom Episodes

Despite being quite difficult summarize in few words the work done, we selected two tasks during the observations in the 4th and 6th grades respectively.

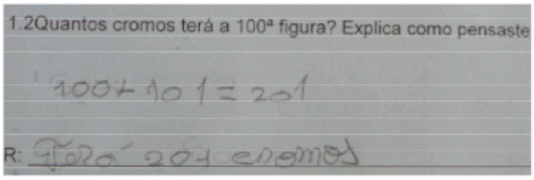
Example 1. This example (part of a math class) intend to illustrate the role played by students and teachers in solving one task, designed under the NPMEB, where you want to develop students own learning. This task is a growing figurative pattern that allowed the exploration of various mathematical topics, with particular emphasis on algebraic thinking that is a new theme for this cycle and used manipulative materials.

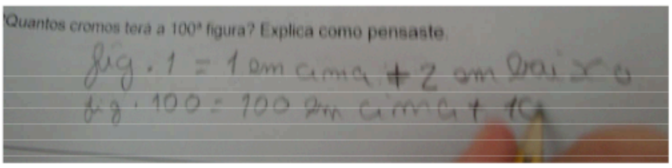
In the third part of the lesson (20 minutes) the students organized themselves into groups of four to solve a pattern task in afigurative sequence context. As a support material the teacher gave each group square to realize the first terms of the sequence.

Emerged as usual in this class, several resolutions and different interpretations and strategies. We conducted a synthesis of all resolutions and each student had the opportunity to share their reasoning with the class.

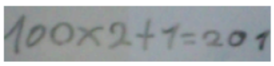
This group was limited to making a statement very concise.

But there were groups that explained their thinking in more detail and clearer.

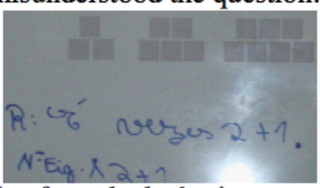




Or through another way to represent the general law of the pattern structure



In the last question, which asked "to determine the number of squares needed to build a picture of any order, "there were many difficulties. A number of students appeared to have misunderstood the question.



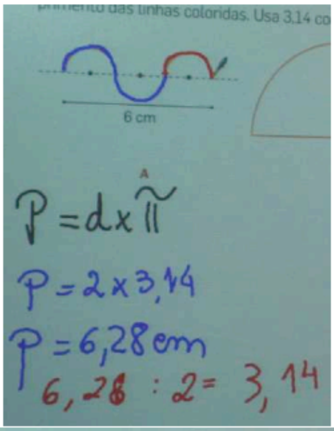
One student, considered the best in the class, explained to colleagues that it was "another two times. " The teacher asked him to complete then he write: "Number figure x 2 + 1".

This, for these young students, it is not easy, it was made *far generalization*, using algebraic thinking and a representation near the formal algebraic expression. Here could have been using the term "double" but in the meantime, the class was nearing the end. (Notes from observation)

If it is true that one of the roles of students is undoubtedly be involved in discussions in small or large group, this is due in very large to the actions of the teachers in order to encourage mathematical communication in classrooms. In fact, the teachers try to provide time and space so that students could present their own resolutions, ask questions and discuss with colleagues. Thus, it is never left to explore issues even in the preparation of the final synthesis.

Example 2. The communication that it was more often observed during classes , it was in the form of dialogue between teachers and students that were always invited to explain and verbalize their thoughts and reasoning. It was noted also that teachers were concerned with questioning on the work and on the arguments presented, providing clues and alternatives in order to supplement and enrich their work. Teacher seemed to have driven discussion of the strategies used in problem solving and presentation of the main findings. The following episode shows that students had opportunities to express themselves, share their thoughts, and present their questions or to submit alternative solutions which could be explored by the whole class.

One student, that usually has problems of being concentrated, read the task. Then the teacher asked to a student:



- How do you think we can find half of the perimeter of a circle?

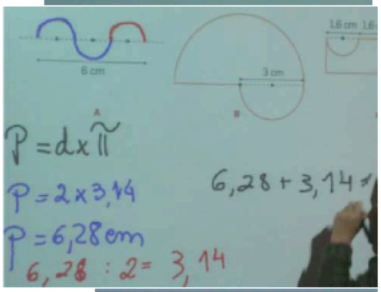
The student answered at once

- Radius times pi.

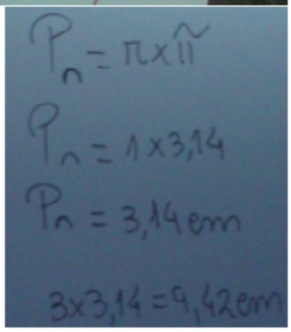
The teacher asked to explain his answer

- What is the diameter of each?
- 2.
- Why?
- Because 2 x 3 = 6.
- Or because 6 : 3 = 2.
- So to know the length of the blue line what I have to do is :

P = d x p ; P= 2 x 3,14 ; P = 6,28



- Anne, do you agree with what William said?
- No, I think it's worth here.  (and makes a draw)
- Why?
- Humm ...
- I wanted to understand ...
- I also wanted to explain ...
- Look, if I took this line and put it here  ...
- Ah! Yeah.
- And now what is missing?
- I think we have to divide 6,28 by 2.
- Let's do the calculation? We can do it but this relationship here (pointing to the calculation 2 x 3,14) ...

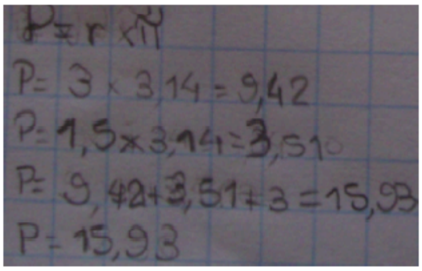


Students painted each line of the drawing with the color of the calculation.

- And now?
- Now we add the two parts.
- Come on. Use another color.

At the end, another student intervened:

- If we make the radius times pi gave half and then we multiply by 3.
- Excellent! Let's register.



Then, the students respond to the other questions by themselves, in pairs. A pair of students presented its solution as shown in the next picture.

This description illustrates the dynamics of work and communication, used often by participating teachers, to help students learn to work in solving problems raised by the tasks presented. However, as mentioned by one of the teachers is not always possible to explore every task or topic with the desired attention and depth, because of the time to carry out all the themes of the program.

Conclusions

The main main conclusions and reflections in this second phase of the evaluative study, can be summarized in the following ideas. 1) *A successful stake* - despite some difficulties it was possible to establish a support system and monitoring which contributed decisively to the creation of new and innovative dynamics in areas such as teacher training and the participation of students; 2) *A well interpretation of the program* - contrary to what appears to be usual, the participating teachers seem to have a well understood on basic content areas of the program, the fact of textbooks are no available may have contributed to the teachers need to study in detail the program; 3) *The planning and collaborative work of teachers* - the planning of lessons, their analysis and discussion led to dynamic collaborative work that contributed to teachers felt more confident of their performance; 4) *The presence of transversal capabilities* - the transversal capacities constant in the program were deliberate and systematic part of the daily concerns of the participating teachers, which eventually settle as a routine in the classroom observed; 5) *Structure of classes and pedagogical routines* - The practices of the teachers were generally well articulated with the methodological guidelines set in the program. Typically, the classes, focusing on tasks followed from in accordance with the next four phases: a) Presentation and appropriation of the task, b) Resolution of the task, c) Discussion of solutions and results, and d) Reflection systematization and synthesis; 6) *Teachers well aware of his role* - all teachers, even with styles and very different experiences and attitudes, seemed to have well-established routines. Perhaps one could say that in general, the teachers participating in this study appear have learned to listen more carefully to a greater number of students; 7) *The problem of time management* - teachers have great difficulty in managing time and when they could, they used the time away from other curricular areas; 8) *The problem of students assesment* - the concepts and practices of teacher assesment participants seemed to be articulated with their teaching practices. For most participating teachers there are issues that are not resolved, such as the very concept of assessment, its purposes, functions, modalities and their nature; and 9) *Student oriented, cooperative and aware of your paper* - students participated with relative ease in the dynamics established in the classroom, were well-oriented by teachers to the tasks to which they were proposed and participated, not all equally, in developing their own learning.

In short, the preliminary results suggest that some improvements for school mathematics are already visible, namely regarding students' attitudes and mathematical competences and teachers' practice.

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Mathematical modelling in classroom: The importance of validation of the constructed model

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Abstract
The present paper, where the basic models used to describe the mathematical modelling process in classroom are mainly discussed, is actually the theoretical supplement of a workshop, in which we shall examine the importance of the last two stages of mathematical modelling process concerning the transition from the solution of the model to the real world (validation of the model and implementation of the final mathematical results to the real system) by presenting a number of suitably chosen examples.

Introduction

It is well known that the reformation attempted from the beginning of the 1960's in mathematics education with the introduction of the *modern mathematics* in school curricula, was proved to be a complete failure. The attempt to teach the fundamental generalizations before presenting the objects to be generalized, had as a result the despoliation of curricula from examples and applications connecting mathematics with the real situations of everyday life and the other sciences using it as a tool and giving birth to very many new mathematical problems and theories.

Thus, and after the rather vague movement of the *back to the basics*, considerable emphasis has been placed from the late 1970's on the use of problem as a tool and motive to teach and understand better mathematics, with two coordinates: *Problem-solving*, where attention was given to the use of the proper heuristic strategies for solving pure (mainly) mathematical problems (Polya 1963, Schoenfeld 1980, etc) and *Mathematical modelling and Applications*, a process of solving a particular type of problems generated by corresponding situations of the real world (Pollak 1979, Niss 1987, etc).

Although views appeared later disputing the effectiveness of using problem solving as a "learning device" of mathematics and giving emphasis to other cognitive aspects, like acquisition of the appropriate schemas and automation of rules (e.g. Owen & Sweller 1989), it is generally acceptable nowadays that through problem solving processes one can give students a balanced view of mathematics and can face effectively the false opposition between *learning mathematics* and *learning to apply mathematics* (e.g. Voss 1987, Lawson 1990, Matos 1998 etc). Even Marshall (1995), the introducer of the current schema theory, presented schemas as vehicles for problem solving that can simplify and reconstruct a problem in order to make it more accessible to the solver.

Models for the mathematical modelling process

Concerning mathematical modelling, the transformation from a real world situation to a mathematical problem is achieved through the use of a *mathematical model*, which, roughly speaking, is an idealized (simplified) representation of the basic characteristics of the real situation through the use of a suitable set of mathematical symbols, relations and functions.

Pollak (1979) was the first who described the process of modelling in such a way that could be used in teaching mathematics. He represented the interaction between mathematics and real world with the scheme shown in Figure 1, which is known as the *circle of modelling*. In the *universe of mathematics* classical applied mathematics and applicable mathematics are two intersected, but not equal sets. In fact, they are topics from classical mathematics with great theoretical interest not having (at least for the moment) any visible applications, while they are also branches of mathematics, which are not usually characterized as classical, with many practical applications (e.g. probability and statistics, linear programming, fuzzy sets and logic, uncertainty theory, chaotic dynamics and fractals, etc).

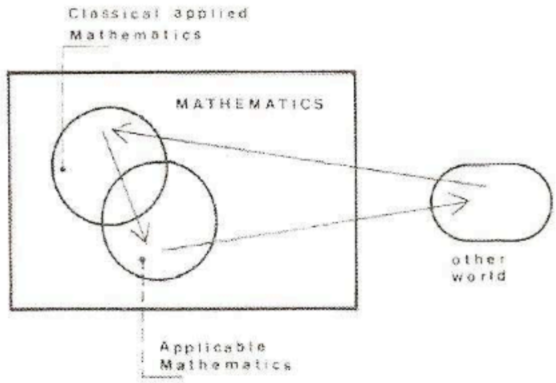

Figure 1: The circle of modelling

The most important feature in Pollak's scheme is the direction of the arrows, representing a looping between the *other world* (including real world, the human activities of everyday life and all other sciences) and the universe of mathematics: Having to solve a problem connected with a real situation we transfer from the other world to the universe of mathematics, where we use or develop appropriate mathematical methods to describe and solve the problem. Then we return to the other world interpreting the mathematical results obtained and implementing them to the real system. If these results are proved to be inadequate in describing properly the real system, we return back to the universe of mathematics making the proper modifications to the model. The modelling circle could be repeated several times until we find the proper solution.

From the time that Pollak presented his scheme in ICME-3 (Karlsruhe, 1976) much effort has been placed by researchers and educators to analyze in detail the process of mathematical modelling. The cyclic representations developed in the late 1970's in undergraduate engineering mathematics courses focussed on student activity at six discrete stages with the addition of a seventh reporting stage (Berry & Davies 1996). Transition between the stages did not at that time receive much attention. Developments in mathematical modelling in schools, since the mid 1980's, prompted a didactical and/or conceptual focus on representations of modelling within a cognitive prospectus. Blomhψj & Jensen (2003) provided a helpful and comprehensive visualization of the mathematical modelling process linking modelling to a perceived reality through six stages, whilst recognizing that theory and data, and associated linking of knowledge, impact on the process. A model similar to that of Blomhψj & Jensen is the modelling cycle of Blum & Leiβ (2007), a key feature of which is its emphasis on understanding the real situation and problem, moving through a situation model and a real model and problem whilst still in the other (extra-mathematical) world. Greefrath's (2007) model is also orientated towards the same direction. The starting point is a real situation that is not the whole reality, but an already structured situation from real life, which obviously must be chosen by someone (e.g. the teacher or the students) to deal with mathematically. This should be transformed into a real model, i.e. a simplified version of the real situation, which can now be transformed more easily into a mathematical model that should lead to a mathematical result. The transformations between the four stages are not named in this model and are unidirectional. For more details about the above models and graphical representations of them the reader may look at Haines & Crouch (2010) and at Meier (2009).

Summarizing all the existing ideas and in order to make the ideal model a bit clearer for teachers, we could say that the main stages of the mathematical modelling process involve:

- s_1 = *analysis of the problem* (understanding the statement and recognizing the restrictions and requirements of the real system).
- s_2 = *mathematization*, which could be divided to *formulation* of the real situation in such a way that it will be ready for mathematical treatment and *construction* of the model. The former involves a deep abstracting process, in order to transfer from the real system to the, so called, *assumed real system*, where emphasis is given to certain, dominating for the system's performance, variables.
- s_3 = *solution of the model*, which is achieved by proper mathematical manipulation.
- s_4 = *validation (control) of the model*, which is usually achieved by reproducing, through the model, the behaviour of the real system under the conditions existing before the solution of the model (empirical results, special cases etc). A model is valid, if despite its inexactness in representing the real system, gives a reliable prediction of the system's performance.
- s_5 = *Implementation* of the final mathematical results to the real system.

The flow-diagram of the mathematical modelling process in classroom, when teacher gives such problems for solution to students is represented in Figure 2. The modeller starting from s_1, which is always the initial state, moves through s_2 to s_3 .From there, if the mathematical relations obtained are not suitable to allow an analytic solution of the model, he (she) returns to s_2, in order to make the proper modifications to the model. If he (she) finally fails to construct a solvable mathematical model giving a reliable prediction of the real system's performance and being unable to make any other "movements" for the solution of the problem during the time given by teacher, returns from state s_2 to s_1 waiting for a new problem, to be given for solution. Otherwise he (she) returns to s_3, to continue the process. After the solution of the problem within the model the modeller returns to the real system, in order to check the validity of the model (state s_4).

Figure 2: The flow-diagram of the mathematical modelling process in classroom

If the model does not give a reliable prediction of the system's performance, (e.g. if the solution obtained is not satisfying the natural restrictions resulting from the real system, or if it is not verified by known special cases etc), the modeller returns from s_4 to s_2, in order to correct the model. From there he (she) will return through s_3 to s_4 to continue the process. After ensuring that the model "works" well, the modeller from s_4 reaches the state s_5, where he (she) interprets the final mathematical results, "translates" them to the natural language and applies the conclusions obtained to the real system. When the process of modelling is completed at state s_5, it is assumed that

a new problem is given by teacher for solution and therefore the process starts again from s_1.
A central object of the educational research taking place in the area of mathematical modelling is to recognize the attainment level of students at defined stages of the modelling process. For this, we have constructed a *stochastic model* describing (Voskoglou 2007), where we introduce a finite Markov chain having as states the main stages of the mathematical modelling process and we calculated its transition matrix in terms of the flow-diagram presented above (Figure 2). Applying standard results from theory of Markov chains we succeeded in expressing mathematically the gravity of each stage of the modelling process (where greater gravity means more difficulties for the students at the corresponding stage) and we obtained a measure for students' modelling capacities. A classroom experiment was also performed to illustrate the application of our model in practice. Two are the main outcomes of this experiment:

- There is a comparison between two groups of students with indications that the teaching of one group might be more effective than that of the other one.
- The analysis shows that mathematization possessed the greater gravity among the stages of the modelling process for both groups of students.

The second outcome was logically expected, since the formulation of the problem involves, as we have already seen, a deep abstracting process, which is not always an easy thing to be achieved by a non expert.
Mathematics does not explain the natural behaviour of an object, it simply describes it. This description however is so much effective, so that an elementary mathematical equation can describe simply and clearly a relation that, in order to be expressed with words, could need entire pages. We believe that this is exactly the main advantage of our stochastic model compared with qualitative methodologies used by other researchers for similar purposes, such as the analyses of questionnaire's collected answers by using respond maps (Stillman and Galbraith 1998), or multiple choice tests (Haines & Crouch 2001) and the related discussion activities, etc.
Models for the mathematical modelling process like the above, and the analogous ones that we have briefly described before, are helpful in understanding what is termed as the *ideal behaviour* (Haines and Crouch 2010), in which modellers proceed from real world problems through a mathematical model to acceptable solutions and report on them. However life in classroom is not like that. Recent research, (Galbraith & Stillman 2001, Doer 2007, Borroneo Ferri 2007), reports that students in school take *individual modelling routes* when tackling mathematical modelling problems, associated with their individual learning styles. Students' cognition utilizes in general concepts that are inherently graded and therefore fuzzy. On the other hand, from the teacher's point of view there usually exists vagueness about the degree of success of students in each of the stages of the modelling process. All these gave us the impulsion to introduce principles of fuzzy logic in order to describe in a more effective way the process of mathematical modelling in classroom (Voskoglou 2010).
In our *fuzzy model* the main stages of the modelling process are represented as fuzzy sets in the set of the linguistic labels of negligible, low, intermediate, high and complete success of students in each stage. The concept of *uncertainty*, which emerges naturally within the broad framework of fuzzy sets theory, is involved in any problem-solving situation, especially when dealing with real-world problems. Uncertainty is a result of some information deficiency. In fact, information pertaining to the model within which a real situation is conceptualized may be incomplete, fragmentary, not full reliable, vague, contradictory, or deficient in some other way.

Thus the amount of information obtained by an action can be measured in general by the reduction of uncertainty resulting from the action. Accordingly students' uncertainty during the modelling process is connected to students' capacity in obtaining relevant information. Therefore a measure of uncertainty is adopted as a measure of students modelling capacities (Voskoglou 2010). Our fuzzy model, apart from the quantitative information, gives also the possibility of a qualitative analysis by providing all the possible students' profiles during the modelling process.
Mathematical modelling appears today as a dynamic tool for the teaching of mathematics, connecting mathematics with our everyday life and giving to students the possibility to understand the usefulness of it in practice. It has also the potential to enhance the performance of students in mathematics generally (Matos, 1998). A special didactic methodology was developed across these lines by De Lang in Netherlands, named by G. Kaiser (Hamburg) as the *application – orientated teaching of mathematics*. But we must be careful! The process of modelling could not be considered as a general, and therefore applicable in all cases, method for teaching mathematics. In fact, such a consideration could lead to far-fetched situations, where more emphasis is given to the search of the proper application rather, than to the consolidation of the new mathematical knowledge!

About the workshop

As we have seen above mathematization seems to be the most difficult among the stages of the modelling process for students. Crouch and Haines (2004, section 1) report that it is the interface between the real world problem and the mathematical model that presents difficulties to the students, i.e. the transition from the real word to the mathematical model and vice versa the transition from the solution of the model to the real world. The latter looks rather surprising at first glance, since, at least for the type of modelling problems solved usually at school, a student who has obtained a correct mathematical solution of the model is normally expected to be able to "translate" it easily in terms of the corresponding real situation. However things are not always like that. In fact, there are sometimes modelling situations, where the validation of the model and/or the final stage of the implementation of mathematical results to the real system, hide surprises that force students to "look back" to the construction of the model and possibly to make (again) the necessary changes to it. The purpose of this workshop is to present some characteristic examples illustrating these situations and to discuss teacher's proper reactions in order to orientate students to act effectively in these cases. We shall see, for example, how it is possible to obtain two mathematically (not practically!) correct solutions when it is asked to construct a channel to run the maximum possible quantity of water by using a given metallic leaf, or why it is possible to find no solution (although there exists!) when it is asked to find the cylindrical tower with the maximum volume among those having the same total surface, etc.

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An Investigation into the design of Advanced Certificates in Education on Mathematical Literacy teachers in KwaZuluNatal

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Abstract

The aim of this paper is to describe an ongoing study into the design of two Advanced Certificates in Education in Mathematical Literacy (ML) offered by two Higher Education Institutions in the KwaZuluNatal (KZN) province in South Africa. Mathematical Literacy is a relatively new subject that has been introduced into the grade 10, 11 and 12 school curricula as an alternate to Mathematics. Mathematics and ML are two distinct subjects with different objectives. There is thus an urgent need to train pre-service teachers and re-skill in-service teachers to teach ML competently in schools, and not to treat ML as a less sophisticated version of Mathematics. This paper looks at the design of the two qualifications and the results of the students who studied for the qualifications.

Introduction

All learners in South Africa have a choice of either Mathematical Literacy or Mathematics for their last three years of schooling. As ML is a relatively new addition to the curriculum, studies on the effectiveness of programmes targeting teacher training are imperative. By comparing and contrasting in-service programmes offered by two Higher Education Institutions (HEIs), methods, strategies and results are brought to light and evaluated. Because of the pressing need to have qualified and effective ML teachers in the classroom, the KZN Department of Education (DoE) tasked two HEIs to deliver Advanced Certificates in Education (ACE:ML) programmes throughout the province. The in-service teachers, who were prospective students, were identified by the DoE and the HEIs, therefore, had no part in specifying mathematics ability, other than the entrance requirements for the qualification. Each programme spanned two years' of study, and extended to a third year, when students were given an opportunity to repeat modules if they had failed to reach the Universities' required standards. The centres dictated by the DoE were in rural, peri-urban and urban areas.

The study described in this paper focuses on approximately 2 200 in-service teachers who have studied ACE:ML on a part-time basis over a period of two years with either of the two HEIs operating in KZN. The method of the broader study (which is preliminary at present) will follow an explanatory mixed method approach where quantitative data on the students' demographics and performance will be collected and analysed to identify any particular trends relating to uptake and success in these programmes. Module templates (providing information about the learning outcomes, modes of delivery, assessment strategies and module evaluation requirements), learning materials and tutor training guides will be analysed to provide information about the nature and design of the two ACE programmes. A questionnaire consisting of closed questions as well as a limited number of open ended items will be sent out to all current students, drop outs and graduates of the programmes. The purpose of the questionnaire will be to gather data about the teachers' perceptions of the effectiveness, relevance and utility of the two programmes as well as to solicit suggestions on how the models of delivery could be improved. Semi-structured interviews will also be conducted with samples of students from each programme to triangulate the data.

ACE programmes from all South African HEIs are currently being placed under the spotlight. According to the November 2010 Draft Policy on the Minimum Requirements for Teacher Education Qualifications selected from the Higher Education Qualifications Framework (HEQF), ACEs will have to be recurriculated into either Advanced Diplomas in Education (ADE) at National Qualifications Framework (NQF) level 7 or Advanced Certificates in Teaching (ACT) at NQF level 6 (Department of Higher Education and Training, 2010). The quandary HEIs are presented with is that the HEQF is under review and there is speculation as to the bredth and depth of content and pedagogical content knowledge that should be included in the proposed qualifications.

Literature overview

Steen (2003) has written extensively about Quantitative Literacy in the United States. He is of the opinion that learners need to be flexibly prepared for life and to this end he suggests teaching a blend of numeracy, mathematics and statistics. De Lange (2003) posits that being mathematically literate has differing definitions depending on the needs of the community; however, in his description of a balanced mathematical literacy curriculum he identifies topics similar to those in the South African NCS, soon to be superseded by an adjusted curriculum:

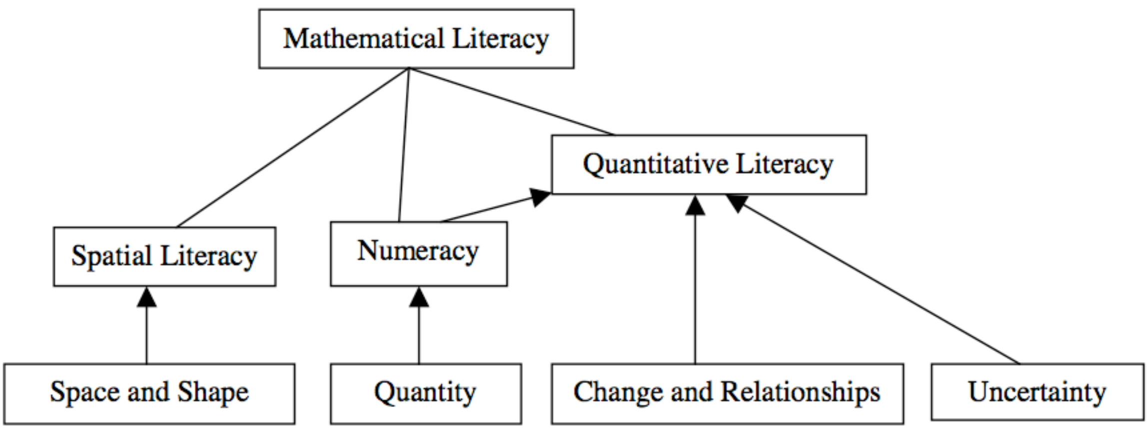

Figure1: Jan de Lange's (2003) conception of a balanced Mathematical Literacy Curriculum

Adler, Pournara, Taylor, Thorne and Moletsane (2009) surveyed the field of mathematics education in South Africa and one of their conclusions was that learning science and mathematics (and, in this instance, mathematical literacy) for teaching is not a simple matter. Many initiatives have not made the required impact on teaching or learner performance. They suggest examining the practices of teacher education with respect to "breadth and depth of domain knowledge; subject content and pedagogy" (p.39)

Schmidt, Cogan and Houang (2011) report on the Teacher Education and Development Study in Mathematics (TEDS-M), an international comparative study of teacher education. The researchers in the TEDS-M study assessed mathematical content knowledge, general pedagogy and mathematical pedagogy as they identified these as three key areas associated with teacher preparation. Although the TEDS-M study focused on the preparation of future teachers, the opportunities to learn are similar to those of in-service teachers who are facing the challenges of a new content area. The TEDS-M study revealed that there is little agreement among HEIs in the United States as to what constitutes teacher preparation, possibly because there is no shared vision of what a highly trained teacher should know (Schmidt et al., 2011). The situation in South Africa with teacher training for ML is similar as there was an urgency to train a massive cohort of teachers before the learning area was implemented in schools, even though the implementation was phased in over three years. HEIs developed programmes in isolation, and perhaps without due regard for Adler et al.'s caveat concerning both breadth and depth of mathematics content. The TEDS-M study further revealed that achievement is related to curricular differences in terms of content cover. Data gathered in the TEDS-M study indicated that the mathematical content offered in teacher training courses was positively related to professional competencies. In the comparison of the two ACEs in this study the majority of modules were ML content-based as professional competencies were deemed to have been acquired through previous teaching experience.

Curriculum contrasts

The requirements for entry to ACE: ML for University A recommended that students have at least a pass for mathematics standard grade at Senior Certificate level, a category C (M+3) teaching qualification at NQF level 5 and at least three years' teaching experience. Similarly, University B required an approved initial teacher education qualification or diploma of at least three years' duration and specified that the prospective students should have at least attempted mathematics at Senior Certificate level. In reality 42% of the 1 048 students studying ACE: ML in KZN from University A and 31% of the 691 students from University B had either no mathematics or failed mathematics in their senior certificate examination.

The curriculum for University A consisted of four 30-credit modules. Three modules focused on ML content and were divided into grade levels – grade 10, grade 11 and grade 12 ML content knowledge. The coherence of the curriculum was addressed by using a commercial ML textbook to underpin the curriculum content. The fourth module was designed as an ML pedagogical module that linked content and context. General pedagogy was deemed to be an aspect of recognition of prior learning (RPL) as the teachers all had a minimum of three years' teaching experience. The emphasis was on reskilling the teachers in a new subject area.

In contrast the curriculum at University B consisted of eight 16-credit modules. The ML content knowledge was covered in four modules that were horizontally aligned to ML curriculum learning outcomes – number and number relationships, functional relationships, space and shape, and data handling. The faculty developed their own study material and utilised school textbooks as supplementary material. There was one module devoted to the ML pedagogical aspects of teaching mathematics and mathematical literacy, while another was a research module designed to improve the teachers' reflective practices. In addition, there were two general pedagogy modules, studied by all ACE students in the faculty regardless of the ACE discipline, and two generic research modules –theory and practice of research.

The ML content modules from both HEIs were devised as a combination of content and context. For example, the one grade 11 examination that was perused was set in the context of a game reserve and the mathematical content. In one question a map was given of the park and a sub-question was,

> "Having watched the elephants at the Hapoor water hole for a while, you realise that it is 17:50 and the game gate closes at 18:30. Sketch on the map the shortest route out of the park estimate the distance in km and calculate how long it will take you if you stick closely to the given speed limit of 40km/h."

Other contextual questions included calculations concerning cell phone billing, loans, interest rates and house renovations. The balance between content and context was similar in all the ML content modules at both institutions.

In both universities delivery was through a cascade method of training tutors to teach students throughout the province. The tutors at University A attended a central block of training then went into the field to tutor on Saturdays whereas University B utilised a mixed delivery approach of block sessions and Saturdays. For both ACE programmes modules were delivered over a semester and there was continuous assessment through assignments and tests throughout the semester. In both programmes all modules, except the practical research module at University B, were summatively assessed with an examination. Class marks contributed towards the final examination mark.

Coherence

The statistical data from the study are in the process of being analysed but a few interesting preliminary results are emerging.

The first investigation was to plot the coherence of the curricula according to Steen and de Lange's recommendations. As the South African NCS seems closely aligned to de Lange's diagram both curricula cover all aspects recommended, even though the University B curriculum is horizontally aligned along learning outcomes whereas the University A is vertically aligned according to grades.

The correlation among the pass rate performance of the students in the three ML content modules at University A ranged between 0.751 and 0.826. Correlation was deemed to be significant at the 0.01 level in a two-tailed test. The correlation of the performance in the mathematical pedagogy module as compared with that of the content modules ranged from 0.712 and 0.769. This indicates that if a student achieved well in one of the four modules, there was a strong possibility that the student would also achieve in the other three modules – and vice versa.

At University B the correlation among the performance in the ML content modules ranged from 0.676 to 0.807; however, the correlation was not as strong between the content modules and the ML pedagogical module (minimum correlation was 0.634), the reflective practice research module (minimum correlation was 0.415), and even less between the content modules and the general pedagogy modules (correlation ranged between 0.385 and 0.567). These correlation statistics imply that students who did well in any one of the ML content modules, performed well in the others. However students who did well in the content modules may not have done well in the general pedagogic modules or the research module, and vice versa. Thus, these statistics indicate that different skills are required for mathematical content, general pedagogy and reflective practice. Thus Schmidt et al.'s (2011) recommendation is strengthened – that mathematical content knowledge, general pedagogy and mathematical pedagogy should be included in teacher training as they focus on different skills.

Further research is required to ascertain whether the two programmes have made an impact on ML teaching in KZN. The objective of the larger study is to investigate teachers' efficacy in the classroom.

Correlation between students' ACE and school leaving results
An interesting aspect of the data collection was the link between the students' achievement in mathematics in the Senior Certificate examination and the probability of passing either qualification in the minimum time period of two years. As all the students, registered at both institutions, were in-service teachers with more than three years' experience, they had all written either Mathematics Higher Grade or Mathematics Standard Grade, or had no Mathematics in their final school examination. Their senior certificate symbol for mathematics was converted to a scale variable ranging from zero (No mathematics or failed mathematics at Senior Certificate level) to 7 (a B-symbol for Higher Grade mathematics at Senior Certificate level.

At both institutions there was a high correlation between the students' mathematics point in the senior certificate examination and their propensity for completing the qualification in the minimum time period. In University A the correlation was 0.805 and in University B the correlation was 0.883. The fact that some students had left school many years before did not appear to affect the data. Not surprisingly, this indicates that a measure of mathematical ability is a prerequisite for success in ML modules.

Conclusion
In this paper we have looked at the composition and student results of two ACE: ML programmes that have been delivered in KZN. It appears that if students are able to achieve well (or vice versa) in a module focusing on ML content from one learning outcome or grade, this is a reliable indication that they will be able to show good progress (or poor) in modules focusing on other learning outcomes or grades. In this study it appears that the students' ML content knowledge was fairly constant over all learning outcomes and grades in both programmes. Those students who achieved in one content module generally achieved in the other content modules as well.

Another outcome of the study is the positive correlation between mathematics marks in the school leaving examination and the students' achievement in the ACE programmes. Results suggest that despite intervening years, school mathematics marks are a strong indicator of success in completing an ML qualification successfully in the minimum time period. Further interviews are planned with the students which may triangulate this supposition.

The expedience and urgency of the delivery of ACE:ML qualifications throughout the province precluded including more than curriculum content knowledge in the ML content modules of both curricula. There is ongoing uncertainty and debate about the extent of mathematical depth required to become an effective ML teacher. Should students who studied ML in their senior certificate examination be excluded from training to become ML teachers? If not, what depth of mathematical knowledge should they explore, considering that they do not have a pure mathematics background? However, if ML learners are excluded from ML teacher training, how will South Africa populate schools with ML teachers if the pool of prospective teachers is limited to those who studied pure mathematics successfully at school?

Both international and national research has indicated that teacher efficacy is a result of balanced teacher training. The implication is that the areas of ML content knowledge, general pedagogy and ML pedagogy should be included in ML teacher training curricula. The issue of the breadth and depth of domain knowledge has not been addressed; however, it is a pertinent issue to discuss during the forthcoming qualifications recurriculation process.

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USING A MODELLING TASK TO ELICIT REASONING ABOUT DATA

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Abstract

Completing modelling tasks not only develops prospective teachers' mathematical knowledge and problem solving competencies, but also prepares them to implement mathematical modelling later in their own practice. I present the preliminary findings of one investigation in a longitudinal project in South Africa where 188 prospective teachers completed a modelling task requiring reasoning about data. Responses show a variety of models on different levels of sophistication. Analysis has not been completed yet.

Introduction and theoretical background

Mathematical modelling is increasingly recognised as feasible teaching and learning perspective in schools (Mousoulides, 2009; Wessels, 2009). Mathematical modelling means "applying mathematics to realistic, open problems" (Maaß & Gurlitt, 2009) and engages students in open, non-routine problems that elicit powerful mathematical models which are extended and refined into systems that can be generalised for use in other contexts (Lesh & Doerr, 2003). The mathematical modelling process entails a number of steps that students iteratively go through, usually jumping between the different stages in a non-cyclic way (Ärlebäck & Bergsten, 2007). Each of these phases are also characterised by multiple cycles of "interpretations, descriptions, conjectures, explanations and justifications that are iteratively refined and re-constructed by the learner, ordinarily interacting with other learners" (Doerr & English, 2001). Mousoulides (2009) describes six processes in the solving of modelling problems: the understanding of the specified task with its constraints and alternatives; the identification of the relevant constraints; exploring and representing possible alternatives; choosing between alternatives; evaluating the choice; and communicating and defending the decision. The reality of the real-world problem is therefore progressively cut away to convert the real-world problem into a mathematical problem. The mathematical solution is then in the end evaluated against its usefulness in the reality of the situation. Modelling problems require students to make sense of the situation and use quantities and operations that they understand and find useful (Doerr & English, 2001). Students' exposure to mathematical modelling tasks cannot be a once off experience – they need multiple experiences to explore mathematical constructs and to use their models in new contexts to be able to generalize it.

Open, non-routine or model-eliciting problems (MEA's) provide the opportunity for students to access and process in the classroom complex mathematics problems at different levels of intellectual sophistication and solve these problems through the interaction between their informal and more formal mathematical knowledge. The paradigm shift from traditional teaching and learning of mathematics to a problem centred approach and a mathematical modelling perspective represents a shift to a more equitable situation in mathematics education. Usable models can originate from solution strategies on different levels, affording achieving students, non-achieving students and students from disadvantaged school environments the same opportunity to build successful models. Students who are exposed to MEA's often change their beliefs about mathematics positively and enjoy these activities, resulting in a shift to positive dispositions.

International reform movements in mathematics have shown that mathematical modelling may be an effective instrument in bringing about change. We know from research (Clarke, Breed & Fraser 2004; Kim, 2005; Riordan & Noyce, 2001; Schoen, 1993) that mathematical modelling students do at least as well, and often better, on standardised tests; are more able to transfer mathematical ideas into real world; are more confident in mathematics; value communication in mathematical learning more highly than students in conventional classes; and, developed a more positive view about the nature of mathematics than their counterparts. A mathematical modelling perspective therefore brings us a step nearer to turning the dreams of children, parents and teachers into reality where all can achieve in and enjoy mathematics. The use of mathematical modelling tasks in teacher education affords prospective teachers the opportunity to learn worthwhile mathematics while they develop their ability to apply the mathematics they already know in the development of powerful mathematical constructs (Niss, Blum & Galbraith, 2007). Prospective teachers are at the same time prepared to implement mathematical modelling in their future practice. The extension of a mathematics modelling perspective to the education of prospective Foundation Phase (K-3) teachers is not common. In traditional classrooms the focus is on what the teacher teaches and not necessarily on what children understand. Problem solving abilities are usually also not focused on in the early years. A modelling perspective in the Foundation Phase (FP) represents another paradigm shift as it emphasises the importance of mathematics education that fosters understanding as well as the development of problem solving abilities in the primary years.

To be able to implement a modelling perspective in the classroom, it is crucial that prospective teachers are exposed to mathematical modelling in their own education at undergraduate level (Garcia, Maaß & Wake, 2009). Successful implementation of a mathematical modelling perspective further depends on teachers

- being familiar with the key concepts of mathematical modelling;
- having appropriate beliefs about the nature of mathematics education; and
- being aware of their own competency to implement this perspective in practice (Maaß & Gurlitt, 2009).

Prospective teachers need to be made aware of the nature and spectrum of modelling competencies and how they are used in problem solving. Modelling competencies involves cognitive, metacognitive and affective competencies which are applied in an integrated way in the solving of mathematical modelling tasks (Biccard, 2010).

Method

The investigation described here is part of a longitudinal research project to prepare prospective FP teachers to implement a mathematical modelling perspective in their classrooms when they start teaching; to determine whether and how they implement this approach; and what the reasons and challenges are for not implementing MEA's. This paper reports on the modelling cycles prospective teachers went through and the models they created while solving the second of the MEA's they were exposed to during the project. Hundred and eighty eight prospective Foundation Phase teachers completed the MEA as part of their mathematics education module. Second, third and fourth year undergraduate prospective teachers (n= 88, 75, 25 respectively) solved the modelling task in groups of two to six in class during 3 to 4 lectures (three to four hours).

The task, "Making Money" (Lesh, Amit & Schorr, 1997), is about an entrepreneur employing nine vendors to sell popcorn and drinks in an amusement park during the summer months. She has to cut the number of vendors employed and asks for recommendations of which six vendors she should rehire full-time and part-time for the next summer. She supplies tables showing the number of hours worked and the money earned by each vendor when business was busy (high attendance); steady and slow (low attendance). The task is to make recommendations of who she should rehire in a letter, describing in detail how the vendors were evaluated and giving clear explanations so that she can decide whether the method is a good one for her to use.

The task met all six requirements for a MEA, which differs radically from traditional textbook word problems (Lesh, Amit & Schorr, 1997):

- The *reality principle*: the task focused on a 'real' problem that needs to be addressed – not a contrived cleaned-up school textbook problem
- The *model construction principle*: the task required the construction of a model and the description of assumptions and conditions in the justification of decisions made while constructing the model
- The *self-evaluation principle*: the task explicitly stated for what purpose and by whom the results were needed to enable the student teachers to evaluate their own solutions and decide whether it needed improvement
- The *model-documentation principle*: the task required explicit explanations of their thinking about the situation and the solution paths they followed
- The *model generalisation principle*: the model(s) created could be adjusted and applied to other situations and contexts
- The *simple prototype principle*: the problem was designed to elicit the creation of a model while still being as simple as possible.

Mathematical modelling essentially involves group work as ". . . competencies of the group are likely to be greater than those of individuals" (Biccard, 2010; Mousoulides, 2009; Zawojewski, Lesh & English, 2003). Individuals verbalise and criticise ideas more spontaneously in a small group setting, resulting in a higher level of creativity and the development of more sophisticated systems.

Data sources included the researcher's field notes, student teachers' rough notes, mind maps and workings as well as their final reports describing their recommendations, their solutions paths and reasons for specific decisions made along the way. The analysis is still in progress. The project is being conducted with problem centred teaching and learning as context – the researcher did not suggest any strategies or solution paths, neither did she prompt student teachers to use the different conceptual models. She provided resources and gave ample time for students to complete the activity. They were allowed to use technology such as pocket calculators and computers.

When student teachers in the project had to solve a MEA for the first time, they could not believe that they were not going to be told how to solve the problem as was the case in school mathematics. Some groups only started working productively on the task after almost an hour and most students took 3 to 4 hours to complete the task, some even took longer. With the second MEA students did not start right away working on the task, but this time they knew that they had to do it on their own and that the researcher would not give hints or prompt strategies and solutions. By the time they had to solve the third MEA, they immediately started working on the problem and most groups finished after about two and a half hours.

Preliminary findings

Responses fall into two big categories (See Table 1). In the first category models were developed without combining the two data sets, i.e. by totalling earnings and totalling number of hours worked (n=6) and focusing on averages (N=1). Two groups calculated only totals for

income and for hours worked, while 4 groups totalled income an hours worked for the different shifts (busy, steady and slow times) and ranked the vendors accordingly. One group used average income and average hours worked to develop their model.
In the second category the data set with number of hours worked was combined with the data set with earnings to calculate rand-per hour (n=41). This category showed 12 different interpretations of the problem (see Table 1), with groups inventing decision-making rules on different levels of sophistication.

Table 1: Analysis of responses

	Category	Description	Number of group responses
No combining of the 2two data sets	**Total income and total number of hours worked** N=6	Totals only	2
		Totals for different shifts, & rankings	4
	Average income and average number of hours worked N=1	Average hours & average income	1
Combining the two data sets	**Income per hour (R/h)** N=41	R/h for months separately	3
		R/h over 3 months	15
		R/h for different shifts & R/h overall	6
		R/h for different shifts & R/h overall & ranking	4
		R/h for different shifts & R/h overall & ranking using ratio	1
		R/h & consistent high scores for different shifts	1
		R/h & R/h for slow times	1
		R/h & R/h in busy times (for deciding part time vendors)	2
		R/h & weighting (points system)	5
		R/h & % of average R/h of all vendors together	1
		R/h & average of all vendors together – deviation from average	1
		R/h & average of all vendors together – above this average	1
		Total	48

Prospective teachers mostly used tables and bar graphs to represent their interpretations of the task, although a few (inappropriate) broken line graphs and pie graphs were used.

Discussion

Realistic problems require working with and understanding different kinds of quantities and operations than in traditional tasks (Doerr & English, 2001). The "Money Making" task entailed statistical analysis and included multiple views and representations of the data (mostly tabular and graphical), measures of central tendency (averages and deviation from averages), as well as the combining of data and analyses of trends. Operations needed in the task therefore differ from operations in traditional problems: additional to adding and dividing quantities and calculating averages, student teachers also needed to sort, organise, select, combine and transform the entire data set rather than just working with single data points.

Groups had to describe in detail the solution paths they followed in the process of developing the model. They were required to identify different alternatives and evaluate and justify their choices of alternatives. One procedure was to brainstorm together and then trim away less useful suggestions to pursue one or two ideas. Another procedure was that each group member had to come up with an own idea and do the preliminary working out. Ideas were then compared and useful alternatives further developed. A third modus operandi was to first explore just one idea initially and then work towards other alternatives. Group 38 for example used income per hour to identify the best six vendors and then compared income per hour for the slow shifts because ". . . if one shows that they can bring in the most money in the slow times then they are good vendors. Not everyone enjoys working slow times and this suggests that Thandi, Jose and Maria are willing to work and are more dedicated to their jobs than the rest. They should be hired full-time". Group 41 ranked vendors by a weighting scheme and remarked: "Using this method, each vendor's performance can be clearly compared against that of the others, and a fair decision (one that takes into account all factors – the number of hours they worked, how much money they collected, as well as when they worked) can be made".

Different solution paths for similar interpretations were common. Some groups for example first worked out the average income per hour of each vendor for each shift per month and then added it all together to calculate an overall average for the individual while other groups worked out total earnings and total hours for each individual and then calculated the total average.

Although the prospective teachers did not receive specific instruction in these ideas, some groups developed quite sophisticated models. Group 47 for example combined the two data sets by calculating income per hour per shift of each vendor, but also focused on the average productivity (income per hour) of the whole group. They ranked vendors according to deviations from this average to decide which vendors to recommend for full-time and which to recommend for part-time employment.

Groups who used only totals or averages of income and/or hours did not critically evaluate their solution paths and stuck with one idea. Most other groups explored the usefulness of different models before deciding on a specific one. A few groups compared conclusions reached through the use of earlier models to check the conclusions reached with a later more sophisticated model.

Summary

The models described above were developed without any interference or guidance from the researcher. Most groups went through a number of modelling cycles through which they progressed from focusing on subsets or isolated pieces of the data to considering combined data sets and underlying trends and regularities.

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Comparing the Use of Virtual Manipulatives and Physical Manipulatives in Equivalent Fraction Intervention Instruction

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Abstract

This paper describes a study designed to identify differences in learning trajectories related to the use of virtual and physical manipulatives during equivalent fraction intervention instruction. Changes in recent years in the approach to mathematical intervention have increased the need for research which identifies effective methods for the teaching of students with mathematical learning difficulties. Recommendations made from current intervention research emphasis the importance of students developing a proficiency in the use of representations through the use of manipulatives. Research indicates that virtual and physical manipulatives are effective tools of instruction. However, each manipulative type has its own unique affordances which affect the learning of specific concepts. The purpose of this study is to identify differences in students' learning trajectories related to the type of manipulative (virtual and physical) used during intervention instruction of equivalent fraction concepts.

Background

Due to the globalization of markets, advances in technology, and the overwhelming spread of information in today's society, mathematical skills have becoming increasingly important for success. Yet, there are a number of students who fail to acquire the needed mathematical skills during regular classroom instruction and the number of students needing special education services is increasing (Singapogu & Burg, 2009). As a solution, many programs, to more effectively support students with mathematical learning difficulties, have begun to shift from the traditional remediation approach to a response to intervention (RtI) approach. As an intervention approach, RtI focuses on the early identification and support of students who are struggling academically. The most common form of RtI intervention is a three tiered design. The first tier of intervention takes place in the regular classroom setting where all students receive research proven effective instruction. It is expected that at least 80% of the students will master the concepts taught during Tier I instruction (Fuchs, Compton, Fuchs, Bryant, & Davis, 2008). In Tier II intervention, students who did not achieve mastery in Tier I are given additional assistance. Tier II intervention is content specific and typically conducted by the classroom teacher in small group settings. Students who do not respond to Tier II intervention are considered to be "non responders" to intervention and will receive additional Tier III special education services (Fuchs et.al. 200).

Preliminary research results of RtI for mathematics intervention, although limited has been positive (Glover & DiPerna, 2007). Yet, developers of RtI programs have reported that the lack of available Tier II instructional materials and tools is limiting program implementation and research (e.g.: Fuchs, Seethaler, Powell, Fuchs, Hamlett, & Fletcher, 2008; Gersten et.al., 2009; Glover & DiPerna, 2007). This study focuses on the use of virtual and physical manipulatives in equivalent fraction intervention instruction.

Representations

In a review of RtI literature, Gersten, et al.(2009) made eight research based recommendations for setting up effective RtI programs. The fifth recommendation reads:

> Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas (p. 30).

Gersten, et al. (2009) explained that one of the most common difficulties experienced by students with mathematical learning difficulties is the ability to connect abstract symbols to visual representations. They suggested that in regular classroom instruction, representations are not emphasized strong enough nor presented systematically enough to facilitate the scaffolding of learning for students with mathematical learning difficulties.

The purpose of using external representations (e. g. manipulatives, drawings, mathematical tables, etc.) in instruction is to aid students in their development of internal representations (Behr, Lesh, Post & Silber, 1983). Students' internal representations are: a) verbal /syntactic images, b) mental images, c) formal notation, and d) affective images including emotions, attitudes, beliefs and values (Goldin & Shteingold, 2001). As student's conceptual understanding of mathematical concepts develops, the power and flexibility of their internal representations grows. Students who have developed only partial internal systems of representations often experience difficulties in learning new concepts (Goldin & Shteingold, 2001). Physical and virtual manipulatives are tools of representation used to scaffold students' learning of mathematical concepts.

Physical and Virtual Manipulative Effectiveness

Physical manipulatives are concrete objects which students use to explore mathematical concepts through the students visual and tactile senses (McNeil & Jarvin, 2007). Virtual manipulatives are tools which are "interactive, web based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard & Spikell, 2002, p. 373). Advocacy for the use of manipulatives stem from the learning theories of Piaget, Bruner and Montessori that students develop and build knowledge as they move from concrete experiences to abstract thinking (McNeil & Jarvin, 2007). Piagetian theory suggests that children need to physically manipulate objects and then should be encouraged to reflect upon the meaning of the results of their physical actions (Baroody, 1989). These theories are supported by the research of studies such as the Rational Number Project, a project focusing on the development of representations through the use of manipulatives. Study results indicate that students using manipulatives significantly outperformed students taught using the more symbolic approach (Cramer & Post, 2002). The researchers described four ways manipulatives facilitated students development of fraction understanding: 1) students used the manipulatives to develop mental images of fraction meaning, 2) comparing manipulative objects helped students form correct methods for compare fractional sizes, 3) students use the manipulatives as references when justifying their answers, and 4) students using manipulatives developed less misconceptions.

The effectiveness of using physical manipulatives in mathematical instruction has been the focus of a large number of research studies. A review of the literature identified three meta-analysis reports. Suydam and Higgins (1977) reported that 11 of 23 studies reported finding significant differences in student achievement favoring the use of manipulatives, two studies favored not using manipulatives, and ten studies reported no significant differences between use and non use of manipulatives. Parham (1983) analyzed 64 studies and obtained 171 effect size scores comparing the use of manipulatives with non use on student achievement. Their analysis yielded an average mean effect size of 1.03, indicating a large effect size favoring manipulative use. Sowell (1989) reported, in their analysis of 60 studies that although manipulatives generally are found to be more effective than other types of instruction, but that there is a lot of variability in results, with some studies yielding large effect size scores while a few yielded negative effect size scores.

Moyer-Packenham, Westenskow and Salkind (2011) conducted a meta-analysis evaluating the effect of virtual manipulatives on student learning. The analysis of 70 effect scores obtained from 29 studies yielded a moderate average effect size of 0.37 when compared with the use of other methods of instruction. When virtual manipulatives were used alone as the primary tool of instruction and compared with instruction using physical manipulatives and with traditional classroom instruction, the averaged effect scores were a small effect of 0.18 (33 effect scores) and moderate of 0.73(18 effect scores) respectively. Their analysis of the qualitative data suggested that virtual manipulatives have affordances of focused constraint, creative variation, simultaneous linking, efficient precision and motivation.

Students with Mathematical Learning Difficulties.

A search of the literature identified five studies investigating the effectiveness of instruction using physical manipulatives with students having mathematical learning difficulties. In two studies, Butler, Miller, Crehan, Babbit, and Pierce's (2003) and Witzel, Mercer, and Miller (2003), students with mild to moderate mathematical disabilities who participated in instruction using physical manipulatives scored significantly higher than students who did not use manipulatives. Similarly, results from three studies involving students with learning disorders' reported that achievement scores improved after students participated in instruction using manipulatives (Cass, Cates, Smith, & Jackson, 2003; Maccini & Hughes, 2000; Moch, 2001).

Four studies investigating the effectiveness of instruction using virtual manipulatives with students having mathematical learning difficulties were identified. Suh, Moyer, and Heo (2005) reported that when using virtual manipulatives, higher achieving students were more efficient and used more mental processes for finding answers while lower achieving students were more methodical in their use and more dependent on using the visual models when scaffolding between the pictorial and symbolic. Three studies reported positive effects when students receiving special education services used virtual manipulatives and two of the studies reported that the students using virtual manipulatives outperformed the students who did not use the manipulatives (Guevara, 2009; Hitchcock & Noonan, 2000; Suh & Moyer-Packenham, 2008). In summary, all of the identified studies in which manipulatives were used with students of differing abilities reported that students with mathematical learning difficulties benefitted from the use of manipulatives.

Combining Use of Virtual and Physical Manipulatives

Although limited, research indicates that there may be an advantage to combining the use of physical and virtual manipulates in instruction. In a meta analysis, Moyer, Westenskow and Salkind (2011) identified 26 effect size cases of instruction combining the use of virtual and physical manipulatives. When instruction with combined use was compared with instruction using only virtual manipulatives, only physical manipulatives, and traditional classroom instruction, results produced a moderate averaged effect of 0.26 (9 effect size cases), a small effect of 0.20 (11 effect size cases), and a moderate averaged effect of 0.69 (6 effect size cases) respectively. These results indicate combining the use of virtual and physical manipulatives may be advantageous to student achievement.

Physical and virtual manipulatives have distinct affordances and disadvantages (i.e. many virtual manipulatives have explicit symbolic pictorial links, physical manipulatives involve tactile senses). Several researchers have reported that the affordances of each type of manipulative have resulted in variations of learning unique to the type of manipulative (Izydorczak 2003; Moyer, Niezgoda, & Stanley, 2005; Takahashi, 2002). As suggested by Behr et al. (1983) while a manipulative may be the most effective tool to use in teaching one concept, it may when used to teach a different concept impede student learning. They suggest more research is needed to identify which manipulative will best facilitate the learning of each concept. The purpose of this research study is to identify differences, related to manipulative type, in students' learning of the concepts of equivalent fractions. The research questions guiding the research are:

1. Are there variations in student achievement unique to the use of different instruction tools (virtual manipulatives, physical manipulatives or a combination of virtual and

physical manipulatives) in the learning of equivalent fraction concepts by students with mathematical learning difficulties?

2. Are there variations in the learning trajectories unique to the use of different instructional tools for intervention (virtual manipulatives, physical manipulatives or a combination of virtual and physical manipulatives) in the learning of equivalent fraction concepts by students with mathematical learning difficulties?

Methods

To answer these research questions, a study will be conducted providing preliminary Tier II intervention to fifth grade students who have not mastered equivalent fraction concepts in the regular classroom. The study uses a mixed methods approach of triangulating evidence from quantitative and qualitative data collected from pre/post/delayed post tests, session assessments, instructors' logs and session artifacts (task sheets, explore papers and video tapes). Data analysis will focus on the development of learning trajectories to be used as models of the progress made by students as they construct equivalent fraction understanding through the use of virtual and physical manipulatives. Not only will the overall effect of equivalent fraction learning be determined from the analysis of pre and post treatment data, but the researcher will also analyze data at the concept level, the individual lesson level, thus making it possible to identify effects of manipulative tool on the spectrum of students' development of equivalent fraction understanding.

Participants will be selected through a screening process. All students in the fifth grade classes of the participating schools will complete an equivalent fraction pre test. Students, except those participating in special education services for mathematics, scoring 70% or lower on the pretest, will be invited to participate in the intervention. Using an ability stratified method, students will be assigned to three groups, virtual manipulatives alone (VM), physical manipulatives alone (PM) or virtual and physical manipulatives combined (PM/VM). Intervention instruction groups will consist of two to four students. Participants will receive 5 days of equivalent fraction instruction, during which they will use their assigned treatment tool. Daily sessions will be approximately 45 minutes in length. The same lesson structure and content will be used for all three treatment groups, with minor adaptations made for tool differences. Each lesson will consist of five phases; pre-assessment, explore, apply, practice and lesson assessment. The lesson assessment consists of two parts; three lesson concept questions and eight cumulative fraction knowledge assessment questions. Lessons will be videotaped and all task sheets and student work will be collected for further analysis. At the conclusion of the intervention treatments students will complete a post test.

Analysis of the data will follow two paths: 1) analysis to identify differences in student achievement at the lesson, concept and summative levels, and 2) the development of learning trajectories showing knowledge and skill growth including the resolution of misconceptions and errors. The learning trajectories will be compared to identify differences related to manipulative type.

The following hypotheses are anticipated outcomes of this intervention study:

- Differences in the type of manipulative (physical or virtual) used creates differences in student's learning trajectories of equivalent fractions.
- Affordances of manipulative (physical and virtual) use are specific to each concept within the content domain of equivalent fractions.
- The type of manipulative (physical or virtual) used in instruction affects the occurrence and resolution of misconceptions and errors which are frequently experienced by students in their development of fraction understanding.

When developing intervention curriculum, teachers and curriculum developers must make important decisions about the type of instructional tools to be used. Although the research literature indicates that both physical and virtual manipulatives are effective tools of instruction, research showing the affordances of each manipulative type for the learning of specific concepts is limited and does not answer the question of how to effectively combine the use of physical and virtual manipulatives in intervention instruction. The purpose of this study is to provide teachers and curriculum developers of intervention instruction, research that can be used in their selection of manipulatives in the different phases of students' equivalent fraction learning trajectories.

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Workshop title: A new rational approach to the teaching of trigonometry in schools and colleges

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Workshop summary: I will outline a new approach to the teaching of trigonometry and geometry in schools and colleges. This approach has much in common with the work of the ancient Greeks, in paritcular Euclid's Elements. The workshop will introduce you to the main concepts of rational trigonometry, which are purely algebraic and much simpler than conventional trigonometry, and show you how to apply it in a variety of practical situations. The series of YouTube videos WildTrig found at user njwildberger goes into a lot more detail, this workshop can be viewed as an introduction to that series.

Comprehensive indicators of mathematics understanding among secondary school students.

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Abstract

The Malaysian science and mathematics curriculum has undergone several significant changes within the last two decades. Correspondingly, the approach to learning and teaching science and mathematics has also changed drastically. From the perspectives of science and mathematics education, the teaching of science and mathematics has shifted from the normative to descriptive (naturalistic) view of mathematics. That is, from the absolutist (behaviorist) tradition to the constructive tradition. However, what students have actually acquired in terms of problem solving, science process skills, communication, reasoning, thinking skills and abilities in seeing the interconnectedness of ideas, as stated in the mathematics and science curriculum are still not clearly articulated or defined. Therefore, there is a need to study what our students have actually acquired based on the above aims (or standards). Nevertheless, if these standards are not achieved, there is also a need to study the gap that exists between those ideals and those attained by our students. This paper reports the research study that aims to identify the levels of mathematics understanding amongst secondary school students, related to their abilities in terms of problem solving, communication, understanding the interrelatedness of mathematical ideas, and mathematical reasoning. It takes a comprehensive look and simultaneously explore into students' attainment both in terms of skills and levels of understanding using the School Science and Mathematics Indicators Program (SSMIP) designed by the research team. The methodology and procedures of the research project consist of document analyses of curriculum materials and guidelines to produce some indicators on the levels of understanding students are expected to attain as they proceed through the schooling system, conducting the tests to be used in describing the levels of achievement in specified subject areas, conducting task analyses based on the questions designed by the research team and finally conducting in-depth clinical interviews on selected students. Started in June 2010, the research is still ongoing and expected to be completed in another year. Initial findings indicated that there seems to be significant differences between curriculum expectations and students' levels attainment and understanding as defined by the curriculum standards.

Introduction

Within the perspectives of science and mathematics education, the teaching of science and mathematics has shifted:

- From the perspective normative to descriptive (naturalistic) view of mathematics.
- From the absolutist tradition to the constructive tradition (from the behaviorist to the constructivist approach).

One can safely conclude that the approach to learning and teaching of mathematics has changed drastically (Noor Azlan Ahmad Zanzali, 1995) over the last five decades. Thus the question of how much the students have benefited from these improvements in the curriculum is relevant. Several studies that aimed to look at what the students have acquired from the curriculum have been conducted. These studies, however, have been generally looked at skills acquired at specific area or level and thus are limited in scope.

This proposed study aims a more comprehensive look and simultaneously probe into students' attainment both in terms of skills and levels of understanding. The inquiry into and creating the School Science and Mathematics Indicators Program (SSMIP) will produce comprehensive and computerized guidelines on school-leavers achievement indicators in both science and mathematics. Potential users will include all higher institutions of learning both public and private institutions, and individual science and mathematics educators. While this type of school achievement indicators are quite common in developed countries, they are, however, new in the Malaysian scenario.

The basic principles of assessment

The word assessment refers to the process of collecting and using evidence about students' learning. Assessment and evaluation both describe the processes of collecting and interpreting evidence for some purpose. They both involve decisions about what evidence to use, the collection of that evidence in a systematic and planned way and the interpretation of the evidence is to produce to produce some of judgment (Harlen, 2007, Harlen2008, Khodori, 200; Salvia, J. &Ysseldke. J. E. 2001).This description is illustrated by the following diagram:

In a nutshell "Educational assessment is formal attempt to determine students' status with respect to educational variables of interest (Popham, 2006; pg 6)

In recent years educators have been urged to broaden their conception of testing so students' status determined via a wider variety of measuring devices – a variety extending well beyond the traditional paper-and-pencil tests. Thus they are many worthwhile learning outcomes not best measured by paper-and-pencil tests. Assessment is a broader descriptor of the kinds of educational measuring teachers do – a descriptor that, while certainly including traditional pap er and pencil tests, covers many more kinds of measuring procedures.

Noor Azlan Ahmad Zanzali (2005) emphasized assessment must be based and address several critical issues of teaching and learning. They are

- Issue 1: Underlying assumptions about the philosophy and goals of the curriculum
- Issue 2: Assessment must be in consonance with current learning and instructional considerations
- Issue 3: Specifications of performance standards
- Issue 4: Developing authentic tasks
- Issue 5: Assessment should measure status and growth
- Issue 6: Scoring and what form?
- Issue 7: Reporting –making public

Objectives of the Research

This research program aims to identify

- the levels of mathematics understanding amongst secondary school students, corresponding to their levels of schooling students' abilities to acquire a variety of mathematical concepts,

- The abilities to carry out a variety of mathematical procedures and to use them to solve problems in both familiar and unfamiliar situations.
- The skills and understanding that students are expected to acquire as stipulated by the curriculum expectations.
- The levels of problem solving abilities, communication, understanding the interrelatedness of mathematical and science, and mathematical and scientific reasoning.

A complete guideline on the levels of students' understanding in science and mathematics according the levels of schooling will be produced.

For each component (mathematics and science) students will be assessed at three levels. Each level is related to the understanding of concepts, application, and problem solving in the respective content areas.

- Level 1 – describing the early stages of mathematical and scientific knowledge typical in a secondary school.
- Level 2 - describing the mathematical and scientific knowledge acquired in the intermediate years of secondary schooling.
- Level 3 - describe knowledge and skills acquired by students who have completed the full range of mathematics and scientific courses typical of a secondary school education

Methodology

The research team will consist of 4 groups, each looking at the areas of mathematics, physics, chemistry and general science areas.

The methodology and procedures of the research project are as follows:

1) Document analyses of curriculum material and guidelines to produce guidelines on the levels of understanding students are expected to attain as they proceed through the schooling system
2) Designing the tests to be used in describing the levels of achievement in specified subject areas.
3) Conduct task analyses based on the questions designed in (2).
4) Conducting in-depth clinical interviews on selected students.

Both the quantitative and qualitative methods will be used.

- **Quantitative Method**: Sets of mathematical questions to be answered by students for each level. Coding procedures will be used to assess levels of understanding. Tests of questions will be designed by each group.
- **Qualitative Method**: Qualitative procedures such document analyses, interviews, observations, task analyses and small group interactions of selected of students

Initial Results:

At this juncture, the process of collecting is still at the initial stages.

Document analyses are seen through the content, the psychological and the pedagogical perspectives. Our initial analyses do indicate that the intended curriculum places heavy emphasis on naturalistic view of mathematics based on constructivist nature of teaching and learning. The use of problem solving, communication, integration and reasoning of mathematics ideas are heavily emphasized. Two questions need to be addressed. First are the elements emphasized in teaching and learning? Second, do the students attain those elements as they go through the teaching and learning processes? The answers to the first questions are discussed in my earlier papers. This research attempts to address the second question

We conducted a qualitative survey on the views and attitude of the students, who are our subjects. Initial findings indicate that they do attain above average attitude as expected by the curriculum.

The results of other procedures is still being conducted and we hope to be able to collect and analyze at the end of the year,

Conclusion

The need to assess students' achievements in terms of the elements as emphasized curriculum, but not evaluated by the paper and pencil tests in the high-stake public examinations, is still very important. This will indicate the attainment of students as expected and regarded as the key elements of learning mathematics by the curriculum.

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The Use of Graphic Organizers to Improve Student and Teachers Problem-Solving Skills and Abilities

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Abstract
This paper reports on the use of graphic organizers to improve student mathmatical problem solving. Graphic organizers are visual and graphic displays that spatially depict the relationships between facts, terms, concepts and ideas within a learning task. Graphic organizers have widespread, and successful use in the areas language arts and special education in communication skills and comprehensions abilities (Hall & Strangman, 2008; DiCecco & Gleason, 2002; Goeden, 2002; Griffon, Malone, & Kameenui, 2001; National Reading Panel, 2000; Ritchie & Gimenez, 1996; Robinson, 1998). In this manuscript, I coalesce the findings of three separate, but related studies using a graphic organizer, *four-corners-and-a-diamond,* specifically designed to aid students in mathematical problem solving. Results found elementary teachers' comfort and self-confidence levels increased dramatically. Results also found third, fourth and fifth-grade students results on open-ended measurement problems increased. Lastly, results found significant improvement on open-ended geometric and algebraic problems with sixth, seventh and eighth-grade students.

Introduction

Think of how you would begin to put together a commercial bathroom vanity cabinet kit. Would you first read the instruction manual? Or would you check the inventory first to see if all parts were included? Would you begin by studying the picture of the completed cabinet? Would you just ask for help from someone more knowledgeable? Would you not even try? Your response to the problem situation above is analogous to student reactions to problem solving. Your previous experiences greatly influence your persistence and approach to a problem. Similarly, we have to accept and utilize student tendencies to increase their problem-solving skills and abilities.

As problem solving is a major, *if not the major*, goal of learning mathematics, how do we assist students develop their skills and abilities (National Council of Teachers of Mathematics, 1989; 1995; 2000)? One approach is to teach Polya's four-step problem-solving heuristics, namely first understand the problem; devise a plan; carry out the plan; and look back (Polya, 1944). However students, and teachers, many times misinterpret the four-steps to be a linear step-by-step procedure, not a process. So teachers and students try to make problems into sequential procedures, which does not work with most problems. And students quit working on problems if they get stuck on a step. This manuscript describes a specific approach to mathematical problem solving derived from research on reading and writing pedagogy, specifically, research indicating that students use graphic organizers to organize their ideas and improve their comprehension and communication skills (DiCecco & Gleason, 2002; Goeden, 2002; Griffon, Malone, & Kameenui, 2001; National Reading Panel, 2000; Robinson, 1998).

Graphic Organizers

Graphic organizers are visual and graphic displays that spatially depict the relationships between facts, terms, concepts and ideas within a learning task (Ellis, 2004). Graphic organizers have widespread, and successful use in the areas of language arts and special education in communication skills and comprehensions abilities (Hall & Strangman, 2008;

DiCecco & Gleason, 2002; Goeden, 2002; Griffon, Malone, & Kameenui, 2001; National Reading Panel, 2000; Ritchie & Gimenez, 1996; Robinson, 1998). Graphic organizers allow (and even expect) the student to:

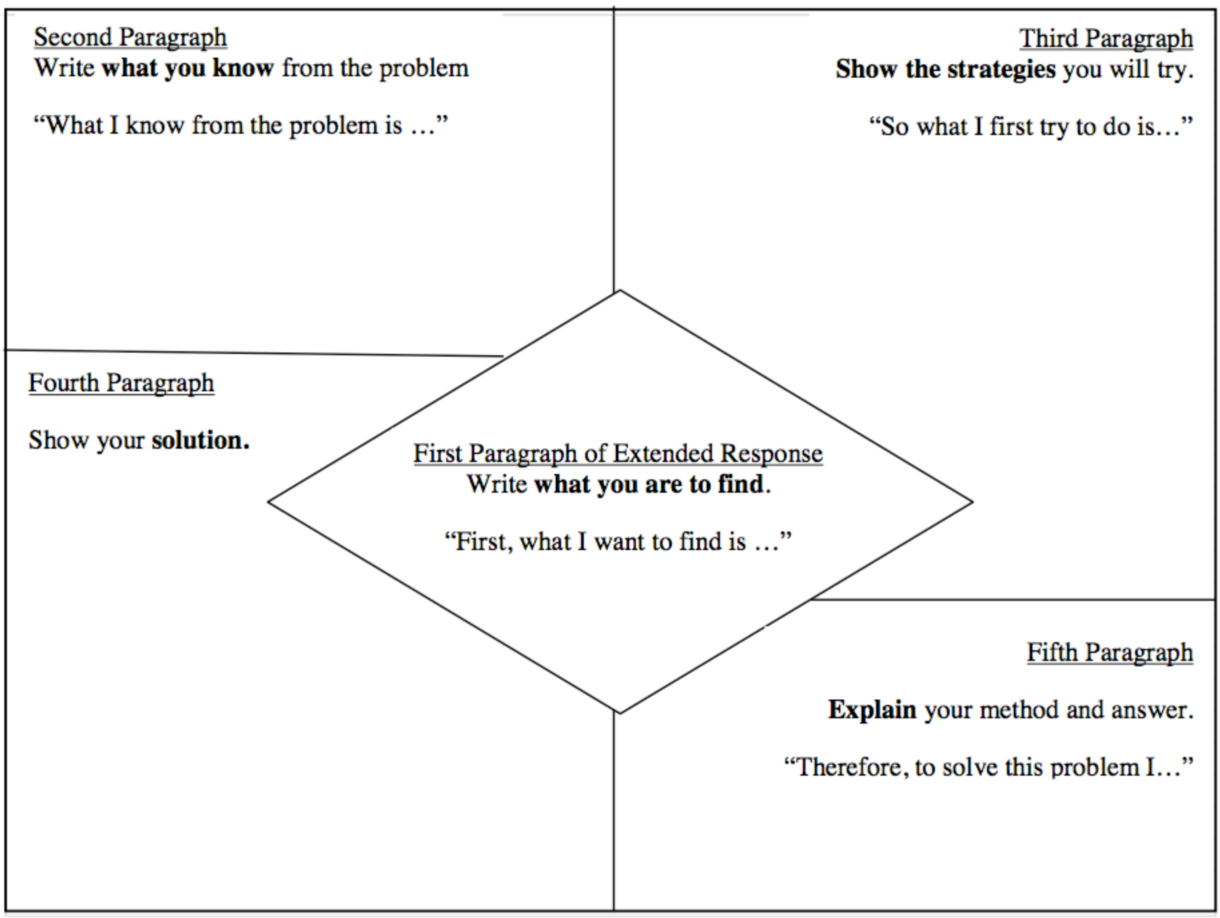
- sort information as essential or non-essential;
- structure information and concepts;
- identify relationships between concepts;
- organize communication about an issue or problem; and
- allow students to utilize their previous tendencies and experiences as a starting point of the problem-solving process (Zollman, 2011; 2009a; 2009b).

Figure 1 depicts the *four-corners-and-a-diamond* mathematics graphic organizer. This graphic organizer is modeled after a *four squares* writing graphic organizer described by Gould and Gould (1999). This *four-corners-and-a-diamond* mathematics graphic organizer (Zollman, 2011; 2009a; 2009b) has five areas: What do you need to find? What do you already know? Brainstorm possible strategies to solve this problem. Try your ways here.

What things do you need to include in your response and what mathematics did you learn by working this problem?

Figure 1

Four-Corners-and-a-Diamond Mathematics Graphics Organizer



The *four-corners-and-a-diamond* graphic organizer encourages students to begin working on a problem before they have identified a solution method. The extended-response written answer is not begun until there exists information is in all five areas. As in the four square writing method, students then organize and edit their thoughts by writing their solution in the traditional linear response, using connecting phrases and adding details and relationships. For the extended-response write up (as used in large-scale assessments) students usually first state the problem, identify the given information, next they propose methods for solving the problem, then show their mathematical work procedures, and finally present their final answer, conclusions and learning. (Zollman, 2011; 2009a; 2009b).

Methodology

These one-year research studies were initiated to improve student mathematical problem solving. There were 240 elementary and 186 middle school students in the studies. The elementary students utilized graphic organizers on the topic of measurement (area and perimeter); the middle school students utilized graphic organizers on the topics of algebra and geometry. None of the students, or their teachers, had previous experience using graphic organizers in mathematical problem solving.

Results

Using the graphic organizer had a substantial impact on the elementary teachers practices. Initially 50% of the elementary teachers reported being uncomfortable teaching problem solving. Afterwards, 100% of the teachers reported being comfortable (80% being very comfortable) teaching problem solving with their students. All the teachers subsequently reported changing their instruction to include much more writing in mathematical problem solving.

On achievement in area and perimeter problem-solving items, the 3rd grade students rose from 55% to 78%. The 4th grade students increased from 40% to 52%. And the 6th grade students improved from 40% to 62% from pretest to posttest scores.

Likewise, the middle school students also significantly improved their achievement scores from pretest to posttest on algebra and geometry problem-solving items. On a four-point scoring rubric, the students went from a pretest score of 0.83 to a posttest score of 2.93 on mathematical knowledge (Z-score -19.8849, p<0.001; effect size 1.94). On strategic knowledge, the student scores rose from 1.52 to 2.79 (Z-score -11.66049, p<0.001; effect size 1.16). Similarly, their explanation scores grew from 0.83 to 2.67 (Z-score -15.3907, p<0.001; effect size 1.51).

Teachers studying their own students reported the use of graphic organizers in mathematical problem solving to be efficient and effective for students at all achievement levels. Teachers saw students who normally would not attempt open-response problems now had partial written solutions. Students who normally did well on problems now had an efficient method of writing and communicating their thinking in logical, complete arguments.

Conclusions

These research studies found that the proper use of the mathematics graphic organizer *four-corners-and-a-diamond* to be an extremely useful instructional method in the middle school mathematics classroom. Students should improve their problem solving abilities with any instructional intervention. The teachers specifically attributed the increase in student performance in mathematical problem solving to the student use graphic organizers. Lastly, the teachers also changed their instructional pedagogy during the study. This may be due to teachers viewing the *four-corners-and-a-diamond* graphic organizer a comfortable method to include writing-across-the-curriculum into their mathematics lessons, and subsequently including more extended-response problem solving into their lessons.

Teaching about problem solving in a hierarchy of procedural steps is neither efficient nor effective. This study concurs with other problem-solving research – teaching *via* a problem solving process is the crucial instructional development (Lester, Masingila, Mau, Lambdin, dos Santon, & Raymond, 1994).

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