syntactical limiters include braces, brackets, wildcards, quotes, and excluders. Each of these narrows the focus of the search.

Along with search engines, there are software applications known as "web crawlers" that locate and make available current events based on selection criteria. These software applications require more programming expertise and are not without cost and thus upgrades, and revisions. These applications are utilities that extract URL, meta-tags, plain text, page size, last modified date value from Web sites, Web directories, search results and lists of URLs from file. Thus, they are heavily dependent upon the depositor of the information's creativity in labeling or timeliness in depositing information.

Each of the humanistic categories must be matched with current events. In other words, the current events that are found using Internet searches and/or web crawler applications are matched with the humanistic categories. Three current examples are as follows:

HUMANISTIC MATHEMATICS CATEGORY: Mathematics of Finance and Economics
CURRENT EVENT: Anthropogenic Oil Spill in the Gulf of Mexico
MATHEMATICS EDUCATION: Algebra
CONCLUSION: Students discover costs of real life anthropogenic societal events

HUMANISTIC MATHEMATICS CATEGORY: Mathematics of Change (and Growth)
CURRENT EVENT: Haiti Earthquake
MATHEMATICS EDUCATION: Pre-calculus
CONCLUSION: Students discover the devastation of an earthquake in relationship to seismic releases of energy/power.

HUMANISTIC MATHEMATICS CATEGORY: Mathematics of Symmetry (Shape and Form)
CURRENT EVENT: State of Glaciers in High Asia
MATHEMATICS EDUCATION: Geometry as Fractals
CONCLUSION: Students discover mathematics relationship to nature.

These three societal events are examples of how mathematics can be taught humanistically. Thus, the curricular and pedagogical environment is alive with events that have both affect, influences on emotion and culture and effect as it demonstrates changes that occur in human situations and circumstances that are of global importance. Thus, students become engaged in and aware of mathematics' relevance, its humanity, and its global ability to make current events a mathematics classroom common experience.

Exploring the challenges of teachers' and learners' understanding of solution strategies using whole numbers

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Abstract
This paper is a qualitative study and reports on work with both teachers and learners in nine schools in the Eastern Cape Province of South Africa on mathematical reasoning and problem solving, specifically related to developing computational fluency with whole numbers. As a facilitator who has been involved with the upgrading of mathematics in-service teachers, the study involved both teachers and learners, being able to calculate in multiple ways by using a variety of solution strategies in order to solve problems. Fluency require of both teachers and learners more than just memorizing a single procedure. It rests on an understanding of the meaning of the operations and their relationship with one another. The work with learners and teachers involved them working individually on solving contextual word problems. The study showed that in most tasks, learners relied heavily on procedural understanding at the expense of conceptual understanding. Traditional standard algorithms appeared to have been learned in isolation from concepts, failing to relate them to understanding.

Introduction
The rationale for choosing whole number computation and studying the solution strategies of teachers and learners was triggered by the poor TIMSS results and my interest in the problem-centred approach that I had used in my own class.
The Third International Mathematics and Science Study (TIMSS, 1996) and the (TIMSS –R, 1998) had found that the performance of South African students was placed at the bottom of a list of over 41 countries that participated in the study. Having worked with many in-service teachers and learners over a number of years, trying to developing their computational fluency and reasoning skills, involving the four basic operations, I have come across a plethora of diverse solution strategies that both learners and teachers use in solving these problems.

As both studies from TIMSS showed no real difference in the performance of learners, a study on whole numbers was developed to ascertain whether learners and teachers were able to use different solution strategies when solving these problems. Encouraging learners to develop and use their own solution strategies is regarded as consistent with a move from being 'teacher-centred' to a more process driven problem solving 'learner-centred approach (Southwood & Spanneberg 1996). The Revised National Curriculum Statement (2002:1) reaffirms this when it states, "the outcomes encourage a learner-centred and activity-based approach to education." With the learner at the centre of the learning process, much more emphasis is placed on learners developing conceptual understanding and learning computational skills (Bransford, Brown & Cocking 1999).

Context
The Rhodes University Mathematics Education Project (RUMEP) is an independently funded non-governmental organisation (NGO), linked to the university, with the specific aim of improving the quality of teaching and learning mathematics in schools. The Project had its beginnings, on an informal, regional level, in 1983 and grew so rapidly in stature and effect

that it became a teacher development institute of the university in 1993, as a formal numeracy project.
A major aspect of the project is the upgrading of both primary and secondary mathematics teachers. These accredited courses represent a direct response to the challenge of reaching the many teachers in deep rural areas who have not had access to in-service training, in the past. The ACE programme is offered through two blocks of university based tuition over two years of part-time study. During these contact sessions, teachers are immersed into all five learning outcomes in Mathematics developing both their mathematics content knowledge and pedagogy.

Theoretical perspective

This study is underpinned by the social constructivist model (Vygotsky 1978) and the problem-centred approach. The social constructivist philosophy emphasises language, culture and the social milieu. In the social constructivist model of the teaching-learning process, four key elements interact and affect each other – the learner, the teacher, the task and the context. As knowledge is socially constructed, the classroom is seen as an extension of the learners' environment. That knowledge which the learner knows is built on the existing knowledge gained through social interactions other than those found in the formal classroom.
Problem solving is consistent with the constructivist philosophy and my submission that learners are encouraged to invent their own procedures as advocated by (McClain & Cobb 2001) so that learners build their own meaning for themselves in order to better understand the concepts and skills of mathematics is pertinent here.

Methodology

The study is qualitative in nature and lies in the interpretive paradigm. It deals with individuals and is interested in describing processes rather than just an outcome or end result (Cohen & Manion 1994; Mwira & Wamahui 1995). Grounded theory was the underpinning methodology selected for the study. It was postulated by Glaser & Strauss (1967) and appropriate for the study as it was a small scale investigation into the solution strategies of learners. The initial justification for this research method was that I intended to develop categories of children's solution strategies. However, when examining the strategies, I found that the level of each task only allowed me to go as far as the first type of coding, namely open coding.
To ascertain the problem solving ability of learners and teachers and their ability to use different solution strategies, nine schools (both teachers and learners) in the Northern region of the Eastern Cape Province took part in the study.

The research instruments consisted of a test, individual clinical interviews with each of the nine learners and teachers and a structured teacher interview schedule.

The majority of the learners were isiXhosa speaking. However for transparency sake, all the questions were translated from English into Afrikaans and isiXhosa. The test consisted of 15 multiple choice type questions and 12 problem type word problems. Grade 7 learners were chosen because it was felt that learners at this age are able to articulate their thought processes and their communication skills are also sufficiently developed at this level.

My interest in the test was to find out how the learners and teachers confronted the problems and what strategies they had used which made sense to them. Further I was interested to see whether the solution strategies chosen, were the same or different from those their teachers had used.

The individual interview schedule was semi-structured in nature (Bogdan & Biklen 1992) and contained a checklist of suggested questions. However, not all questions were pre-determined. This kind of interview allowed me to have more flexibility and freedom to explore the solution strategies adopted by the participants and whether they could explain carefully what they had done. The goal and emphasis of using the semi-structured interview was to probe for understanding.

A short, additional structured interview schedule was drawn up asking participant teachers to comment on aspects of the test. The intention behind doing this was to gauge whether the tasks were too difficult for their classes and whether translating the tasks into their mother tongue had any effect on how learners approached them.

Interpretation and discussion of solution strategies

The TIMSS Curriculum Framework was used in this study to place each task into categories and into performance expectations. The performance expectation component refers to the cognitive dimension and describes the kinds of performance or behaviours that might be expected of learners. I shall discuss three whole number problems from the test.

Task 1

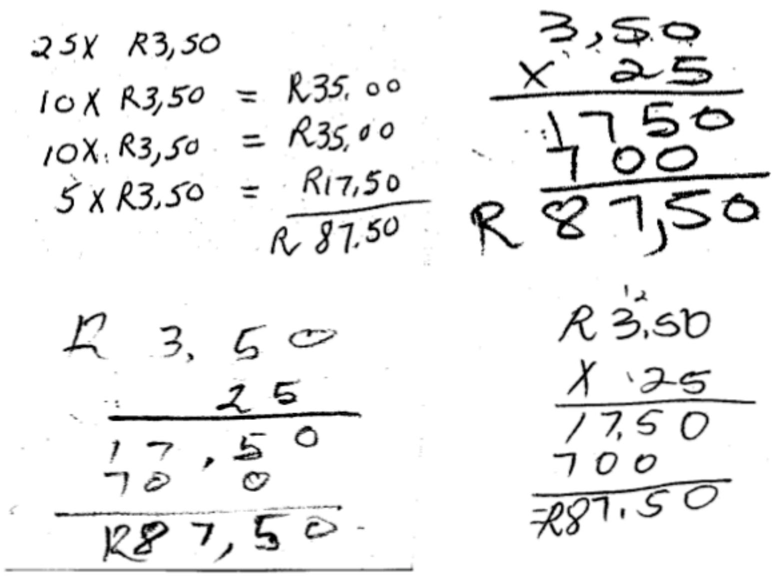
25 learners go on an outing to the beach. They each buy an ice-cream which costs R3, 50.
How much must they pay altogether?
Open coding was used and the following strategies were developed.

Learner	CS	IS	Strategies	Teacher	CS	IS	Strategies
1	√		Vertical algorithm	1	√		Vertical algorithm
2		×	Counting strategy	2	√		Decomposition of the multiplier
3		×	Vertical algorithm	3	√		Vertical algorithm
4	√		Mathematical model	4	√		Decomposition of the multiplier
5	√		Vertical algorithm	5	√		Vertical algorithm
6		×	Vertical algorithm	6	√		Fraction multiplication
7		×	Fraction multiplication	7	√		Decomposes multiplier
8		×	Counting strategy	8	√		Vertical algorithm
9	√		Vertical algorithm	9	√		Short method of X

Content area category	Number
Subcategory	Decimal Fractions
Subordinate subcategory	Properties of operations
Performance expectation	Using routine procedures
Subcategory	Performing routine procedures

Only 44% of the learners managed to solve this problem with the majority using the traditional vertical algorithm strategy. Only a few were able to explain the process involved. Many got lost along the way because they lacked the mastery of the multiplications tables. Strategies identified were: decomposing the multiplier, using the vertical algorithm, use of a mathematical model, a counting strategy and fraction multiplication. This example was correctly answered by all nine teachers (100%).

Teacher and learner strategies



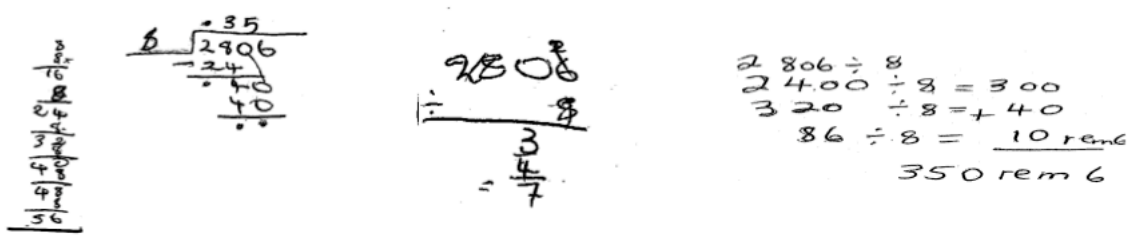
Task 2

Farmer Zodwa and his workers pick apples. They pick 2 806 apples and place them in packets with 8 apples to a packet. How many packets do they fill? Are there any apples left?

Learner	**CS**	**IS**	**Strategies**	**Teacher**	**CS**	**IS**	**Strategies**
1		×	Vertical algorithm	1		×	Vertical algorithm
2		×	Unclear strategy	2		×	Partitioned dividend
3		×	Unclear strategy	3	√		Partitioned dividend
4		×	Factors of 2 and 4	4	√		Partitioned dividend
5		×	Partitioned dividend	5		×	Vertical algorithm
6		×	Vertical algorithm	6		×	Vertical algorithm
7		×	Partitioned dividend	7		×	Partitioned dividend
8		×	Guess work	8		×	Partitioned dividend
9	√		Vertical algorithm	9	√		Vertical algorithm

Only 11% of the learners solved this problem while only 33% of the teachers accurately answered it. Most learners relied on memorisation but got stuck halfway as they tried to solve the problem. Teachers still teach division as a ‘goes into’ model which we know is an ineffective model as digits are treated separately (Fosnot & Dolk 2001). Teachers tend not to emphasis the reciprocal relationship between multiplication and division or what the meaning of a remainder is.

Learner and teacher strategies



Task 3
Mrs Khumalo has a bag of sweets to give to her Grade 7 classes. She gives the first class 167 sweets and the second class 248 sweets. She then has 35 sweets left in her bag. How many sweets were in her bag at the start?

Learner	**CS**	**IS**	**Strategies**	**Teacher**	**CS**	**IS**	**Strategies**
1	√		Vertical algorithm	1	√		Vertical algorithm
2	√		Decomposition of numbers	2	√		Vertical algorithm
3	√		Vertical algorithm	3	√		Vertical algorithm
4	√		Vertical algorithm	4	√		Vertical algorithm
5	√		Vertical algorithm	5	√		Vertical algorithm
6	√		Vertical algorithm	6	√		Horizontal and vertical algorithm
7	√		Vertical algorithm	7	√		As above
8	√		Vertical algorithm	8	√		Vertical algorithm
9	√		Vertical algorithm	9	√		Vertical algorithm

There was a 100% success rate with this problem. This shows that some teachers spend more time teaching this operation at the expense of the other operations.

Learners' strategies

Findings

Most of the solution strategies that the learners used were straight forward procedures that they had learnt. They relied mostly on procedural understanding at the expense of conceptual understanding. The solution strategies of whole numbers adopted by the learners in the study were similar to the whole number solution strategies used by their teachers. A few teachers and learners did employ their own constructed solution strategies. They were able to make sense of the problems and to 'mathematize.' Language played a role in that learners sometimes struggled to communicate their thought processes in a coherent manner, even though the problems had been translated into mother tongue as well.

Challenges

Teachers should ensure that learners be given sufficient opportunities to solve problem-solving type word problems. Besides addition, the other whole number operations must also be given the recognition they deserve. In each lesson planned, time should be given to allow learners to master basic skills to enable them to use this information when planning their solution strategies. The need to take cognisance of language problems is a further aspect that teachers need to take into consideration. They should note that if word problems are written in English and mother tongue, reading, comprehension and encoding errors would be lessened. To make sense of the mathematics, it is important for English second or third language speaking learners to be allowed to rely on their own language, since background knowledge is the basis of any learning process in mathematics. The implication is that teachers need to spend time allowing learners to look at the processes involved in arriving at a solution, rather than just focusing on the solution. By allowing learners to develop their own solution strategies, teachers will be sensitized to the thinking and reasoning of learners as they strive to make sense of the mathematics. Teaching algorithms as fixed procedures restricts the thinking ability of learners to reason, communicate and consequently, their ability to do mathematics. Teaching for understanding should be emphasized at the expense of 'teacher taught procedures'. The idea of being computationally fluent will result in learners being able to explain and analyse their methods and this will result in their developing efficient, accurate and flexible strategies. As their knowledge of different strategies grows, so does their computational fluency.

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Stepping into Statistics: Providing a Head Start for students

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Abstract

The major aim of this paper is to discuss the learning design of a head start introductory statistics module made available to students on-line. It explores the combination of different kinds of resources in particular genres of video to support learning and the learning of discipline content and processes. Different mechanisms for facilitating communication between students, and production issues and issues in merging the head start program with the introductory statistics subject.

Introduction

For the past 15 years the development of an introductory first year level university statistics subject, STAT has been guided by a quality cycle of planning, acting, reviewing and improving. Each year students valuing of resources has been evaluated in terms of their usefulness in helping students learn and understand statistics. Resources that have not been considered valuable terms of helping students learn have been reworked or replaced. For some years students have placed a high value on their laboratory manual, lectures, assessment, provision of marking criteria and fully worked solutions. Innovations have included the collection and use of real data and working topics of social significance. Students have since the 1990s have been provided with access to all resources and communication with other students on line through an E-Learning system.

The subject fits a traditional subject teaching profile in that it provides three weekly lectures (numbers 80-300 students) and two hours in a laboratory class (numbers 20-30 students). In this subject lectures are highly interactive even with large class sizes. New topics are started with an activity to elicit student ideas and these are then refined and formalized in terms of the appropriate statistical language and theory. Students work with a mix of real data gathered by them selves and data supplied by others. Strategies for learning and learning issues that arise are also explored. In recent years there has been an increasing numbers of students using online resources rather than face-to-face. In 2010 students with no previous failures were allowed to complete their laboratory work at home or in their own time. Only the lecturer of the subject is avaible outside of class time to assist students.

In the search for the best approach a variety of assessment tasks have been employed over the years. These have included assignments involving the collection and analysis of data, portfolios, summaries, tests and presentations to class and final examinations. Different weightings have been applied. The approach that has been adopted involves a final examination worth 50 per cent, a presentation where the student in a pair collects their own data and makes a presentation to class worth 10 per cent and four competency "tests" commencing in even weeks from the fourth week of a thirteen week session. Typically only the first of these tests is held within the class and the remainder completed by the students in their own time. The "tests" involve analysing data and working with theory, like an assignment but the allowed time is a few days. The first tests have a rapid turn around in terms of marking and students are provided with fully worked solutions. In odd weeks the students who do not gain 70 per cent are required to undertake a similar test, and those not passing the second test are identified as "at risk" students. The "at risk" students are given more support through directing them to resources, interviewing the student, or direct comments on the work. In the final week, make-up tests are available to all students.

In 2009 the online resources were expanded to include a variety of video clips relating to different topics. Early approaches to providing resources to students included grouping all similar resources for example lectures, video clips and data.

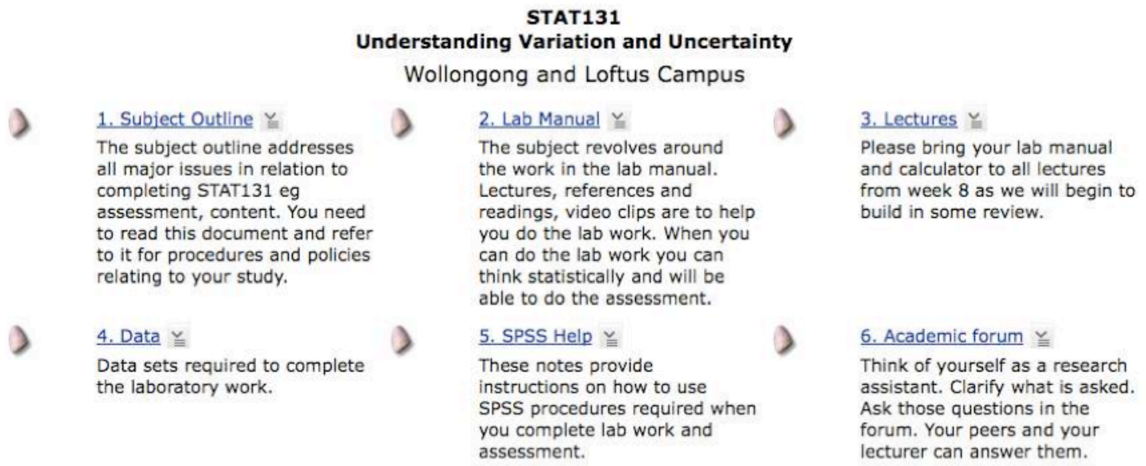


Figure 1. Organisation of resources for students by common category

An alternative approach has been to Group resources by week as in Figure 2.

Figure 2. Organisation of resources by week.

A third option has been to group both by category of resources and by week.

Student change evaluations revealed that when the students used the video clips they were considered extremely useful in helping students learn and understand statistics. However not all students *found* the video clips. As a consequence in 2010 the innovation focus was on the learning design itself. In this introductory subject students were provided with a *learning design visual sequence* illustrating the "chronology of tasks, resources and supports" (Aghostinho et al, 2009). Originally conceived of as a learning design from the perspective of the teacher, in this subject it was used to communicate with the students, the tasks they had to complete, the primary resources and support materials if they needed additional resources, see for example Figure 3.

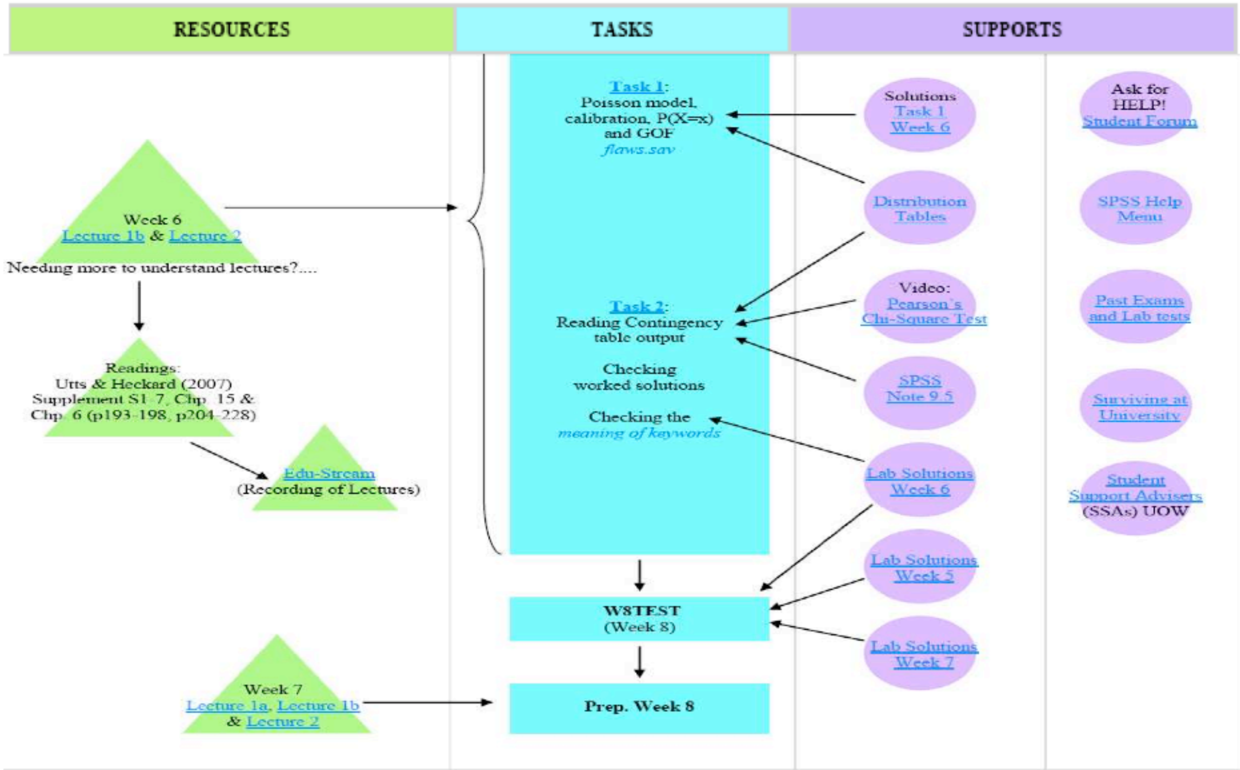


Figure 3. An extract of a learning design visual sequence for a week of STAT (Porter & Baharun, 2010)

The visual learning design maps linked all student supports, resources and tasks. The competency based assessment system allowed early identification of students at risk, that is those who did not satisfactorily complete any test/re-test sequence. Video clips were used to provide students with discipline support and this was supplemented by direct communication, both email and in-person, between the lecturer and at risk students. The map provided a clear picture of what was required, when and what help was available.

One outcome of trying new approaches is that as a teacher one can see what was previously not evident. It makes no sense that students would enroll in a subject, pay fees and walk away with a zero, low mark and a fail grade. There are students who are inactive, do not come to class and do not submit assignments that feasibly deserve such a grade. For some years there has been an increase in zero/low mark fails typically interpreted as students who cease to be active in the subject but who do not withdraw. How to engage these students has inspired innovations in the statistics subject for the past fifteen years. As teachers we have worked with the mantra "If we can get students to class we can assist them through the learning." At a graduate level the innovations with on-campus, distance and international students have been highly successful with many learning outcomes including a low failure rate (Baharun & Porter, 2009). At the undergraduate level the learning design map teamed with competency-based assessment highlighted another group of students, seemingly with learning difficulties, namely students who attend lectures and laboratory classes, are active in class answering questions, but who have difficulties submitting assessment even after guidance and with the possibility of re-submission.

Non-submission of assessment is the key indicator of pending student failure. For this group of students being able to communicate what they know through the assessment system appeared problematic. However, at the end of session students with the exception of one, unanimously endorsed the assessment system. Contrasting with this, approximately two-thirds of completing students indicated that they would like access to a head start program. Students wanted more time to cover the curricula. Hence, the development of a Head Start module.

Head-Start Module

The Head-Start module implements a draft-final assignment model rather than the test-retest model of STAT, potentially a more positive experience than “failure” feedback repeat test. This should assist more fragile learners. The module is accessed on-line after students have enrolled in the subject. For students electing to complete the module, they will have approximately three weeks head start and will be able to complete a first piece of assessment which can substitute for the first in-class test in the subject proper. The first two weeks of session then become consolidation of the Head Start program (See Figure 4.)

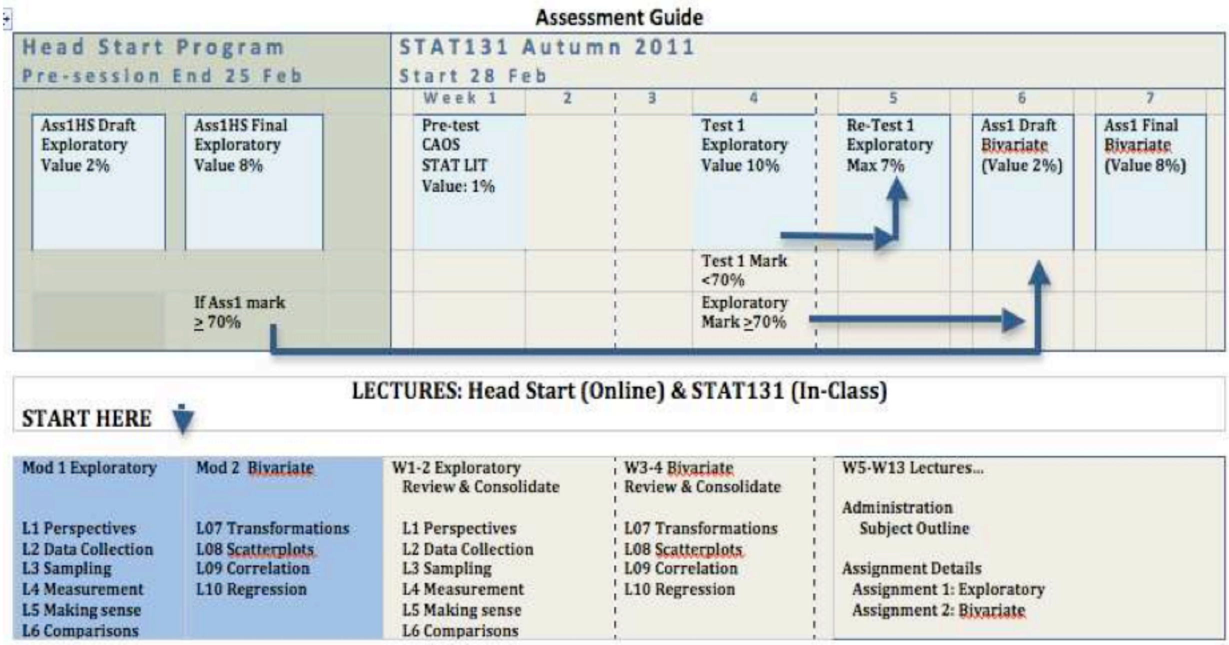


Figure 4. Merging the head-start module and STAT subject

From the outset a pattern is established for the Head Start on-line lectures.

1) Orientate the learner with text and video

> ***Orientation:*** *In 2010 approximately two thirds of the students completing STAT131 indicated that they would like to be able to commence STAT131 before the official start to classes. They felt too rushed to take on board the new skills. The Head Start Program Stepping into Statistics allows students to …*

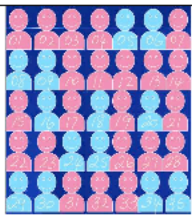
2) Provide an activity to get students communicating with each other

Task 1 Meeting colleagues and identifying your pre-conceptions about Statistics Introduce yourself to other students in the academic forum. Without looking at any text or what other students say write down: What you think statistics involves? Do you have any concerns about studying statistics? What you are looking forward to in studying statistics? See what others have to say. Review your answers at the end of session.

3) Provide an activity to address a relevant learning issue

Task 2 Experience the rush Many students find complex, multistep math problems particularly difficult. The problem set on the next page is designed to evoke in you the feelings a student might feel working out a problem that requires combining and using several mathematics skills in a short period of time. Give yourself **two minutes** to solve all three problems. **Adapted from** **http://www.pbs.org/wgbh/misunderstoodminds/math.html**	Follow all four instructions below to solve each of the three problems. Record your answers. A. Multiply the third number in the first row by the seventh number in the third row. B. Add this result to the fifth number in the second row. C. Add to this total ten times the fourth number in the third row. D. Subtract the eighth number in the first row from the result. Problem 1: 6 5 8 7 4 5 6 8 4 3 2 1 9 5 6 4 2 1 6 5 1 5 1 3 2 3 5 etc Debrief

4) Provide a sequence of tasks for the students to complete that cause them to interact with the discipline issues, content and processes.

Task 3: Not comfortable accepting statistical results Many people are uncomfortable when research outcomes are based on a **sample** or portion of the entire **population** for which data on some **variable** are collected. The video clip has an exercise where you conduct a census. Do this and answer the following: Why it is often acceptable and maybe even better to collect data using a sample rather than a census?	
Task 4: Simple Random Sample For a sample to represent the population you must use a probability sampling plan. Here we have a population of 35 men and women, they represent your sampling frame. Your task is to take a simple random sample of size 7 using a random numbers table. What observations are in your sample? How did you get them?	 Population of 35 Random Numbers Table

Conclusions

At this stage evaluation remains at the design level. From the perspective of Boud and Prosser (2001) to have the potential for high quality learning the design must address engagement of learners, consider the implementation within the broader learning context, challenge learners seeking active participation supporting students ampliative skills and provide practice. The Head Start module relies heavily on a variety of resources combined with different types of activities to connect students with their discipline and everyday life decision-making. The video resources include orientation chats, worked examples to demonstrate calculations, theory snippets, stimulus materials, procedural matters such as taking a sample (Task 4) or using statistical packages and interpreting output . The opening forum communication, Task 1, sets the scene for students to begin to explore their perspective as to what statistics involves and this is followed by activities that challenge narrow definitions of what statistics is about. Activities such as those in Task 3 where students conduct a census and discover that they have probably not "counted" correctly and that maybe sampling is a good approach to collecting data are to engage students, through providing an activity that challenges their perceptions. Throughout the module students are provided with practice, working with data and theory and debriefing through video clips.

Students have only just commenced the head start module and further evaluation will be required to explore whether the uptake of the program and the benefits to students has warranted its development.

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Transforming Mathematical Tastes: a Twist of Lemon – or a Pretzel?

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Abstract

Since the early 1990s, the nature of students enrolling in universities around the world has been changing [(1)]. In New Zealand, the initial government-led focus was "bums on seats" with a drop in prerequisites, open entry and a rise in the average age of students. A subsequent focus, underway now, is "success and retention" – in order to attract the maximum funding, tertiary institutions need to ensure the students achieve. Meanwhile secondary school mathematics has been changing its attention from abstract to more utilitarian content [(2)]. When students arrive at tertiary institutions, many have insufficient mathematics understanding and skills to cope with the demands of such courses as economics and engineering. "Service mathematics" courses have been designed to counteract this lack of mathematical ability. This presentation considers two such courses offered at Bay of Plenty Polytechnic in Tauranga, New Zealand, taught in a manner mostly traditional for secondary but not tertiary level, but with a "twist", in an effort to bring students up to speed. A survey of the students over two years provides evaluations to complement this reflective overview.

Introduction

Bay of Plenty Polytechnic is a small institution, situated in Tauranga in the North Island of New Zealand, with approximately 8000 full and part time students. Tauranga (population 140000) is not a university city but the Polytechnic has degree agreements with several institutions across New Zealand. I am in my 12th year as a mathematics tutor and advisor attached to the Kahurangi Student Services department of the Polytechnic. During this time I have received two peer-nominated sustained teacher excellence awards (2009 and 2005) and one student-nominated exceptional adult-educator award in 2002. I work with students from all programmes where mathematics is involved. The range is broad; from learning multiplication tables and understanding division to calculus and complex numbers and often in context. I have a passion for mathematics; many of the students have a "maths phobia" and I have been particularly interested in analysing how best they learn. In researching this, I became aware of the "mathematics problem" which appears to be world-wide. Students have been arriving at tertiary institutions, enrolled in courses which require a reasonable level of mathematics competency, but without the necessary skills and often with negative attitudes. I have investigated within our Polytechnic with my two classes that I tutor to research how to best teach to alleviate this problem. In my presentation I will discuss my experiences, the papers I teach, and the results of surveys I have administered.

Transforming Mathematical Tastes: a Twist of Lemon – or a Pretzel?

"I hate maths. I can't do it. I'm only taking this paper because it's compulsory. So long as I get a C, I'll be happy", announced a student (I'll call her Marie) as she entered my management mathematics class for the first time. As I learned later, Marie was an A+ student in all her business, management and accounting subjects. What a tragedy that such an obviously bright person had such an attitude towards mathematics!

This attitude can often be traced back to one particular mathematics teacher at secondary or even primary level. Duncan remembered a teacher in year nine "torturing" him with his sarcasm when he gave a wrong answer; Lauren recalls her teacher telling her at the age of six to "go home and play with your dolls. You will never be good at maths". Yet both of these people now have high qualifications, but certainly not in mathematics. Of course, not all students dislike mathematics. In many cases the students arriving at tertiary institutions "like" mathematics but have either never advanced to any great level for a variety of reasons or they last attended school many years ago and have long since forgotten most of the mathematics they learned. Other students, more recently at secondary school, have become so dependent on graphics calculators (poorly taught and poorly used) that mathematics has become like the "Yellow Pages" – their fingers do the walking but their thinking brain is not involved. When they can no longer reach for their crutch, they topple. Added to this is a unit-based assessment system at secondary level which partitions mathematics and fails to produce a coherent overview. Secondary students can achieve sufficient credits, yet still be inadequately prepared for tertiary demands.

The "Mathematics Problem"

Students are enrolling in programmes such as business and engineering which require mathematics content well above their level of expertise. There have always been some students in this category but over the past 20 years the problem has grown exponentially. This "mathematics problem" is not confined to my institution, or to just New Zealand; it is of global proportions. In the UK in 1995, the Engineering Council [3] concluded that "students are now accepted on engineering courses with relatively low mathematics qualifications". In the U.S., also in 1995, a Survey on Remedial Education in Higher Education Institutions [4] found that 78% of higher institutions had found the need to offer remedial courses in reading, writing or mathematics. In 2004, also in the US, the Committee on the Undergraduate Programme (CUPM), reporting on four years of findings, discovered that twice as many students were enrolled in "remedial" or "introductory" mathematics courses compared with those in calculus or statistics courses [5]. They noted that the remedial courses were particularly challenging to teach because of the diversity of mathematical backgrounds of the students, who also brought with them negative attitudes created by past experiences with mathematics.

From a diversity of countries, e.g. Ireland, Australia and Canada, there is a body of literature that documents the "mathematics problem". Neither is it confined to English speaking countries. At the International Congress on Mathematical Education (ICME-9) held in Japan in 2000, a group which was very broadly international, identified as a serious issue the challenge posed by non-specialist mathematics students requiring service mathematics [6]. Here in South Africa, Engelbrecht and Harding in 2008 acknowledged that university lecturers could no longer assume that students have certain mathematics skills [7]. Also in South Africa, in 2010, Winnips, Brouwer and Mwambakana confirm there is a "mathematics problem" and have held workshops on using e-learning to improve the situation [8].

In New Zealand the problem has become even more pronounced because of a change in government policy. Originally the attitude was "get them off the dole and into education", the so-called "bums-on-seats" policy. The current government has refocused and the policy almost underway is "funding according to success and retention". In fact, the government now requires us to have a "good idea" that students have the capability of succeeding before they can be enrolled. The government has spent millions of dollars investing in the development of "The Tool" which diagnoses students' literacy and numeracy at an elementary level. The Tool is designed to (a) help prevent students from enrolling in courses for which they have little hope of success; and (b) indicating to both tutors and students the personal specific mathematics skills requiring extra attention.

Institutional Intervention

What steps have we been taking at Bay of Plenty Polytechnic to alleviate this "mathematics problem"?

Unfortunately, due to IT constraints, The Tool is not administered to students until after they are enrolled. However, difficulties and knowledge gaps are highlighted, tutors can become more informed about their students and assistance is offered by our Student Learning Centre (Kahurangi). The drawback of The Tool is that it is limited to an elementary level and assesses whether students have sufficient mathematics to succeed at levels 1 - 3 programmes such as carpentry and electrical level 3. It does not assess whether students have the skills to achieve at higher level Diploma or degree papers.

The current policy within Kahurangi is to focus on providing assistance within the classroom or in workshops specific to an identified problem area. There are fewer one-to-one appointments available due to resource limitations. The main focus is on levels 1 - 3 as per a government directive. This makes the two papers that I teach even more important in helping remedy the mathematics problem at higher levels.

Four years ago, as a mathematics advisor in Kahurangi, I supported students enrolled in a Management Mathematics paper which is taught under an articulation agreement with Waikato University in Hamilton, NZ. It became obvious that some of the students were inadequately prepared to meet the demands of the university paper, which itself was a "service mathematics" paper and had been designed specifically to support those students who needed a higher level of mathematics to cope with their business, economics or management degrees. Subsequently, I wrote a "Mathematics Bridging" paper, tailor-made for the Management Mathematics students, to be taught over three weeks at Summer School. Interestingly, although the paper targeted Management Mathematics students, it has proved to be suitable in content for Engineering students as well. This paper, worth ten credits, is a "short course" for which government has cancelled all funding but I am still able to run it as I am paid to support mathematics and it is classified as "support". It is free for the students and they receive a certificate upon successful completion, although the credits do not count towards any other programme. Ten topics are covered in ten days, and there is a test each day on the previous topic with a chance at a resit in each topic halfway through and at the end. It is an intensive pressure paper and serves as a great refresher for those students who have studied successfully at year 12 level or equivalent somewhere in their past (year 13 is the last year of our secondary schooling).. For those who had never reached that standard, the paper is formidable indeed. With absolute dedication, some students with little background do achieve a pass but it has become obvious the fast pace is not suiting the needs of all the students.

In response to this, I am now also running the Mathematics Bridging paper across a whole semester in the evening and the students have a two-week time span in which to digest the concepts and content of each topic. This mode of delivery will be evaluated at the end of the year. It may be that there is a need for a simpler paper to be provided for those who struggle even at the slower pace.

Over the past three years, while I have been teaching the Management Mathematics, all students who completed the paper passed except for one student who rarely attended lectures and was dependent on his (poorly taught) graphics calculator. The paper is assessed using ten assignments (one per week), two short tests and an examination of three hours. Of the ten assignments, the two lowest are discarded, a feature which the students felt was very fair, allowing for their pressure times or personally difficult topics. (As a Polytechnic, a good proportion of our students are more mature, returning to education to either upskill or begin a new career, but often with family commitments making a demand on their time). Alternatively, the pass mark can rely entirely on the examination, a helpful situation if there has been a crisis in the student's life. The teaching programme was also planned to allow for in-class practice time before tests and the examination. The classes were small (9 to 14 students) and this lent itself to a tutorial style classroom where questions could be raised and answered.

These two papers are a response to the mathematics problem. As well, in a dual project with a colleague, I am helping develop an online diagnostic system specific to each paper that has a mathematical content. Students will be provided with an analysis of their readiness to cope with the mathematics involved in their prospective courses of study. They will then be supported by eMathematics workshops; this should be a successful intervention method for some students. There will still remain a body of students who need an oral explanation and Kahurangi will provide the back-up if the students have enrolled.

The Survey and Key Results

I surveyed the students at the end of the semester using an online survey. Not all students returned the surveys, with non-returns mainly in 2010 when I was delayed from sending out the surveys. In the survey I explored their mathematics background including their feelings towards mathematics at the start of the paper and what changes had occurred by the end of the paper. I investigated their learning needs and styles, their thoughts on the teaching style, whether they used the extra support from Kahurangi, their opinion on the assessment structure and any external factors that had impacted on their progress. The Bay of Plenty Polytechnic also undertakes an evaluation by the students regarding the paper and the teaching. This is completely anonymous and provided not just further data but a check on the reliability of the survey responses.

The mathematics backgrounds of the participants were varied, ranging from being successful at year 12 in secondary school to never being successful in mathematics. Accordingly, feelings towards mathematics ranged from "I love it" to "I am terrified of it". By the end of the paper, everybody had positive feelings towards mathematics and felt confident approaching it. "I feel far more confident in all areas" was a typical comment. Students were quite clear that their learning styles were a mixture of, particularly, visual and aural but often kinesthetic as well. They were happy that all of these had been catered for in the teaching. They particularly liked the build up on the whiteboard of the processes for solving problems, then having the time to practice in class, even discuss it with another student. No one wanted PowerPoint presentations, (and none were given), which is probably a reaction to many hours spent in other lectures where they were subjected to a misuse of PowerPoint. (It can be a great tool when used imaginatively). They were not aware that I analysed every step of the process and identified the basic skills embedded. It takes just a moment to run over something like $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$ before proving the completion of the square to find

maximum/minimum values. If they lose steps like these, they lose the whole proof while they are worrying over basic algebra.
The students really appreciated the relaxed atmosphere in the class, feeling comfortable enough to ask any question. In the first class, I always run an Ice-breaker so everybody knows a little about everyone else. The teaching language used was important and I made a conscious effort to identify new vocabulary, especially in a topic like calculus. Time spent in class on revision was much appreciated; it helped "pull it all together". Students having difficulty grasping concepts were able to book extra time with me through Kahurangi on a one-to-one basis early on before the problem became insurmountable. According to their survey responses, this was essential. The students recognised the sound organisation, the provision of extra resources for practice and the prompt return (by the following lecture) of their assignments. Of course, this last factor was made possible by the size of the class.

Conclusion

Although the classes surveyed were small, extrapolating the findings to larger groups underscores the usefulness of studies such as this. In many ways, the lectures were very similar to the "chalk and talk" often seen in secondary school. The differences are subtle, such as ensuring everyone feels they can achieve despite their backgrounds. The students learn early on that their progress is important to both them and me and are willing to be helped, even if they have a negative attitude towards mathematics itself. The content is thoroughly analysed and considered and taught well. There is time to be assisted, whether in class from peer or tutor, or through Kahurangi.
It is not just one magic factor that has made this class successful, it is a conglomeration of factors interwoven like a pretzel. In my experience, it eventually depends on the attitude of the students. Those who enjoy mathematics and have had success in the past, given that there is "quality teaching", will achieve a good result. For those who arrive with negative backgrounds, learning that their progress is valued and that there is help available can be sufficient for students to overcome their personal barriers – and perhaps that is the dash of lemon that makes the difference. The last words must go to my student, Marie, who passed with her usual A+, not the C she was aiming for. She wrote "I now realise I can do anything I put my mind to". That is a true transformation!

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Tangram-base Problem Solving in Radical Constructivist Paradigm: High School Student-Teachers Conjectures

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Abstract
A series of tangram-based problem solving tasks, focusing at visual geometric construction and justification by high school teacher candidates, are reported. Sociocultural and psychological components of von Glasersfeld Theory of *radical constructivism* have been utilized. The purpose was to describe and analyze how students' cognitive constructions have been initiated, modified, and re-modified as they were proceeding in their attempts to solve and justify spatial tangrams-based problems.

Background
The 'tangram' has been originally referred to as the 7-pieces dissection (or tangram problem) consisting of seven flat shapes forming together a square shape (five triangles: two identical large, two identical small and one medium triangles; a small square and a parallelogram). It was originally invented in China at some unknown year in history, and then carried over to the world by trading ships in the early 19th century to become well-known since then (Wang & Hsiung, 1942; Read, 1965). In particular, assuming that the small square has an area of one unit square then each large triangle has an area of two unit square and each small triangle of an area of half unit square and medium triangle of an area of one unit square and a parallelogram have an area of one unit square too. The objective of the tangram problem, often called tangram puzzle, is to form a specific shape, given only its outline or description, using all seven pieces with no overlapping (Figure 1).

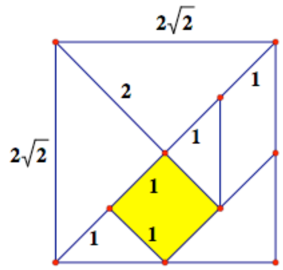


Figure 1: Seven pieces tangram set resembled into a square of side $2\sqrt{2}$ units

Clearly a given tangram set is not confined to a particular rigid and fixed shape formation such as the case of a "jigsaw" puzzle set of pieces, rather the pieces can be rearranged into several geometric shapes each of which is area equivalent to the original tangram square. Meanwhile the new constructed shapes may be convex: a shape is convex if the line joining any two points within the shape falls entirely in the shape; otherwise it is a non-convex shape. This tangram shapes formation represents a rich environment through which shape-to-shape, shape-to- parts and part-to-part interrelationships would be explored by the learners and possibly at all levels of schooling, from pre-kindergarten to university teacher training classes.
The literature provides a variety of examples using the seven pieces tangram set. The pieces may be rearranged into other distinct shapes each of which contains also seven pieces yet are of equal

area of the original seven pieces tangram set whether a resultant shape is convex or not. Below are the *only* thirteen convex shapes that are the resultant of the shape formation using the seven piece tangram (see Wang & Hsiung, 1942, p. 596; Scott, 2006, p. 5).

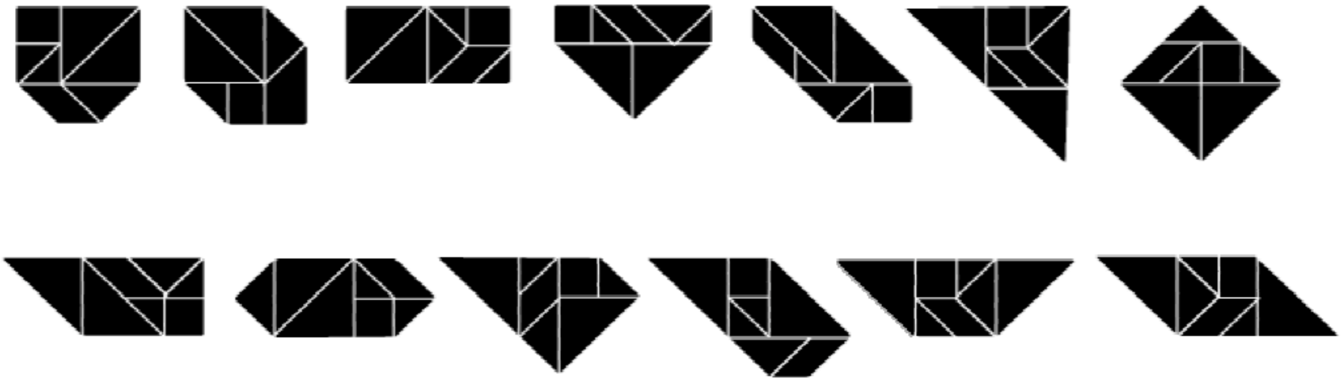


Figure 2: The thirteen convex shapes made out of the seven pieces tangram set

Also, tangram shapes formation may be carried out by using only two, three, four, five or six pieces in addition to the whole seven pieces.

Radical Constructivism: Sociocultural and Psychological Mechanisms in Justification

It has been suggested by von Glasersfeld (1995) that, based on his radical constructivism theory known as "*Radical Constructivism: A Way of Knowing & Learning*", when individuals deal with the physical world, their minds construct, through certain mental mechanisms and collections of cognitive structures, their conceptualization, reason, and coordination of their engagements (von Glasersfeld, 1995; 1984; 1974). Battista (1999) has described the notion of *abstraction* as the process through which the mind selects, coordinates, unifies, and registers in memory a collection of mental acts that appear in the attentional field (p. 418). Further, Battista (1999) has referred to von Glasersfeld's (1995, p. 69) ideas of *abstraction* and added that abstraction has several levels: At its *perceptual level* (most basic), abstraction isolates an item in the stream of an experience and seizes it as a unit. Battista added that material or entity is said to have reached the *internalized level* whenever it has been sufficiently abstracted so that it can be re-presented (re-created) in the absence of its perceptual input. Material or entity is said to have reached *interiorized level* whenever it has been disembodied from its original perceptual context and it can be freely operated on in imagination, including being "projected" into other perceptual material and utilized in novel situations (Battista, 1999, p. 418). Earlier, Steffe & Cobb (1988) asserted that *interiorization* is "the most general form of abstraction; it leads to the isolation of structure (form), pattern (coordination), and operations (actions) from experiential things and activities" (p. 337). von Glasersfeld, in presenting his radical constructivism theory, stated that *understanding* requires more than abstraction; it requires *reflection* which is the conscious process of re-presenting experiences, actions, or mental processes and considering their results or how they are composed. *Reflective abstraction* takes mental operations performed on previously abstracted items as elements and coordinates them into new forms or structures that, in turn, can become the content -what is acted upon- in future acts of abstraction (von Glasersfeld, 1995, p. 69). Battista (1999), in reporting his 3D cube arrays' study, suggested that besides von Glasersfeld's (1995) list of mental mechanisms that includes abstraction and reflection mechanisms there are three additional mechanisms that are fundamental to understanding students' reasoning. They are *spatial structuring, mental models, and schemes* (Battista, 1999; Battista & Clements 1996). *Spatial structuring* is the mental act of constructing an organization or form for an object or set of objects. It determines an object's nature or shape by identifying its spatial components, combining components into spatial composites, and establishing interrelationships between and among components and composites. *Mental models* are nonverbal recall-of-experience-like mental versions of situations; they have structures isomorphic to the perceived structures of situations they represent (Battista, 1999, p. 418; 1994). Mental models consist of integrated sets of abstractions that are activated to interpret and reason about situations that one is dealing with in action or thought. A *scheme* is an organized sequence of actions or operations that has been abstracted from experience and can be applied in response to similar circumstances. It consists of a mechanism for recognizing a situation, a mental model that is activated to interpret actions within the situation, and a set of expectations (usually embedded in the behaviour of the model) about the possible results of those actions (Battista, 1999). Meaningful learning occurs as students make adoptions to their current cognitive structures as a result of their reflection on an experience (Steffe, 1988; Battista, 1999). An accommodation is triggered by a *perturbation* which is described as a disturbance in mental equilibrium caused by an unexpected result or a realization that something is missed or does not work (von Glasersfeld, 1995, p. 67). Perturbation arises when students interact with other individuals or with the physical world (Battista, 1999). von Glasersfeld (1995) made a clear distinction between teaching and training stating that, "From a educator point of view one of the most important features of radical constructivism is the sharp distinction it draws between teaching and training. The first aims at generating understanding, the second at competent performance" (p. xvi). Further, von Glasersfeld in referring to learning mathematics stated that "To know mathematics is to know how and why one operates in specific ways and not in others, how and why the results one obtains are derived from the operations one carries out" (p.xvi).

The Classroom Sessions

***First Session*:** Through the classroom problem solving sessions using the seven pieces tangram set, the set was introduced to the students by providing each of them with a colored plastic made model of the seven pieces tangram. The students were instructed that these seven pieces together can be manipulated through motions of translation, rotation and/or reflection to formulate other geometric shapes without overlapping, where the area of each of the resultant shapes made by the seven pieces, is invariant. The students were encouraged to use these seven pieces collectively to form new shapes of equal area. Each of which must be a simply polygon. The instructors explained the concepts of a simple polygon as opposed to a non simple polygon. The students were curious to know the difference and for that the instructors elaborated in saying that a simply polygon is any polygon in which each vertex is created by only two external sides. That is a vertex can have no more than two sides passing through it. This emphasis on the concept of simple polygon is essential since the concept of non simple polygon is rarely included in the middle/high school curriculum. Further, the concept of convex and non convex was also introduced.

Second Session: The students started forming as many shapes as they can within the instructions given above using the plastic tangram pieces. They were encouraged to ask questions and were advised to trace and make copies of their constructions. As the students being familiar with the Geometer's Sketchpad (GSP) they were encouraged to use GSP to construct digital images for their shape formations based on the seven pieces tangram.

Third Session: This session was a continuation on the second session but through the computer lab applying GSP. During the computer lab session, the students were challenged with the following question: ***"Given a simple polygonal region and assume it is dissected into a finite number of sub-regions, then what is the maximum number of sides for a simple polygonal region that can be constructed using all sub-regions? Make a conjecture."*** The students then were allowed to take the tangram pieces home for further shape construction, refinement, and show-and-tell opportunity at the following class session. Further, the students were told that they were about to introduce a *new mathematical proposition* should their answer be verifiable. At the tail of the session, one student has asked: "so how far we can go with the number of sides of the new tangram?" The instructor replied: "you may go as far as you see it possible; as a hint, the maximum number of sides possible for a simple polygon is more than twenty!"

Students' Uses of Radical Constructivist Paradigm
Due to the space restriction, two case studies are presented.
Case Study 1: Student, Lee, using the tangram pieces, she has come up with an answer to the question: "*What is the maximum number of sides for a simple polygonal region that can be constructed using all the sub-regions? Make a conjecture as an answer to this question.*" Through a series of constructions started with a square, rectangle, right triangle, parallelogram, trapezoid, pentagon, hexagon, heptagon, octagon, nonagon, decagon, hendecagon, dodecagon, but then Lee emerged with an answer to the question above by stating: "shape with maximum number of sides that can be produced with tangram set is 23 sided polygon." Lee also stated: "the sum of the number of sides of all 7 pieces tangram is equal to the number of sides of the shape that contains the maximum number of sides that can be possibly created using these pieces". She further stated: "Conjecture: **Several shapes can be combined to form a simple, closed shape with maximum number of sides n, where n is the sum of all the sides contained by the sub-shapes altogether**" (Lee's bolding). Lee then offered the figures shown below in Figure 3 a & b.

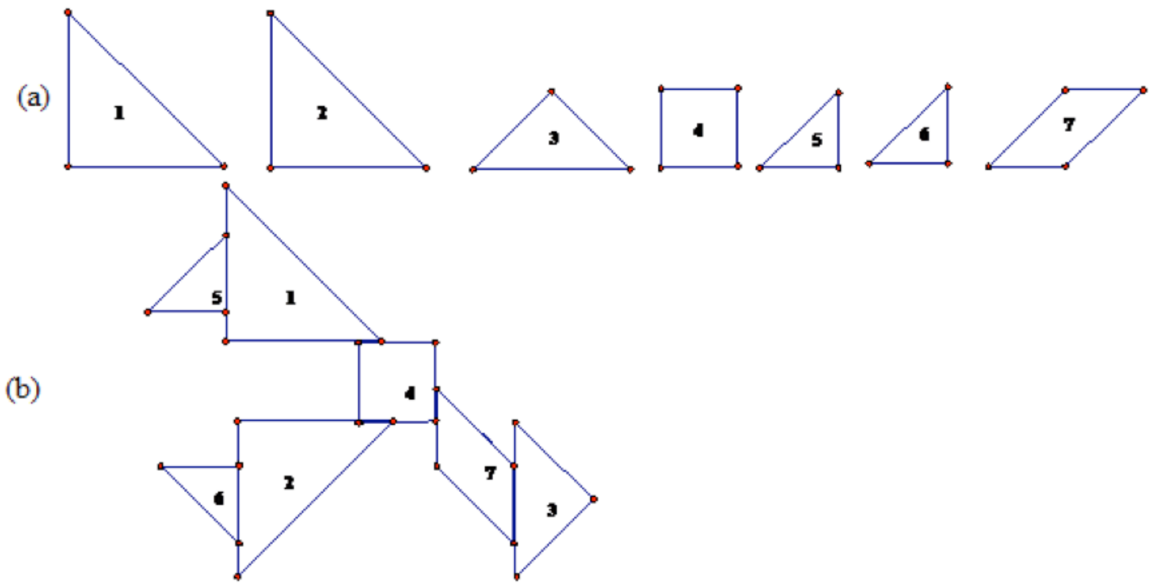
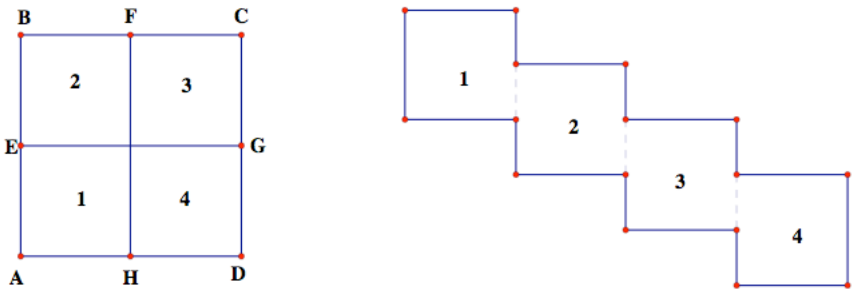


Figure 3: Lee's conjecture

Case Study 2

Student, Brooks, came up with an interesting method for his conjecture in stating that, "for a given square ABCD when dissected into four congruent squares symbolized as 1, 2, 3 and 4 as shown in Figure 4a, the four squares can be rearranged into the shape shown in Figure 4b, and hence, in his words: "*The highest sided polygon is equal to the cumulative number of sides of all of the individual shapes*." Clearly, Brooks meant by his statement that the ultimate created shape using the resultant pieces by dissecting the given square has to have its number of sides to be equal to the total number of the sides of all resultant pieces due to the dissection process. Brooks did not further elaborate on the case when the square ABCD contains the 7 pieces tangram set. Nonetheless, his spatial structuring presents an elegant answer to the question presented above.



(a) **(b)**

Figure 4: Brooks Conjecture

Epilogue

As a reflection on the two cases, out of several cases, within the experimental observations reported, it would seem clear that the von Glasersfeld's radical constructivism theory has been present throughout the students' work on the tangram 7-pieces set activities. Our interpretation of the student, Lee, is that: Lee's processes described in Case Study 1 above (Figure 3 a & b) was directly exemplifying Battista (1999) and Battista & Clements' (1996) interpretation of *spatial structuring* concepts. Evidently, her mental acts of constructing an organization or a form for an object (or set of objects) has been by determining the object's nature or shape, identifying its spatial components, combining components into spatial composites, and establishing interrelationships between and among components and composites. Further, Battista (1999) indicated that material or entity is said to have reached *interiorized level* whenever it has been disembodied from its original perceptual context and it can be freely operated on in imagination, including being "projected" into other perceptual material and utilized in novel situations (p. 418) – Lee was virtually acting along this path. The use of *reflective abstraction* through her mental operations, performed on the previously abstracted items (the 7-pieces shown in Figure 3a), seems to be her building elements that she coordinated into new forms or structures (von Glasersfeld, 1995, pp. 69-70). Then, she coordinated these abstracted pieces or elements to come to make the new form or resultant shape, the 23 sided figure, depicted in Figure 3b. Her acts on the 7-pieces were clearly and directly performed exemplifying Battista's (1995) notion of *interiorized level of abstraction* (p. 418). For the question: "*What is the maximum number of sides you can make for your constructed polygons? Make a conjecture as an answer to this question*." Brooks has also used the notion of *spatial structuring* applied on his pieces, 1, 2, 3, and 4 shown in Figure 4a; Brooks used these perceptual pieces to form the newly created shape

shown in Figure 4b. Brooks, in dealing with his 4 squares, seems to have reached "*interiorized level*" of abstraction in disembodied them from "their original perceptual context". Brooks then has freely operated on the 4 squares "in imagination, including being "projected" into other perceptual material and utilized in novel situations" (Battista, 1999, p. 418). It would seem evident that Brooks has been using *reflective abstraction* through his mental operations, performed on the previously abstracted items along the von Glasersfeld theory (1995, p. 69). Finally, the conjecture that is the crux of these mathematical activities is stated below:
For a given polygonal region, when dissected into a 'k' number of sub-regions, 1, 2, 3, ….., k, with the corresponding number of sides $n_1, n_2, n_3, \ldots\ldots, n_k$, the simple polygonal region with the maximum number of sides that can possibly be constructed by all these sub-regions is
$= n_1 + n_2 + n_3 + \ldots\ldots + n_k$.

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VITALmaths – Transforming learning experiences through mathematical video clips

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Abstract

This paper provides an overview of the VITALmaths project (Visual Technology for the Autonomous Learning of Mathematics), a collaborative initiative between the School of Teacher Education of the University of Applied Sciences Northwestern Switzerland (PH FHNW) and the FRF Mathematics Education Chair hosted by Rhodes University in South Africa. This project seeks to produce, disseminate and research the efficacy and use of a growing databank of short video clips designed specifically for the autonomous learning of Mathematics. A dedicated website has been established to house this growing databank of video clips (www.ru.ac.za/VITALmaths) from which the video files can either be freely downloaded or streamed. Specific to the South African context is our interest in capitalising on the ubiquity of cellphone technology and the autonomous affordances offered by mobile learning. This paper engages with a number of theoretical and pedagogical issues relating to the design, production and use of these video clips, a number of which will be shown during the presentation.

Introduction

Among the implications that can be expected from the implementation of the National Educational Standards in Switzerland, as well as a number of other European countries, the following three appear to be most important for Mathematics education (Linneweber-Lammerskitten & Wälti, 2008): (a) It will be necessary to find better ways to deal with heterogeneity, particularly with regard to supporting weaker pupils; (b) It will be necessary to give more attention to the non-cognitive dimensions of Mathematical competency such as motivation, sustaining interest and the ability to work in a team; (c) It will be necessary to deal with aspects of Mathematical competence such as the ability and readiness to explore mathematical states of affairs, to formulate conjectures, and to establish ideas for testing conjectures.

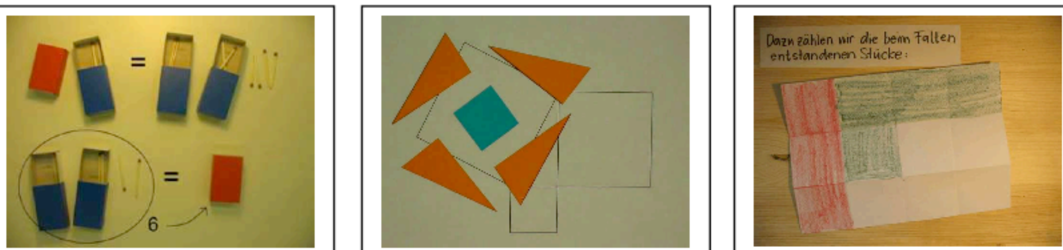
These necessities find resonance with similar implications which have arisen from the implementation of the Revised National Curriculum Statement in South Africa. This is true not only in terms of subject-specific outcomes and assessment standards, but it is also echoed by the social transformation imperatives of the curriculum and the desired attributes of the kind of learner envisaged within the South African education system (Department of Education, 2002, 2003).

Bearing in mind that these educational standards and curriculum statements are representative of the minimum levels of knowledge and skills achievable at each grade, it follows that the establishment of such aspects of competence as minimal standards *for all pupils* can only be successful if appropriate measures are taken to (a) fully integrate weaker pupils in the learning experience and (b) create an environment that is stimulating, motivating, interesting and encourages social competence. The VITALmaths project began as a response to this challenge of developing auxiliary means that could not only release teachers from the frontal introduction to mathematical themes, but also provide an opportunity for learners, particularly weaker learners, to experience genuine and challenging mathematical activities and explorations.

Autonomous learning through video clips

Although the use of video clips in the pedagogical context of the classroom is nothing new, the majority of these videos tend to be instructional in nature and consequently are underpinned by specific pre-determined outcomes and pedagogical imperatives. The VITALmaths project was born out of the identification of a need for short, succinct video material that could be swiftly and easily accessed and used autonomously by both teachers and pupils alike. Common design principles of these video clips are that they are short, aesthetically delightful, and are self-explanatory in the sense that they require minimal instruction. An important aspect of these video clips is that they encourage genuine mathematical exploration that transcends the mathematical content of the film by encouraging a desire to experiment, use trial-and-error, formulate conjectures, and generalise results.

VITALmaths video clips are silent, short in duration (typically 1 to 3 minutes), and are produced using a stop-go animation technique incorporating natural materials as opposed to high-tech graphics. The video clips explore and develop mathematical themes in a progressive manner that supports and encourages genuine mathematical exploration. These themes include, amongst others, striking visual approaches to proving the Theorem of Pythagoras; patterns and symmetry generated through tiling activities; elegant visual support for various results from elementary number theory; interior angles of polygons; equivalence of different area formulae; visual insight into numerical operations. A dedicated website has been established to house this growing databank of video clips (http://www.ru.ac.za/VITALmaths) from which the video files can either be freely downloaded or streamed. A selection of screenshots is shown in Figure 1.



This video clip models the solving of simultaneous equations through a process of logical reasoning without the introduction of algebra.

A proposal is made for a visual proof of the Theorem of Pythagoras. The question is raised as to whether or not this constitutes a general proof.

A model of a rectangle is used to visualise the sum of two fractions, a quarter and a third.

Figure 1. A selection of screenshots from three VITALmaths video clips.

One of the guiding tenets behind the project is that of autonomy. Autonomy represents an inner endorsement of one's actions – a sense that one's actions emanate from within and are one's own (Deci & Ryan, 1987 as cited in Reeve & Jang, 2006:209). As Mousley, Lambdin and Koc (2003) succinctly comment, "Autonomy is not a function of rich and innovative materials themselves, but relates to genuine freedoms and support given to students" (p. 425). Thus, teachers cannot directly provide learners with an experience of autonomy (Reeve & Jang, 2006), but rather they need to provide genuine opportunities that encourage, nurture and support autonomous learning. Being sensitive to this, critical elements of the design principles of the video clips take into account both cognitive and non-cognitive dimensions.

Design principles & design process

Of fundamental importance to the VITALmaths project are the design principles on which the video clips are modelled, since this plays a critical role in terms of how students are likely to interact with the technological medium. At the heart of this design process is the notion that designers design the *experience*, not simply the *product*. The basic design cycle is shown in Figure 2.

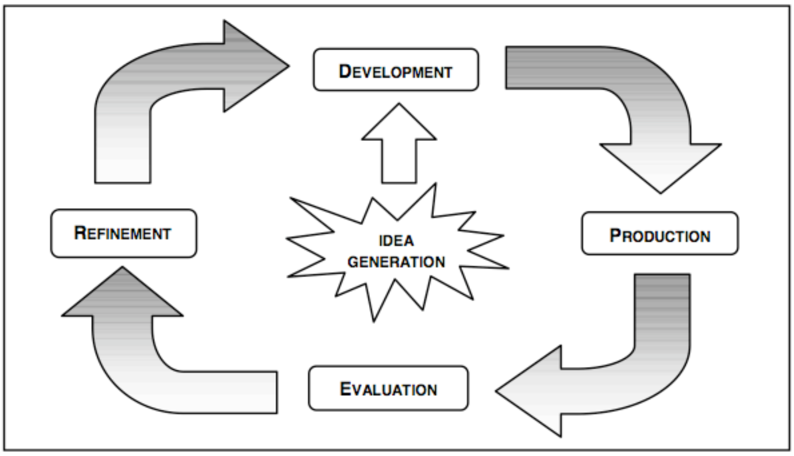


Figure 2. The design cycle.

The idea generation process is multi-faceted. One aspect of the VITALmaths project relates to the production of video clips specifically aligned with certain textbooks used in Switzerland. The idea here is that teachers will be able to use these clips as an auxiliary means to the introduction of new mathematical themes thereby allowing more time to focus on weaker pupils. However, another important aspect of the project is the production of video clips that are purposefully *not* aligned with the mathematical content of school curricula. It is envisaged that these video clips will be used in the preparation of exploratory lessons, for personal conceptualisation of mathematical concepts, and as motivational and explanatory tools, with the emphasis lying on teachers and learners using them as autonomously and independently as they wish. Ideas of appropriate topics are sourced from teachers, pupils, and experts in the field. In addition, we are exploring the possibility of a group of pupils conceptualising and creating their own video clips as part of a school Design & Technology project.

Once an idea has been generated it is developed into a workable video clip. In terms of the design principles that relate specifically to the video clips themselves, a purposeful decision was taken to eschew high-tech graphics animations in favour of using natural materials. This design consideration supports autonomous learning on two levels. Firstly, in terms of cognitive access, the use of natural materials should allow for a more direct and personally meaningful engagement with the content of the video clips when compared with the additional abstract dimension associated with high-tech graphics animations. Secondly, learners will be able to personally source all the required material to explore identical or similar scenarios, thus encouraging hands-on mathematical exploration that will have personal meaning for each learner. The production process presently used incorporates a stop-go animation technique using the free software *VideoPad Video Editor*. Once the video files are created they are then converted from AVI format to both MP4 and 3G2 formats. The MP4 format is appropriate for PCs, laptops, iPhones and most cellphones. The 3G2 format is appropriate for older cellphones that aren't MP4-compatible.

The evaluation process relies on feedback from teachers, learners and experts in the field, and is continuously used to reflect on and refine both the production process itself as well as the design principles that inform the conceptualisation of the video clips. This feedback forms a critical component of the refinement stage of the design cycle in which video clips are either modified or reconceptualised.

Mobile technology

Specific to the South African context is our interest in the use of cellphone technology as the primary distribution platform for these video clips. Our interest lies not only in the use of cellphone technology as a means of viewing the video clips, but ultimately as their primary distribution platform. Not only will cellphone technology enhance and support the autonomous learning objective of the enterprise, but it will greatly facilitate access to these video materials. In addition, it is anticipated that this innovation will have a significant positive impact for teachers in deep rural settings where access to mathematics resources is very limited.

There are a variety of mobile devices that have found application within the education arena - Personal Digital Assistants (PDAs), tablet PCs, iPods, and some games devices. However, fuelled by the development of powerful telecommunication networks which support an ever increasing range of data access services, coupled with technological advances and steadily declining costs of cellphones themselves, cellphones have emerged as a viable option for mobile learning. The VITALmaths project aims to capitalise on the flexible and versatile potential of cellphones for mobile learning.

The educational potential for mobile learning afforded by cellphone technology is diverse (Kolb, 2008; Prensky, 2005). Within South Africa a number of projects have already harnessed the ubiquity of cellphone technology to support the learning of Mathematics (e.g. ImfundoYami / ImfundoYethu, *M4Girls, MOBI*™ *maths*, and *Dr Math*). Selanikio (2008) makes the pertinent comment that "for the majority of the world's population, and for the foreseeable future, the cell phone is the computer". This sentiment is echoed by Ford (2009) in her pronouncement that "the cellphone is poised to become the 'PC of Africa'". The challenge for educators is thus "to capitalize on the pervasive use of cell phones by younger students for educational purposes" (Pursell, 2009:1219). The VITALmaths project aims to take up this challenge and to capitalise on the flexible and versatile potential of cellphones for mobile learning.

Conclusion

The VITALmaths project began as a response to the challenge of developing auxiliary means that could not only release teachers from the frontal introduction to mathematical themes, but also provide an opportunity for learners, particularly weaker learners, to experience genuine and challenging mathematical activities. The desire for teachers and learners to make autonomous use of the video clips is supported by the broad and open philosophy embraced by the design principles on which the video clips are conceived. The video clips are short, succinct, visually and intellectually appealing, relevant and mathematically inspirational, and a growing databank of video clips has been established from which they can be freely downloaded (http://www.ru.ac.za/VITALmaths).

The broad and open philosophy embraced by the design principles on which the video clips are conceived aims to encourage teachers and learners to use the video clips as autonomously as they desire. Continued research into the use and impact of these video clips seeks to develop a base for sustained growth and development, while at the same time contributing and participating in the academic discourse surrounding the use and development of visual technologies in the Mathematics education arena.

Acknowledgements

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Figural pattern generalisation – the role of rhythm

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Abstract

This paper uses a theoretical backdrop of *enactivism* and *knowledge objectification* to explore the role of rhythm in the generalisation of pictorial patterns. Through an analysis of two vignettes, rhythm is shown not only to be an indicator of the conscious or unconscious perception of structure, but it is also shown that rhythm can be an artefact born out of specific counting processes, an artefact which in turn can *lead* to structural awareness.

Introduction

The use of number patterns, specifically *pictorial* or *figural* number patterns, has been advocated by numerous mathematics educators as a didactic approach to the introduction of algebra and as a means of promoting algebraic reasoning. From a pedagogic standpoint, French (2002) comments that the introduction of algebra through what is potentially a wide range of pattern generalisation activities may be effective in assisting pupils to see algebra as both meaningful and purposeful right from the earliest stages. Although this route to the introduction of algebra is not without its problems (see e.g. Warren, 2005), pattern generalisation activities nonetheless present a meaningful way of arriving at algebraically equivalent expressions of generality. This lends itself well to exploring the notion of algebraic equivalence in a practical context where pupils would experience the process of negotiation towards meaning (Mason et al., 1985), a process which in itself has the potential to develop and support pupils' *productive disposition* (Kilpatrick, Swafford & Findell, 2001). Vogel (2005) also suggests that patterning tasks have the potential to develop important metacognitive abilities. This paper focuses specifically on the role of *rhythm* in the process of figural pattern generalisation.

Enactivism

Enactivism is a theory of cognition that draws on ideas from ecology, complexity theory, phenomenology, neural biology, and post-Darwinian evolutionary thought. As Nickson (2000:8) discusses, the basic tenet of enactivism is that there is no division between mind and body, and thus no separation between cognition and any other kind of activity. Enactivist theory brings together action, knowledge and identity so that there is a conflation of *doing*, *knowing*, and *being* (Davis, Sumara, & Kieren, 1996).

Within an enactivist framework, there is a purposeful blurring of the line between thought and behaviour, with the focus on the dynamic interdependence of thought and action, knowledge and knower, self and other, individual and collective (Davis, 1997:370). Cognition is not seen as a representation of an external world, but rather as "an ongoing bringing forth of a world through the process of living itself" (Maturana

& Varela, 1998:11). Thus, cognition is viewed as an embodied and co-emergent interactive process where the emphasis is on *knowing* as opposed to *knowledge*.

From an enactivist stance, we need to consider not only the formal mathematical ideas that emerge from action, but to give close scrutiny to those preceding actions – "the unformulated exploration, the undirected movement, the unstructured interaction, wherein the body is wholly engaged in mathematical play" (Davis et al.,1996:156). Within an enactivist framework, language and action (e.g. rhythm) are seen not merely as outward manifestations of internal workings, but rather as "visible aspects of … embodied (enacted) understandings" (Davis, 1995:4).

Knowledge Objectification

To *perceive* something means "to endow it with meaning, to subsume it in a general frame that makes the object of perception *recognizable"* (Sabena, Radford & Bardini, 2005:129). Radford (2008:87) refers to the process of making the objects of knowledge apparent as *objectification*, a multi-systemic, semiotic-mediated activity during which the perceptual act of noticing progressively unfolds and through which a stable form of awareness is achieved. Use of the word "objectification" in this context needs to be interpreted in a phenomenological sense, a process whereby something is brought to one's attention or view (Radford, 2002:14). Knowledge objectification is premised on the notion that semiotic means such as gestures, rhythm and speech are not simply epiphenomena, but are seen to play a fundamental role in the formation of knowledge (Radford, 2005:142).

Rhythm, the "ordered characteristic of succession" (Fraisse, 1982:150), is an important element in the process of knowledge objectification. Rhythm, whether in speech or gesture, is not merely the conscious or unconscious perception of order, but crucially it creates a sense of expectation or the "…anticipation for something to come" (You, 1994:363). There is thus an inherent sense of expectancy associated with rhythm, and it is seen as a crucial semiotic device in the process of generalisation (Radford, Bardini & Sabena, 2006). Rhythm is a subtle yet powerful semiotic device since it is able to operate on multiple levels – verbal, aural, kinaesthetic and visual. In the process of generalisation, rhythm aids and supports the move from the particular to the general by enabling pupils to project and make apparent a regularity or perception of order that transcends the specific cases under scrutiny (Radford et al., 2006).

Discussion

Two vignettes are presented to highlight the different roles that rhythm can play in the process of figural pattern generalisation. In the first vignette, rhythm is identified as an indicator of unconscious structural awareness. In the second vignette, a specific counting procedure leads to a rhythmic pulse which in turn leads to the development of structural awareness.

Vignette 1

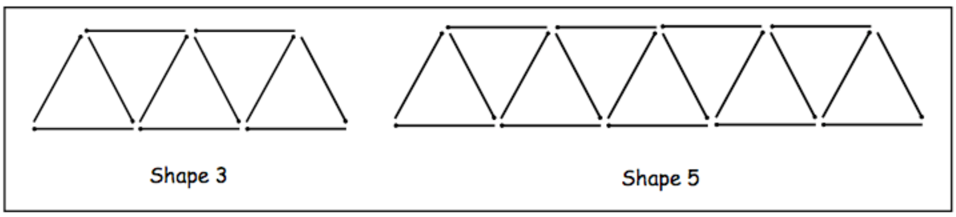

Figure 1 Visual stimulus (two terms of a pictorial pattern) presented to Grant.

Grant was presented with the two terms shown in Figure 1. Grant began by counting the forward-leaning parallel matches of Shape 5 from left to right (see Figure 2). After a brief pause he then worked his way back from right to left counting the backward-leaning parallel matches. He then counted the remaining top and bottom matches in pairs, rhythmically alternating between top and bottom as shown in Figure 2: 1,2…3,4…5,6…7,8…9. This counting procedure was central in alerting Grant, whether consciously or unconsciously, to the non-paired match in the bottom row. Based on this counting procedure, Grant was able to arrive at the following general expression for the n^th^ term of the sequence: $n+n+2n-1$.

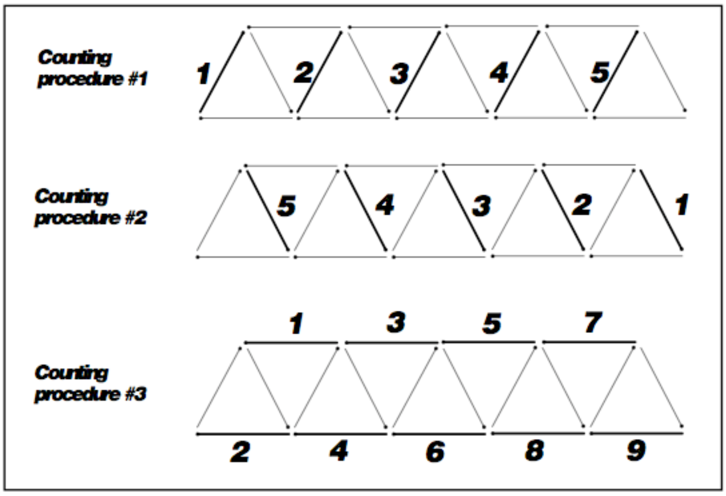

Figure 2 Grant's different counting procedures.

Grant was able to justify his general expression ($n+n+2n-1$) by relating the $n+n$ portion to two sets of "parallel central matches", while the $2n-1$ he associated with what he referred to as the "outside matches". Just prior to writing the $2n-1$ part of the expression, Grant made use of indexical gesturing - he first gestured a horizontal line across the top of Term 5 and then a second horizontal line across the bottom of Term 5. As Grant wrote down the $2n-1$expression he commented that he was just simplifying $n+[n-1]$. When asked to articulate how he was "seeing" it, he was insistent that he saw the structure as $n+[n-1]$, i.e., in terms of n matches along the bottom and $n-1$ matches along the top, and that the $2n-1$ portion of his expression was in fact an algebraic simplification of $n+[n-1]$. Grant then re-wrote his expression for the n^th^ term as $n+n+n+[n-1]$ which he explained as being a truer representation of his visual apprehension of the pictorial pattern.

Grant's initial formula ($n+n+2n-1$) suggests that the "outside" matches have been split into pairs - one match of each pair forming part of the upper horizontal row with its paired match positioned below it in the bottom row. The $2n-1$ "outside" matches in Grant's initial formula seem to represent n pairs of matches, making $2n$ matches in total, the "-1" being an adjustment required due to the right-most pair missing a match in the upper row. Although his final expression ($n+n+n+[n-1]$) suggests that the "outside" matches were in fact sub-divided into two distinct horizontal rows, with n matches along the bottom and $n-1$ matches along the top, the $2n-1$ portion of his original expression is likely to have been inspired by his counting procedure shown in Figure 2, in which the rhythmical pairing of the top and bottom matches was central to alerting him to the non-paired match in the bottom row. In this instance the rhythmic counting procedure seems to be an indicator of an unconscious perception of structure which subsequently was manifested in the general algebraic expression for the n^{th} term.

Vignette 2

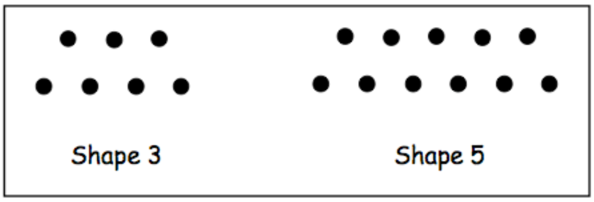

Figure 3 Visual stimulus (two terms of a pictorial pattern) presented to Anthea.

Anthea was presented with the two terms shown in Figure 3. Upon initial presentation of her pictorial pattern, Anthea counted the dots in Shape 3 and Shape 5 in an economical zigzag manner as shown in Figure 4 (a) and (b). She then double-checked her two answers. When she re-counted the dots in Shape 3 she used the same counting method as she used in the initial count. However, when she re-counted the dots in Shape 5 she did so in a slightly modified manner. This new counting procedure is shown in Figure 4(c). After a bit of silent thinking she counted the dots in Shape 3 one final time, using her *initial* counting method, before writing down the formula $n+(n+1)$. She then tested her formula mentally and was satisfied that the formula worked for these two cases.

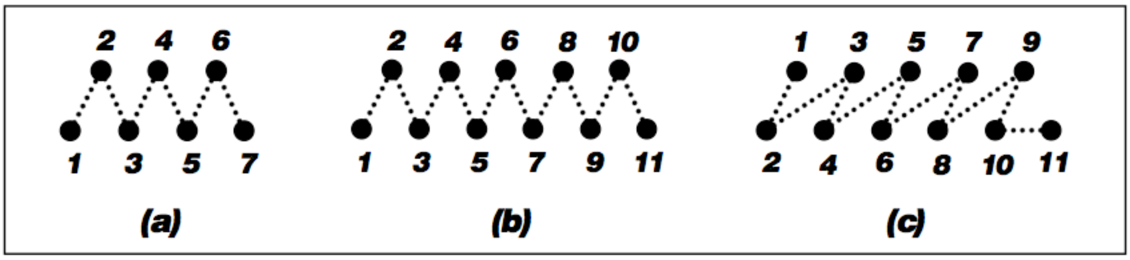

Figure 4 Anthea's different counting procedures.

We now move onto the development of Anthea's second algebraic expression. After sitting silently for about 30 seconds she suddenly wrote down $2n+1$. Anthea explained that her formula $2n+1$ was arrived at through numerical considerations only. However, what is interesting is that the formula $2n+1$ seems to resonate with the counting method shown in Figure 4(c). Although Anthea arrived at her general expression $T_n = 2n+1$ through a process of numeric rather than visual reasoning, it is likely that the second of her two earlier counting processes unconsciously inspired this algebraic expression. With the first counting method the starting point is the dot furthest to the left on the *bottom*. For the second counting method the starting point is the dot furthest to the left on the *top*. Both methods result in an overall zigzag movement from left to right, but the first method is more economical in terms of total distance traversed during the counting process. The second method is uneconomical since each movement from top to bottom has a small lateral component to the left (e.g. the move from dot 1 to dot 2 in Figure 4(c)) with the result that a longer distance needs to be covered when moving from bottom to top (e.g. the move from dot 2 to dot 3 in Figure 4(c)). The distance traversed in the second counting method is in fact just over 20% longer than that traversed in the first method. However, what is crucial to appreciate is that this alternating top-to-bottom and bottom-to-top movement, where the top-to-bottom movement is accomplished slightly faster than the bottom-to-top movement as a result of the two different path lengths, creates a critical sense of *rhythm*: 1,2…3,4…5,6…7,8…9,10..11. The critical distinction here is that instead of the rhythm being an artefact of a counting method inspired by a perceived structural regularity, the rhythm is actually an artefact born out of the counting process itself, an artefact which in turn may *lead* to perceived, albeit possibly unconscious, structural regularity and thus to the development of a new apprehension and associated general algebraic expression.

Concluding comments

The cognitive significance of the body has become one of the major topics in current psychology (Radford et al., 2005:113). Knowledge objectification, premised on the notion that semiotic means such as gestures, rhythm and speech are not simply epiphenomena but play a fundamental role in the formation of knowledge (Radford, 2005:142), proved to be a useful theoretical construct in the exploration of rhythmic aspects of embodied knowing. Rhythm was found not only to be an indicator of the unconscious perception of structure but also an artefact of the manner of engagement with the visual stimulus, an artefact which in turn *led* to structural awareness.

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Probability in Mathematics: Facing Probability in Everyday Life

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Abstract

Understanding chance is essential to understand probability. The aim of this study was to find out how teenagers understand this idiom. We interview students from 6th, 8th and 11th grades. The results from the interviews pointed out that the idiom 'chance' is not clear enough. They understand better theoretical situations like throwing a dice. They understand the random and the certain in this situation. On the other hand, when they connect chance to everyday situations they relate to them as yes/no situations. They have difficulties in defining the random and the certain.

Introduction

Let's look at any typical "text book" activity devised for 8th grad students to understand what 'probability' is. The students are doing an experiment with tossing a coin (or a few coins together). They find that the ratio between the number of heads they got all together, and the number of coin tosses, is stabilized on 1/2. So they define the probability to get head on a random coin toss as this number. Do students, after this activity, understand what probability is? Are they able to apply this notion to everyday life? There is a need to find how teenagers understand probability. More specifically, how they understand 'chance' and what meaning do they apply when they use it in everyday life. These are the goals of this research.

Background

Piaget and Inhelder (Piaget and Inhelder, 1954) said that understanding probability means the ability to produce some quantification assessment (for the probability) that a certain event will occur. In order to give such assessment, one should understand both 'chance' and the usage of the combinatory operations. Piaget and Inhelder also said that understanding 'chance' is to understand total possibilities and randomness. Total possibilities events are events that will never occur (like throwing two dices and getting 14) or, events that one can be sure that they will occur (like throwing two dices and getting a result bigger then 0 and smaller then 13). They defined 3 stages for developing understanding of 'chance' and probability. Only at the third stage (from the age 11/12) one understands 'chance' and is able to use the combinatory operations.

According to the theory of Piaget and Inhelder, understanding 'chance' is essential for understanding probability. Understanding ‘chance’ is a combination of the discrimination between certain and uncertain events, and the understanding the distribution and the variety of the results.

Researchers in the last decade focused on understanding the variety of the results of an experiment that is repeated many times (like: Watson and Kelly, 2003; Watson et. al. 2003; Watson and Moritz, 2000, 2003; Sanches and Matinez, 2006). These researchers claim that understanding the variation of the results reflects actually understanding of the 'chance'. This understanding is combined of 3 aspects: understanding the randomness, understanding the structure of probability, and connecting this structure to empirical results.

The main aim of our research is to find what meaning teenagers, ranging from 11 years-old up to 17 y"o, give to the word 'chance' in everyday situations, including probabilistic situations.

Methodology

56 students (19 from 6th grade, 18 from 8th grade and 19 from 11th grade) were interviewed. In addition to free conversation, each interview included the following assignments:

1) Explain what is 'chance'.
2) Compose a sentence with the word 'chance'.
3) Explain what is 'almost zero chance'?
4) Read the story of the weather man. Did the weather man was wrong when he said that there is a 'almost zero chance' for the next day to be rainy?[1]
5) Playing "Ladders and Ropes" game. It is the same usual game with one additional rule: before throwing the regular dice, each player has to throw a special dice, which has 3 faces marked with a circle and 3 faces marked with a X sign. If the player gets the circle sign, he may proceed as usual and throw the regular dices. But if the player gets the X sign, the turn is passed over to the next player. Before the game, the students were asked what was the chance to get a 'circle' and what was the chance to get 'X'. When one player (or more) got many more Xs then circles (or vice-versa), the interviewer stopped the game and talked with the students about those chances again.

We asked about 'almost zero chance' in addition to 'chance', because we wanted to delve deeper into Piage's "understanding of the random and the certain". 'Almost zero chance' can easily be taken as the equation "chance = zero". While 'almost zero chance' **is fitted to** describe probability of random events, the notion of "chance = zero" is fitted to the probability of certain events. In the case of 'almost zero chance' there is still some probability that the event can occur, though it likely to be very poor.

Findings

Explanations to 'chance':

Most of the subjects said that chance means uncertainness. Like: "something that may happen", or: "a possibility that I'll get one out of two options". Very few subjects added some quantification. Like: "the percentages for something that will occur". (See table 1).

One note that almost 25% of the subjects could not explain what 'chance' is. They said that they understand what 'chance' is but they cannot explain it.

	6th grade		8th grade		11th grade	
	N	%	N	%	N	%
uncertainty	13	68	11	61	10	53
quantification	1	5	3	17	3	16
Could not explain	5	26	4	22	6	31
total	19	99	18	100	19	100

Table 1: Distribution explanation to 'chance'

Composing a sentence with 'chance':

All of the subjects composed a sentence with it the word 'chance', although there were some subjects that could not explain it. All the sentences were connected to everyday situations. In all the sentences, the meaning of 'chance' was uncertainty that an event will occur.

[1] Last night the weatherman said that there is almost zero chance of rain today. Today it did rain. Was the weatherman wrong?

One can classify the sentences into two categories. The first category - sentences with yes/no situations. Will the event happen or not. Like: “there is a chance that tomorrow we will win the football game”, or: “there’s no chance that I’ll eat tomatoes”. The other category - sentences with some quantification for 'chance'. Like: “there’s 20% chance that I’ll win the lottery game”, or: “low chances of having a rainy day tomorrow”. (See table 2.)

	6^{th} grade		8^{th} grade		11^{th} grade	
	N	%	N	%	N	%
Yes/no situation	12	63	11	61	11	58
quantification	7	37	7	39	8	42
total	19	100	18	100	19	100

Table 2: Distribution of composing a sentence with 'chance'

Note that almost 60% of the subjects use the word 'chance' for dichotomy situation (will this-and-that happen or not). There is a slight decline with age. The rest of the subjects (about 40%) added some quantification.

Explanations to 'almost zero chance':

All the subjects explained what is 'almost zero chance'. Most of the 6^{th} grade subjects (54%) said that it means “no chance at all”. Most of the 8^{th} grade subjects (56%) explained it well. The number of subjects from the 11^{th} grade who explained it well exceeded 68%.

The weatherman story:

More than 50% of the subjects from each grade said that the weatherman was mistaken. The rest said that he gave the right information, because their still was some probability that there will be rain the next day.

Consistency:

About 40% from the subjects at each grade showed right consistency. They said that 'almost zero chance' means a very small probability, and the weatherman was right. Most of the subjects from the 6^{th} grade (53%) showed wrong consistency. They said that 'almost zero chance' means no chance at all and the weatherman was wrong. Only 44% from the 8^{th} grade subjects showed the wrong consistency. The number decreased (32%) at the 11^{th} grade. There was also a third group - those who had no consistency. They said that 'almost zero chance' means no chance al all and that the weatherman was right, or the other way around - that 'almost zero chance' means a very small chance and the weatherman was mistaken. This group percentage increased from 11% al the 6^{th} and 8^{th} grades to 26% at the 11^{th} grade.
All the above details are gathered in table 3.

We can conclude that about 40% from all subjects understand correctly 'almost zero chance'. They understand that it means very poor chance, but there is still some chance that the event will occur. They understand it correctly when it is related to everyday situations. This rate does not change with age. However, about 40% from the subjects understand 'almost zero chance' as “no chance at all”, or the equation “probability = 0”. This rate decreases with age. (53% at the 6^{th} grate, 44% at the 8^{th} grade, 32% at the 11^{th} grade). An average of 16% from the subjects is inconsistent. This rate increases with age (from 10% at 6^{th} grade up to 26% at the 11^{th} grade).

		6th grade (N=19) N	%	8th grade (N=18) N	%	11th grade (N=19) N	%	Total (N=56) N	%
Explanation	chance=0	12	63	8	44	6	31	26	47
	very poor	7	37	10	56	13	68	30	54
The weather-man story	Wrong	10	53	9	50	11	58	30	54
	Right	8	42	9	50	8	42	26	47
consistency	wrong.	10	53	8	44	6	32	24	43
	right con.	7	37	8	44	8	42	23	41
	No con.	2	10	2	11	5	26	9	16

Table 3: Distribution of 'almost zero chance'

The 'Ladders and Ropes' game:

Before starting the game all member of each group of players were asked what is the chance to get a circle, and what is the chance to get an X. In each group there was at least one player who counted the number of circled and the number of X-s on the faces of the dice. All the subjects agreed that the chances are 50:50. This means they have the same chance to appear. The subjects then started playing the game, and the interviewer stopped the game when one symbol appeared many times more then the other. At this point the interviewer asked one or two players (those who got one symbol more then the other one) what symbol they gut most of the time? Then he asked them what are the chances to get a circle and to get an X, and if it is still 50/50?

Most of the subjects said that the chances to get a circle or an X did not change and they are still 50:50. Only few subjects (3 from the 6th grade, 2 from the 8th grade and 2 from the 11th grade) changed the probability to set a circle or an X according to the results they achieved. In one of the groups, one of the subjects said that, the chances are still the same 50:50 but he had “a special ability to throw the dice and get a circle”, which explained why he got so many circles.

At this point the interviewer asked them to explain how come the dice-tosses are so unbalanced if the chances are the same. The subjects from the 6th grade said that the differences between the numbers of the symbols are a matter of luck. The subjects from the 11th grade said that the numbers of all circles appeared at the game is very close to the number of the all X-s. Another answer was that in the short run one can get one symbol more times then the other, but in the long run the numbers will be almost equal.

Discussion:

From the findings we can see that most of the subjects understand chance as a word that is connected to uncertain events in the future. Following this one may think that they understand correctly what 'chance ' means. **But the key question is: How they understand uncertain events in the future**. That's why the subjects were asked to combine a sentence with 'chance'. We found that the majority of the subjects described yes/no events, and we also note that about a 1/4 of the subjects could not explain what 'chance' means (yet they could compose a sentence with the word). Had we found that the majority of subject who couldn’t explain it was of the youngest (6th) grade, we could have justified it by the limited ability of young students to express themselves. Yet, almost a 1/3 of the subjects from the 11th grade said that they know what 'chance' is and they cannot explain it. We believe this fact alone gives evidence to the fact that many students do not understand 'chance' at all, even though they often use it in everyday life.

Some connected ‘chance’ with yes/no situations, and if so they connect it to equal probability. For example:

Interviewer: Everything that may happen is “50% chance”?
Student: Yes.
Interviewer: Your friend told me that team so-and-so has no chance to win the next game. Do you agree with him?
Student: Yes, there is a great chance.
Interviewer: So there is a great chance. Is it still 50/50?
Student: Yes, it is 50/50.
Interviewer: Why?
Student: Because, if there is a chance - then it is 50/50.
Interviewer: What is the chance that tomorrow will be a nice day?
Student: Very high.
Interviewer: Very high, do you mean higher then 50?
Student: 50/50.
Interviewer: Still 50/50?
Student: Yes, because it may vary.

We got the same picture when we asked about 'almost zero chance'. From the subjects explanations one might conclude that they understand it well. But their reaction to the weatherman story showed that more then 50% of the subjects said that the weather prediction was wrong.
Almost everyone who explained 'almost zero chance' as ‘no chance’ also said that the weatherman was wrong. This was more common among the young subjects. Alternatively, most of the subjects who explain ‘almost zero chance’ as ‘a very poor probability’, said that the weatherman was right. This was more common among the older subjects. These two groups were almost the same in size – about 40% each. We can see the same phenomena in 'chance' and in 'almost zero chance'. Theoretically, teenagers understand these notions well. But when students connect them to everyday life, they might change their meanings – these notions became synonyms to yes/no situations (will happen or will not happen).

The "Ladders and ropes" game demonstrated that most of the subjects showed a correct understanding of 'chance'. For example:

Interviewer: What did you get?
Student: Majority of X-s.
Interviewer: So what are the chances to get X?
Student: Still 50/50.
Interviewer: So how come you got so many X-s?
Student: Because when you throw the dice 3 times, and it does not tell anything (he threw the dice more then 3 times).
Interviewer: So how many times we need to throw the dice?
Student: about 100 times.
Interviewer: If I throw the dice 100 times then what?
Student: Most of the chances that you will get 50/50.

From this we can see that they understand that in the short run one might get one symbol more times then the other, but in the long run the numbers will be equals.

Earlier we claimed that understanding 'chance' in everyday situations is connected to yes/no situations. Here, the game reflects a well profound understanding. There is a contradiction. One way to settle the contradiction is to claim that only few of the subjects understood these ideas (the subjects were gathered in groups during the game). Even though the interviewer posed the question to one player, the other

answered, causing the first players, who were confused by the interviewer's questions, to adopt the answer and agree the student who answered. This is what Way and Ayres (2002) called the fragile probability knowledge and added that subjects do not feel the need to reason consistently.
Another way to explain the contradiction is that throwing a dice in a game like this presents a situation with many outcomes. Experiencing situations like this can build-up a correct understanding of probability. We have no doubt in our minds about the importance and contribution of a game like this to the understanding of probability. However, we believe that because of the many repetitions, and because students learned probability based on such games, students classify chance and probability as theoretic notions, and do not connect them to everyday life.

In this research, we found that there are two types of understanding 'chance'. One type is to understand 'chance' as a theoretical notion, and the other type is to understand 'chance' in connection to everyday life. Students display better understanding of probability when it is related to theoretical situations, especially older ones. When they are asked about everyday situations they connected them to yes/no situation and usually they estimate them as 50/50 chance. Amit and Jan (2006) pointed out that students are developing intuition to differ between numerical probability situations and empirical probability situations. Numerical probability situations are we here "theoretical situations", and empirical probability situations are everyday situations.

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Teaching Derivations of Area and Measurement Concepts of the Circle: A Conceptual-Based Learning Approach through Dissection Motion Operations

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Abstract

In this article a variety of Dissection Motion Operations (DMO) are presented; they are primarily focused at the derivation of the area formula of the circle through hands-on manipulation for the classroom practices in a *conceptually-based learning fashion*. The National Council of Teachers of Mathematics (NCTM, 2000), within its dedication to the improvement of mathematics pedagogy and assessment, has established the standards for mathematics teachers and policymakers in the United States and beyond. Adopted by the Ontario Ministry of Education (2005), the NCTM's principles have infiltrated the Ontario mathematics curriculum. As such, *conceptually-based learning of mathematics versus procedural-based learning* has had an opportunity to flourish in the classrooms of the province of Ontario, Canada, for the past six years. The movement toward reform based mathematics classrooms finds its philosophy grounded in the concerns about procedural based learning that is taught without conceptual understanding. Several conceptual-based approaches to the origin of the formulas such as the area of the circle will be presented in this article.

Background

In reference to the pitfalls of the procedural-based learning methodology in teaching, Freire (2003) has raised a red flag about this methodology and has long written about teachers' responsibility to do more than "fill" students with the contents of lectures. Freire has directly talked about the status of teacher-directed education where "banking" concepts through which students were seen to have the responsibilities in regards to the storing, receiving and filing of deposits. Freire (2003) further argued that in the process of such rote learning procedures, it is the students themselves that are being filed away (p. 55). Freire's position has echoed the National Council of Teachers of Mathematics Standards (NCTM, 2000) calling for conceptual meaningful understanding versus rote learning and memorization. This article will investigate the literature as it pertains to the numeracy level of secondary school graduates in Canada. Research indicates that both students and teachers in North America have difficulty with measurements involving area and that this difficulty can be traced to area attribute problems (Baturo and Nason, 1996; Battista, 2003; Kouba, Brown, Carpenter, Lindquist, Silver and Swafford, 1988). Battista (2003) further discussed the lack of "spatial structuring" which is the underpinning of area and volume concepts and which result in the inability of students to apply what they have learned (p. 132). Baturo and Naon (1996) have made a specific emphasis at the use of manipulatives as a medium for hands-on manipulation in addition to the specific *dynamic* aspect of the area concept stating that:

> In those schools where teachers do provide their students with concrete experiences in developing the notion of area and its measurement, it is often done in a cursory and disconnected fashion or, if done conscientiously, tends to focus almost exclusively on the static perspective of the notion of area to the exclusion of the dynamic of area (Baturo and Nason, 1996, p. 239).

Conceptual understanding of the concepts and processes involved in area measurement includes "knowing that the area of a shape remains the same when it has been changed by 'Cutting and Pasting' to form different shapes" for concrete knowledge (Baturo and Nason, 1996, p. 239). Further, conceptual understanding includes "knowing that congruence and 'cut and paste' transformations conserve the area of a shape; area is a continuous attribute that can be divided into discrete subunits" (Baturo and Nason, 1996, p. 236).
Baturo and Nason (1996) further discussed the importance of students understanding of how pi was derived and the shapes it is associated with along with how the formulae associated with each shape are developed and connected. In this connection, Rahim (2010) has introduced a treatment for teaching the derivation of area formulas for polygonal regions through Dissection-Motion-Operations (DMO) as a systematic refined process for "Cutting and Pasting" acts cited above. Baturo and Nason (1996) further stated that students who combined shapes when attempting solving problems were more stimulated than if they were simply finding the areas of the shapes. Alarmingly, their study revealed that only two of their 13 students understood the relationship between the area of a rectangle and the area of a triangle. As a result, their use of the formula for area held no meaning for them. The students admitted that they did not remember engaging in activities that were meant to enhance their understanding of this relationship (p. 256). Stephan and Clements (2003) recognized that maintaining area is an important concept that it is often not included in measurement instruction. The authors stated that students have difficulty to understand that when a shape is cut up into a finite number of pieces and when the pieces are rearranged into a different shape with no overlapping, its area remains the same. Further, Stephan and Clements (2003) found that research indicated that children use differing strategies to conceptually comprehend the ideas behind the concept of area.

Historically, Ma (1999) stated that the approximation of the area of a circle using the area of a parallelogram has been known since the 17^{th} century (p. 116), (see Smith & Mikami, 1914, p. 131). Ma has indicated how Chinese teachers have made a full lesson by inspiring students to subdivide a circle into sectors and rearrange the pieces into a parallelogram like shape, then, by imagining the number of the sectors increases, a closer resemblance of a parallelogram shape will be reached. And as the approximation process continues the ultimate shape ends up into a parallelogram (and hence into a rectangle by further cutting and moving) implying the presence of the area formula of the circle (p. 116).

Teaching the Derivation of the Area of a Circle: A Dissection-Motion-Operation Approach

Clearly, the number of equivalent sectors in which a circle can be dissected would be either odd or even. As such, there will be three possibilities of dissecting a circle:
Case 1. The number of the equivalent sectors, n, in which a circle can be divided, is even;
Case 2. The number of the equivalent sectors, n, in which a circle can be divided, is odd; and,
Case 3. The number of equivalent sectors, n (even or odd), is a perfect square.

As Case 1 (will be shown below) deals with the relationship between the circle and the parallelogram, it should be noted that this relationship has been known and used for centuries. Most recent elementary mathematics texts have used this relationship to introduce a visual conceptual understanding of the area of a circle in a dynamic and meaningful fashion.

Incidentally, Rahim (2010) has introduced a decomposition-composition (Dissection-Motion) treatment of showing the origins of all area formulas of polygons of all types. Rahim was focusing at the dynamic teaching approach for the derivation of area formulas for polygonal regions through what is called Dissection-Motion-Operations (p. 195). The following are derivations of the formula for the area of the circle through *shape transforms* for each of the three cases listed below.

Case 1: The number of the equivalent sectors, n, in which a circle can be divided, is ***even***. In this case, the circle is transformed into a ***parallelogram*** like shape of equal area through the following steps:

1. Dissection Step: Consider the dissection of the circle given in Figure 1. Note that, in this case, the circle can be dissected into ***n** number of equivalent sectors where n can be any* ***even*** *positive integer* **≥ 4**. However, for simplicity, assume it is dissected into *eight* equivalent sectors as shown in the left part of Figure1 below.
2. Motion Step: Each of the eight individual sectors is appropriately moved under a transformation operation of translation, rotation, and or reflection to form the parallelogram like shape shown at the right part of Figure 1.

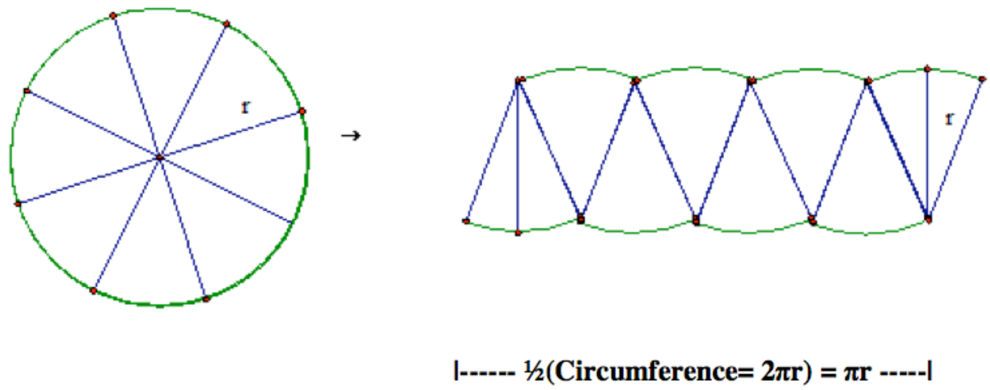

Figure 1. Shape transform of a circle into a parallelogram resemblance through DMO

Then, from Figure 1, it is clear that

$$\text{Area of the circle} = \text{Area of the parallelogram like shape} \approx \tfrac{1}{2}(2\pi r)\times r = \pi r^2.$$

This result holds true when the number of equivalent sectors increases to approach infinity (∞). Accordingly, the shape at the right side of Figure 1 will take the shape of the parallelogram.

Case 2: The number of the equivalent sectors, n, in which a circle can be divided, is *odd*. In this case, the circle is transformed into a ***trapezoid*** like shape of equal area through the following steps:

1. Dissection Step: Consider the dissection of the circle given in Figure 2. Note that, in this case, the circle can be dissected into ***n** number of equivalent sectors where n can be any* ***odd*** *positive integer* **≥ 3**. For simplicity, assume it is dissected into five equivalent sectors as shown in the left part of Figure2 below.

2. **Motion Step:** Each of the five individual sectors is appropriately moved under a transformation operation of translation, rotation, and or reflection to form the trapezoid like shape shown at the right part of Figure 2.

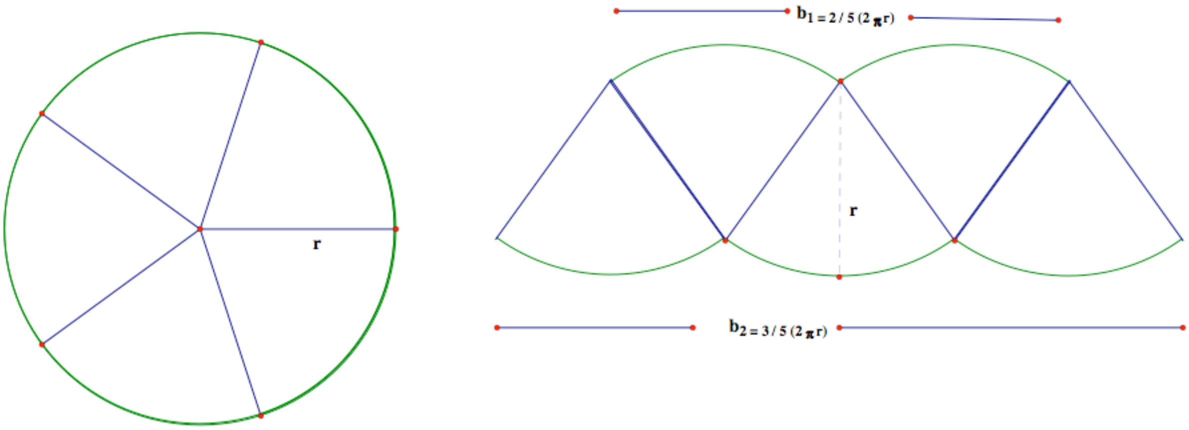

Figure 2. Shape transform of a circle into a trapezoid resemblance through DMO

Then, from Figure 2, it follows that

Area of the circle = Area of the trapezoid like shape

$\approx \frac{1}{2}(h)(b_1 + b_2)$

$= \frac{1}{2}(r)[(2/5)(\mathbf{2\pi r}) + (3/5)(\mathbf{2\pi r}) = \mathbf{\pi r^2}$.

This result holds true when the number of equivalent sectors increases approaching infinity (∞). As a result, the shape at the right of Figure 2 should take the shape of the isosceles trapezoid.

Case 3. The number of equivalent sectors, n (even or odd), is a perfect square. In this case, the circle is transformed into an isosceles triangle like shape of equal area through the following:

1. Dissection Step: Consider the dissection of the circle given in Figure 3. Note that, in this case, the circle can be dissected into ***n** number of equivalent sectors where n can be any* ***perfect square** positive integer* **≥ 4**. Assume the circle is dissected into nine equivalent sectors as shown in the left part of Figure3 below.
2. Motion Step: Each of the nine individual sectors is appropriately moved under a transformation operation of translation, rotation, and or reflection to form the isosceles triangle like shape shown at the right part of Figure 3.

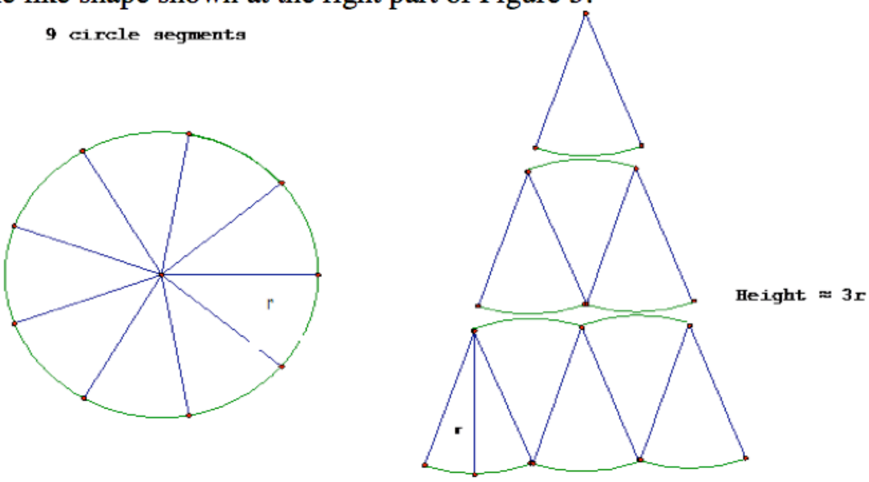

Figure 3. Shape transform of a circle into an isosceles triangle resemblance through DMO

From Figure 3, it follows that

$$\text{Area of the circle} = \text{Area of the isosceles like shape}$$
$$\approx ½ (h)(b)$$
$$\approx ½(3r)[(3/9)(2\pi r) = \pi r^2.$$

And similarly, this result holds true when the number of equivalent sectors increases approaching infinity (∞). Thus the shape at the right side of Figure 3 will take the shape of an isosceles triangle.

Epilogue

Further relationships among 2D shapes can be developed through classroom sessions for building conceptual mathematical understanding of the area and measurement concepts of a variety of shapes in geometry. This type of shapes-to-shape interrelationships is directly related to van Hieles (1985) levels of thought development in geometry. As such, these interrelationships are vital and necessary particularly for middle school levels for a deep understanding of measurement in middle school levels such as grades 8 and 9.

The literature suggests that Canada (and the rest of the world for that matter) is currently struggling with numeracy in general, including in particular, the conceptual understanding of the area of a circle. Battista (2003) has expressed concerns on the lack of spatial comprehension that is resulting in an inability for students to apply what they have learned (p. 131). Ma (1999) and Linquist (1987) have stressed the importance for students to explore multiple perspectives. Stigler and Heibert (1999) have reported how the Chinese teachers in their study were considering frustration and struggle as part of the learning process. On the other hand, Baturo and Nason (1996) reported a failure on the part of teachers in North America to provide concrete experiences that involve the development of the area concept in the classroom. The authors stated further that teachers are teaching "to the exclusion of the dynamic of area" (p. 239). Rahim (1986) have introduced examples of how area can be discussed in a dynamic sense in the classroom.

We have shown in this paper that there are a number of ways for students to discover the area of a circle, offering many opportunities for deep conceptual understanding with which students can move forward along the van Hiele's level of geometric thought development.

The dynamic approach is in no way limited to hands-on manipulations, rather, it can be introduced through the use of Dynamic software too such as the Geometer's Sketchpad (GSP Version 5 – North America-Based software) and Cabri II and Cabri 3D (Grenoble, France-Based software). Furthermore, the dynamic feature in teaching geometric shapes is not restricted to two dimensional objects; it is suitable to 3D objects too. We believe that with a dynamic approach, students are given the opportunity to see themselves as emerging critical thinkers in the process!

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Creating Desirable Difficulties to Enhance Mathematics Learning

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Abstract
We can't make our students into seekers if we aren't seekers ourselves. This research-based, practice-oriented paper explores the nature of "desirable difficulties" and the benefits of creating such desirable difficulties to help students shake naïve or loose thinking and to construct "new" knowledge by encouraging transfer of related prior knowledge to new situations. The paper also discusses certain tasks designed to promote the interconnectedness of mathematical knowledge with respect to mathematical concepts from different branches of mathematics and various representations of mathematical concepts.

The intended purpose of this paper is: (1) to help develop instructional strategies that enable all students to engage in reasoning and mathematical discourse about mathematical ideas; (2) to help teachers understand how mathematical ideas interconnect and build on one another to produce a coherent knowledge base of number, geometry, and data analysis, and (3) to recognize and apply mathematics to solve problems, including those in contexts outside of the "apparent" mathematics currently under study.

Introduction
Not everyone can be a mathematician, but everyone can want to be a mathematician. It is important to understand that being good at mathematics is not evidenced by how many answers you know. Instead, being good at mathematics may be best evidenced by what you do when you don't know the answers. We must help students construct and own "new" knowledge in such a manner that they are then able to apply new knowledge in ways that are different from the situation in which it was originally learned.

Albert Einstein may have captured it best in stating that, "Students that can't explain 'it' simply, don't understand 'it' well enough."

Why do some students successfully learn mathematics and others do not? Successful students internalize processes, connect concepts and ideas, generalize across domains, and develop personal understanding. According to the Piagetian concept of reflective abstraction, students engage in interiorization, coordination, encapsulation, generalization, and reversal. A curriculum that emphasizes these actions will lead to increased likelihood of connections and transfer.

Most agree that teaching is a complex practice and hence not reducible to recipes or prescriptions. If we can also agree that true learning is reflected by the ability to readily transfer knowledge, skills, attitudes and values from one situation to another, then certain assumptions about mathematics teaching automatically deserve attention.

These assumptions include:

Mathematics is more than a collection of concepts and skills.
A goal of teaching mathematics is to help students develop mathematical power.
All students can learn to think mathematically.
WHAT content is learned is fundamentally connected with HOW it is learned.
Content should be explored at different levels of abstraction.

There are no simple means to assure that these assumptions are adequately addressed in all situations. However, the fundamental task of a mathematician is to convey to students not only what mathematicians know, but also what they do, and how and why they do it.

Body

Sometimes learners express a reluctance to look at mathematics in an alternative way to their initial exposure to the topic. Pleas of "You're going to confuse me!" may actually signal an unrecognized, and certainly unacknowledged level of confusion that is ALREADY present – not potential confusion on the horizon.

There are many benefits to be gained by actually creating something that we'll refer to as DESIRABLE DIFFICULTIES designed to encourage thinking about mathematics as well as enhancing both long-term retention and transfer. Out of apparent chaos and confusion emerges a deeper understanding and appreciation. In fact, students who are successful in making sense of mathematics are those that believe that mathematics makes sense.

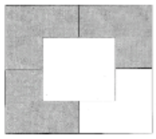
Using the sage principle that "No matter what IT is, the chances of finding IT are dramatically increased if you're looking for IT" we must explore techniques to encourage and reinforce mathematics as a way of thinking. The ideas we gather are like so many pieces of colored glass at the end of a kaleidoscope. They may form a pattern, but if you want something new, different, and beautiful, you'll have to give them a twist or two. You experiment with a variety of approaches. You follow your intuition. You rearrange things, look at them backwards, and turn them upside down. You ask "what if" questions and look for hidden analogies. You may even break the rules or create new ones.

Consider the following incomplete list of students in a class roster:

ANN, Brad, CAROL, Dennis, ???

What name would you propose as the next on this list? Seldom does the question cause "random" answers of "Joe" or Mary" since everyone begins to search for some pattern to apply. Often, the response is along the lines of "Edward" or, perhaps "Edith" – especially those that have self-imposed a rule of alphabetical order. Those suggesting "Edith" may also wish to insist that it should be female to preserve alternating sexes. Then someone chimes in with "Emillie" – with two l's – since the name, whatever it is, should contain seven letters. If the names were also printed in colors, say, black, red, green and blue, then someone else might suggest that the next name should be printed in black to maintain a perceived rotating pattern. Still another would suggest that it should be printed in capital letters. Are we done? How do we know? Have we satisfied EVERY rule? What about the "rule" that calls for three vowels – did you see that? Should we propose "EVELYNN" instead? Does that violate a "rule" that double n's should appear in every fourth entry? And so on and so on… How does one ever know if their rule (or combination of rules) is the right rule (or combination)? What makes an answer correct? Is there more than one? Are there infinitely many? You'll know that you failed in this exemplar if, when finished with the discussion, someone still asks, "But what is the CORRECT answer?" The point here is that it isn't a mistake to have strong views; it is a mistake to have nothing else.

Some "desirable difficulties" challenge us to consider something from a different perspective than initially considered. Such activities shake the foundations of students that believe conclusions "can't be altered." Consider the following graphic as a representation of a fraction. What fraction do you see pictured?



If each student is asked to individually identify a fraction based on this picture you'll find that many different fractions are offered up for consideration. As a challenge for open-mindedness, if student A "sees' ¾ and student B "sees" 3/5" then challenge each to find the other's fraction. Many possible fraction representations can be found in this image. Challenge the students to let go of their first impressions and search for something that they didn't see at first glance. For example, how can this same image also be perceived as 2 ¼?

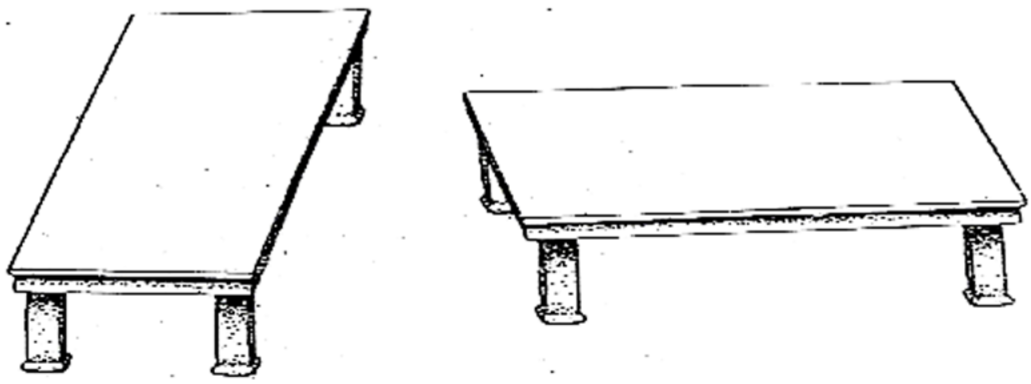
Another example of a "desirable difficulty" is one that involves a situation in which we know WHAT the answer is, we just don't know WHY that is the answer. For instance, suppose a standard deck of cards is arranged into alternating "red-black" order, and a simple cut of the deck is performed once so that the two "half decks" show alternate colors on the bottom card. Follow this by a simple single shuffle, and ask WHY it is that when the cards are drawn off the top in pairs, there will always be one red card and one black card in each pair? If this is NOT the case, then start over – you've done something wrong in the above process. The question is, how can I be so sure? WHY must this always be the case? Why aren't there any pairs of red or pairs of black? Isn't a shuffle SUPPOSED to mix the cards?

A common category of "desirable difficulties" is represented by paradoxes. Unlike most DD's, the main effect of a paradox is that it leaves you scratching your head looking for an explanation. Still, these illustrate the dilemma of unexpected results. A more rewatrding cousin of the paradox is the "desirable difficulty" that has a twist or surprise ending.

Consider the number of segments of length "1" that can be found on a standard 5x5 peg geoboard. A moment's reflection yields the answer of 40 (20 horizontally and 20 vertically). How many segments of length 2? 30 (15 horizontally and 15 vertically). How many of length 3? 20 (10 horizontally and 10 vertically). How many of length

4? 10 (5 horizontally and 5 vertically) So, how many of length 5 on a standard 5x5 geoboard? The pattern shouts for 0, but, in reality, the answer is 8.

One of the most common exemplars of a "desirable difficulty" may be found in an optical illusion. While such illusions neither hold the attention nor have the rich extensions that subsequent exemplars might have, they, nevertheless, provide the reader with a quick emotion of incredulous surprise. Foe example, consider the following illustration.



A2 TURNING THE TABLES

Despite the appearance, the two tabletops are congruent! I'll leave it to the reader to cut a shape to match the table on the left and then rotate it to the table on the right.

The feeling of "impossible" or "that can't be" stays with the reader even after having "proven" the congruence for him/herself. What's missing, though, is the "so what?" of a richer learning experience.

Concepts, terminology, and symbols are the foundations on which mathematics learning and linkages are built. We learn new concepts, terminology, and symbols in many ways by exploring information that is presented to us in many forms. The notion of "having learned something" implies, among other things, that the learner can readily demonstrate the ability to identify, label, use, and transfer the knowledge, skills, and processes learned. While linking mathematics to "real-world" experiences is an effective method of introducing new concepts and emphasizes the "use and transfer" components of learning, student verbalization of connections through desirable difficulties is another significant means of assisting with the "identification and label" components of transfer.

Student exploration emphasizes opportunities for internalizing and anchoring information by having students verbalize – not blindly, but meaningfully - important facts, identifications, definitions, and procedures. Learners internalize information through continued exposure and concerted effort. Students do not learn by doing – they learn by thinking about what they are doing. Mathematics, therefore, is best characterized as a series of action verbs rather than as the rules that those action verbs produce. Students must engage in Modeling, Analyzing, Thinking, Hypothesizing, Experimenting, Musing, Applying, Transferring, Investigating, Communicating and Solving. These practices anchor information and help students absorb and retain the information upon which critical thought in MATHEMATICS is based.

A healthy combination of student action and linking new material to previously learned mathematics concepts, procedures, and practical experiences will set the stage to help students feel more comfortable in their knowledge and understanding of new concepts or procedures. Understanding gained through concept development and linkages, in combination with meaningful memorization of important terms, concepts, and algorithms, gives students power over mathematics. This power leads to confidence and an increased comfort level in their ability to function and reason mathematically.

Linking new material to previously learned mathematics concepts, procedures, and practical experiences sets the stage to help students feel more comfortable in their knowledge and understanding of new concepts or procedures. Mathematics teachers are cognizant that the concepts and skills they teach today are often used later as building blocks for more abstract ideas. It is just as important to be aware of the benefits of using these links as a means of building a spirit of partnership in learning.

It is as important for us to acknowledge as it is for students to realize that they can do the mathematics that we are about to teach -- before we teach it. Students that believe they possess prerequisite knowledge that leads to a "new" concept play a more active role in learning than those who feel the teacher is the sole provider of the knowledge needed to learn a new concept.

Introducing concepts through linkages enables students to relate new ideas to a context of past learning. Students are then more likely to understand and, therefore, absorb new material. For example, students that are being taught to multiply polynomials should be led to the connection between this algorithm and the standard algorithm for multiplying whole numbers taught in the third or fourth grade. Similarly, the student who recognizes "lining up decimal points when you add decimals" as just an extension of “adding like place values from whole numbers” is far ahead of the student who sees this generalization as a NEW rule to learn, unrelated to prior knowledge.

Connecting mathematics to real-world experiences is another effective method of introducing new concepts through linkages. While students too infrequently link their transactions at the store to mathematics class, they often quickly understand that if one candy bar costs fifty cents, then two will cost a dollar. Thus, buying candy at a store can be linked to such mathematics concepts as ratios, proportions, ordered pairs, linear graphs, patterns and functions. But such “simple” links must also be followed by less direct paths to help students find their way to the richness of connections. For example, how might a telephone book help develop an approximation for pi?

Consider this chain of related concepts:

- Subtraction as a difference $[x_1 - x_2]$
- Distance between two points, x_1 and x_2, on a number line
- Distance between two ordered pairs on a grid
  square root of $[(x_1 - x_2)^2 + (y_1 - y_2)^2]$
- Pythagorean theorem $[a^2 + b^2 = c^2]$
- Equation of a circle $[x^2 + y^2 = r^2]$
- Area of a circle $[A = \pi r^2]$
- Monte Carlo probability model $[P(e) = P(A_c)/P(A_s)]$

Since $\dfrac{P(\text{inside circle})}{P(\text{inside square})} = \dfrac{\pi r^2/4}{r^2} = \dfrac{\pi}{4}$

We can then use the last four digits of 200 random numbers in the telephone book. Split each of these into two two-digit numbers. Treat the first pair as "x" and the second pair as "y" and then, Determine how many phone numbers out of the 200 selected yield $x^2+y^2 \leq 10000$ and, using the probability formula, we have an approximation for pi from a telephone book!

Conclusion

Understanding gained through concept development and linkages, in combination with memorization of basic facts and algorithms, gives students power over mathematics. This power leads to confidence and an increased comfort level in their ability to function and reason mathematically. Students exposed to mathematics in this manner are more likely to have the ability to set up problems, not just respond to those that have been previously identified for them. They are more likely to value a variety of approaches and techniques; to have an understanding of the underlying mathematics in a problem; to have the inclination to work with others to solve problems; to recognize how mathematics applies to both common and complex problems; to be prepared for open problem situations; and to believe in the value and utility of mathematics.

Ideally, spending valuable instructional time on desirable difficulties will allow us to better address the expectations of a school mathematics program. As teachers, it is important to recognize that our beliefs about the nature of mathematics influence what students learn and what they perceive mathematics to be. Providing students with a rich array of opportunities to construct meaning from experiences that challenge the ways in which student think and act when confronted with unfamiliar mathematics enhances the ability to make the conceptual leap from concrete to abstract reasoning.

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Why don't we make it our business to teach Business Statistics well?
Some parlous practices and some recommended remedies.

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Abstract

"The Grand Vizier of Randomania wants to know the average number of sheep per family ..." "If the ages of the 4 employees of a pizza shop are 19, 21, 19 and 17, calculate the mean, median and mode." "A bank manager wants to estimate the mean balance for a class of accounts. So, she takes a random sample ..." In university teaching, questions like these do not convey to students the importance and relevance of Statistics in business. They signal that Statistics progresses little beyond content learnt late in primary school and that the discipline has yet to discover the computer database. With slapdash or outdated teaching material, students see abundant errors, inattention to detail, perfunctory analysis and solutions, focus on calculation rather than understanding and application, humdrum repetition of school topics, failure to respond to different learning styles and knowledge levels, and failure to keep pace with current computing practices. These problems are exacerbated as research pressures force academics to further sideline teaching – while encountering a larger and more diverse student body. Although we can't create professional statisticians in a semester or two, we should at least aim to produce critical, well-informed consumers of Statistics. This paper discusses some parlous practices and recommends some remedies.

Three recent events prompted me to write this article. When I told a graduate I had just met that I am a statistician, he politely sympathised with my choice of profession because his experiences indicated that it must be so boring. (Another resounding condemnation of Statistics teaching!) My keenly mathematical son, in planning his Science degree, respectfully told me that he wouldn't do any Statistics because he'd already done it every year in high school. Sharing the lecturing of a colleague's subject caused me to reflect (again) on the teaching of Business Statistics. (The newest edition of the textbook very poor and the colleague was uncomprehending of my criticisms as he was unaware that the edition had changed a year ago and was continuing to teach from the superseded edition.)

Although job opportunities have driven many brighter students into Business, rather than Science, degrees Business students remain fairly innumerate and numerophobic – despite the importance of numbers in business, especially those with $ signs attached. (A coursework Masters student returning to study commented to me recently that the hardest topic of the first two weeks of her Statistics subject was Percentages!)

For those students who don't have the mathematics skills (never obtained them or have forgotten them), lecturers should promulgate an inventory of the prerequisite maths knowledge for their subjects, administer a diagnostic test and provide support material for those who need it. Students may be surprised to be reminded that they don't need mastery of the entire school maths curriculum and that a manageable amount of preparation would greatly enhance their confidence and performance. Such recognition of our students' starting point becomes more important as lecturers increasingly must adapt to the diversity embodied in students from overseas, of low Socio-Economic Status, with mature entry, and of lower tertiary entrance scores.

The same approach should apply to the Excel skills that we take for granted. Despite the apparent ubiquity of computers, undergraduates arrive with little spreadsheeting experience. (Over 6 years of science-based secondary education at a well-resourced, private school, my sons barely saw a spreadsheet.) This is a lack which is very limiting in their university studies in Statistics.

Because many Business students are not from a Maths/Science background, they lack a logical, problem-solving approach to the material. They also lack appropriate study techniques for quantitative content - not appreciating the importance of summarising as a simple, but effective technique, and being apparently ignorant of the vital importance of doing (rather than just reading the solutions of) problems. For a subject using a textbook which contains many problems, I review each problem and produce a list of Basics and a list of Extras – to make a student's task more manageable. I continually encourage them to do Problems – with weekly reminders and with a problem log sheet. I think it incredible that such reminders should be necessary, but was gratified by the following unsolicited email from a student. *"I got 83% ... a big thank you to you since ... your advice "do the problems" is in fact very very very useful. I finished most of the problems ... I've a much better understanding after doing the problems ... Thank you so much for reminding us to do the problems every week."*

We need to provide students with an expectation of something different from the calculation-fest, received annually at high school, and to convince them that there are useful business skills to be acquired. Because of the abiding skepticism about Statistics ("Lies, damned lies ..."), we start with a difficult task – but an important one - and we will fail to convince if we teach poorly. "Figures don't lie, but liars figure." can be an effective rejoinder. In Week 1, I confront my reluctant MBA (Business Insights from Data Analysis) students with the reflection that, if they can't understand and strategically use the data that's relevant to their business sector, then perhaps their competitors can!

Many current(!) textbooks and on-line courses let us down badly. Firstly, they are a cacophony of calculation – often just a set of context-less numbers awaiting the ministrations of a formula! With most calculation done expeditiously nowadays by electronic means, the task of humans is to do the thinking – are students being trained for this? In the year 2011, why do simple linear regression problems still require calculations (even using the "alternate formulae"!) when interpretation should be the focus? Why do textbooks proclaim, "The (arithmetic) mean is the average ..." when three uses of "average" abound in common parlance? Why do textbooks ask students to repeatedly calculate summary measures when we can instead be discussing how these concepts are commonly (mis)used.

- *"I look forward to the day when all Australian workers earn more than the average wage."*
- *"The average salary was $28,467, that is, most people earned about that amount."*
- *"The average Australian pagan is a female Melburnian under the age of 35 who was born in Australia, has a university degree and lives in a de facto relationship."*
- *"Australia is not a country that follows averages; averages are the exception to the rule."* (From a discussion about rainfall.)
- *"A high variance situation lands you back in the head-in-the-oven-feet-in-the-refrigerator syndrome."* (From a discussion about risk in investment portfolios.)
- *"**Ms Average Australian!** According to the latest census, the average Australian is female."* (From a newspaper report.) Does this make me above or below average?

- *"The condition of this book is classified as 3 stars (average)."* (From a website for collectibles – using a 5 star scale.)

Secondly, errors abound in the body of textbooks and in their problem solutions. (What happened to quality control?) I was recently asked to lecture a colleague's subject, for which a 2010 edition of the textbook (textbook B) had recently become available. The publisher had no errata file when I requested it and this turned out to be the most error-prone book I have ever seen! (As the book had already been adopted, a staff member had to be appointed to re-do all solutions.) Perfunctory, procedure-based, error-ridden, incomplete solutions provide no basis for learning. In some books, the examples in the body of the text are not accompanied by electronic data files. So, to work through the examples, students have to manually enter the numbers into a spreadsheet from a table of data on the page.

One popular Business Statistics offering (textbook B) has, through all six of its US editions (dating back before 1997) and its two Australian adaptation editions, an illustration of basic time series properties via a graph which bends back on itself – giving an "interpretation" of as many as three observations at each time period. How does such laxity persist over so many editions, years and co-authors, and so many countries, lecturers and students? (A colleague dismissed my criticism with, "But you read things really carefully!" I replied, "No, I read things!")

Many textbooks (and an on-line course from a very prominent Business school) persistently have trouble, from edition to edition, with defining a symmetric distribution and distinguishing mode and modal class. *"A data set is symmetric if the data set's histogram has a single peak at the center and 'looks the same' to the left and right of the most likely value of the data."* is one offering. (The underlining is mine.) Students are often asked to calculate the mode of a data set which has several equi-frequent modes, the textbook failing to recognise that Excel reports only the smallest of these. The above-mentioned on-line course states, "Let's return to histograms ..." and displays two time series! Textbook A (in its 5th US edition) doesn't have any of the following words in its index: 'average', 'frame', 'statistic', 'parameter', '(regression) coefficient', 'pie chart', 'confidence level', 'level of significance', 'cyclic behaviour', 'seasonality' or 'expected value'.

With such persistently poor teaching materials from highly experienced authors and institutions, it's little surprise if students become dispirited and fail to take our discipline seriously.

Too often, textbooks are not "business friendly" - a histogram with obfuscating bin midpoints (-0.24945, -0.18845, ..., 0.17758, 0.29959), axes with inadequate or no labels, unnecessarily mathematical phraseology ("Hence, …"), a dearth of explanation of the business context and relevance, and so on. This not how we train our students to analyse data and report results professionally and comprehensibly to a lay audience.

A late 1990s review of a newly released Business Statistics textbook referred to *"a 1990s reprint of 1980s publication, written in the 1970s by a 1960s mind"*. It was referring to just one of many texts (then and now) with anachronistic examples that carry over from the author's first edition. Questions like *"A bank manager wants to estimate the mean balance for a class of accounts. So, she takes a random sample..."* look absurd to today's "computer generation". Why are many tables of the standard cumulative Normal distribution (P[0 < Z < z]) not consistent with Excel & current calculators? (P[Z < z])? Why bypass the practical benefits of incorporating the *finite population correction factor* for the archaic justification of simplifying the calculation? Why do the non-heuristic "shortcut" ("alternate") formulae and, often, their derivation still command attention? These anachronisms (and more!) diminish our credibility.

The credibility of our discipline is further undermined by the use of obviously unreal data sets (*viz*., the Grand Vizier of Randomania). *"If the technique is as important as the teacher claims, how is it that he/she has been unable to find a real example?" …Inventing data serves to reinforce the misconception that Statistics is a science of calculation, instead of a science of problem solving*.[1] There are challenges in creating data that are realistic. For example, textbook A provides customer service times at a fast-food restaurant, in minutes, to 15 decimal places!

Many illustrations of the points that I make here will be provided in my presentation. However, they have been omitted here for copyright reasons.

It seems, from much of the currently available teaching material, that we still have a long way to go in taking Business Statistics away from mere calculation to problem solving and plain-language reporting of the results. To this end, we need to pressure publishers for better textbooks (and ancillaries - preferably correct, complete and mutually consistent). Because of wide variations in the content and presentation of teaching material, busy lecturers will still usually need a textbook (or they will be made busier by having to write their own). Through our communities of practice, we can advise each other on suitable choices. (How much credence can we put in reviews that have been written in return for merchandise from publishers?) Sadly, we can't rely on the usual set of "cookie cutter" problems (or their perfunctory solutions) in textbooks for effective teaching. However, fortunately, the daily media provide a fecund source of highly applicable and undoubtedly relevant material.

Some see salvation in the use of on-line materials. However, many of these just repeat the well-worn mistakes of the printed page. Perhaps e-books will introduce new frontiers – perhaps for easier updating, maintaining topicality and correcting errors. We should strive to teach with real business data and problems or we will lack credibility. Admittedly, this may be difficult for reasons of privacy or copyright but we should at least strive for realistic business scenarios. Ideally, teaching departments appoint a moderator to support each subject's coordinator. Part of the moderator's brief should be to bring Business Statistics teaching at least into the 20th century (preferably the 21st).

With recent graduates informing me that they can't implement statistical techniques in the workplace because they fear that their bosses will be intimidated, it seems that we still have a long way to go in teaching Statistics to the business world.

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Using technology to assist Mathematical Literacy learners understand the implications of various scenarios of loan circumstances when buying a house.
(Workshop Summary)

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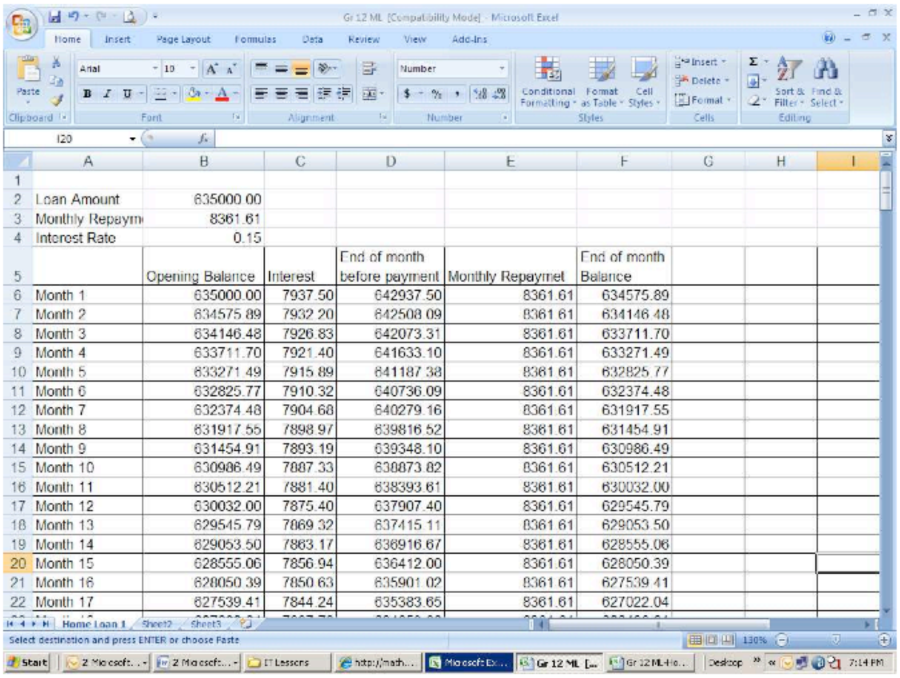
Abstract

Whilst focusing on the contextual topic of buying a house, which is relevant to the South African Mathematical Literacy curriculum. I will be exploring the implications of various scenarios the effects of varying bond terms, changes in interest rates, the impact of additional payments and the consequences of lump sum deposits have on the life of a loan (DoE, 2003). Technology is used to enable the exploration of these various scenarios without the tedious pen and paper method that would be required without an Excel program to extrapolate the data. Excel also allows us to easily change the parameters and see how these changes affect the lifespan of the loan.

Introduction

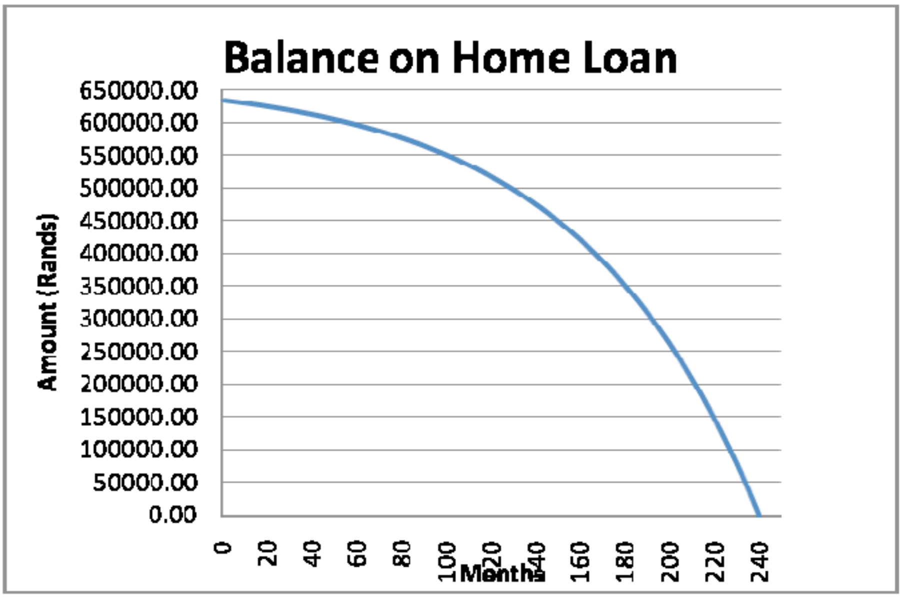
In trying to get learners to understand the intricacies of various scenarios when it comes to buying a house and taking out a bond, I find Excel removes the computational restraints of pen and paper calculations. Alagic (2003) suggests that technology allows learners to engage interactively with problems enabling them to see the immediate effect of changes and therefore comprehending better what happens with parameters are changed.

So the lesson starts with us looking at setting up the Excel spreadsheet, in order to do this the leaner needs to have a basic understanding of the working of Excel and the theory behind how interest and payment calculations work with respect to a loan.



	A	B	C	D	E	F
1						
2	Loan Amount	635000.00				
3	Monthly Repaym	8361.61				
4	Interest Rate	0.15				
5		Opening Balance	Interest	End of month before payment	Monthly Repaymet	End of month Balance
6	Month 1	635000.00	7937.50	642937.50	8361.61	634575.89
7	Month 2	634575.89	7932.20	642508.09	8361.61	634146.48
8	Month 3	634146.48	7926.83	642073.31	8361.61	633711.70
9	Month 4	633711.70	7921.40	641633.10	8361.61	633271.49
10	Month 5	633271.49	7915.89	641187.38	8361.61	632825.77
11	Month 6	632825.77	7910.32	640736.09	8361.61	632374.48
12	Month 7	632374.48	7904.68	640279.16	8361.61	631917.55
13	Month 8	631917.55	7898.97	639816.52	8361.61	631454.91
14	Month 9	631454.91	7893.19	639348.10	8361.61	630986.49
15	Month 10	630986.49	7887.33	638873.82	8361.61	630512.21
16	Month 11	630512.21	7881.40	638393.61	8361.61	630032.00
17	Month 12	630032.00	7875.40	637907.40	8361.61	629545.79
18	Month 13	629545.79	7869.32	637415.11	8361.61	629053.50
19	Month 14	629053.50	7863.17	636916.67	8361.61	628555.06
20	Month 15	628555.06	7856.94	636412.00	8361.61	628050.39
21	Month 16	628050.39	7850.63	635901.02	8361.61	627539.41
22	Month 17	627539.41	7844.24	635383.65	8361.61	627022.04

Secondly we chart the lifespan of the loan (again learners need to be familiar with how to chart graphs using Excel).



Then it is time to investigate what effect changing parameters, such as interest rate, increased payments, lump-sums, additional payments have on the lifespan of the loan. Learners are now able to change these parameters to see what happens to their charts.

Once learners have an understanding of loans I usually then take them onto a banking website to introduce them to the interactive software that can be found there to see what criteria must be met in order to qualify for a loan. It highlights the ratio of earnings to repayments and gives a clear indication of how much one can afford.

This is a good example of real world applications of mathematics through graphical representation of information through the use of technology. It addresses the curriculum criteria of being able to draw graphs by hand/ technological means as required by situations and critically interpreting tables and graphs in real life situations. This enables learners of Mathematics Literacy to critically use their skills to prepare themselves for real life situations. It also moves away from the traditional talk and chalk and becomes more explorative for learners in which to grapple with the real-life problems that they will be confronted with.

Conclusion:

When teaching my grade 12 learners I do feel that the use of the above method helps them gain deeper understanding as to the effects of various scenarios on loans. They themselves can manipulate the loan setup to see what happens when they do what. It also allows them to take the initiative as to what to change to see what happens and if it is as they predicated. This is something that I feel has worked quite effectively in my classroom.

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Developing Skills for Successful Learning

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Abstract
Many of the teachers in the less advantaged schools in South Africa lack skills, varied teaching techniques and capacity. Multiple changes to the curriculum have left teachers confused about content and application of teaching methods and strategies. Many of these educators are demoralised and confused, which has negatively impacted on their confidence and self-esteem. Many teachers see each support intervention as a new and stand alone entity, and many educators often do not have the skill or experience to either use or integrate the resources that are present in the schools. The focus at Edupeg is to add value to the provincial and national educational endeavour, through facilitating improved curriculum delivery, by increasing material resources, enhancing intellectual capacity and promoting the confidence and self-esteem of both teachers and pupils. Through sensitively exposing teachers to more effective teaching methods and strategies we are able to create opportunities for children to be actively and creatively engaged in their own learning. Our goal is to achieve improved quality of teaching in the classroom, which will lead to improved pupil comprehension, and ultimately, performance.

INTRODUCTION – A practical and interactive session
Many teachers think that "good teaching" consists of a teacher dominating the class proceedings, predominantly through talking, to impart knowledge to the quiet and attentive pupils. A pre-requisite for this teaching style, is that pupils pay attention to what the teacher is saying. As often is observed, the outcome of such an approach is that the children become passive and eventually inert to the extent that they are virtually mute – with virtually no participation in their own learning process. Teachers rely on the more skilled and alert pupils, to answer the occasional posed questions. This process actually masks the inadequacies of the weaker learners, who become further withdrawn due to their lack of participation and their growing perception of their lack of skills and competence. A sense of low levels of confidence and self-esteem tends to become the result of this process. "The biggest obstacle standing in the way of the achievement of young people is what they believe about themselves." Professor Jonathan Jansen, "How to build excellence in a culture of mediocrity", U.C.T. February 2011.

Children should be seen as thinkers, not empty vessels which need to be filled with knowledge. They are indeed active participants in the construction of their own knowledge. It is during these early years, when the first networks of knowledge based on experiences are formed, that the foundations for all later learning are laid. The environments of home and school enhance and support each other in learning. They provide the context in which learning takes place and the opportunity for learning to be effective. These environments obviously influence the child's growth and development, and are the foundation for all learning situations. Extensive studies by Piaget concluded that knowledge is created actively and is not received passively. It is thus vital to create and provide an exciting and stimulating learning environment.

The Edupeg programme provides stimulating, interactive opportunities for young learners themselves to interact with learning resources. While practically engaging with Edupeg activities, children learn to recognise, evaluate and cope with educational and developmental challenges. Edupeg activities also help to develop concentration, perseverance, memory skills, social skills, self-confidence self-esteem, and cognitive skills, including thinking and reasoning.

This comprehensive, learner centred educational programme, is fun and challenging, but not threatening. The activities encourage a positive attitude towards learning and the self-corrective aspect promotes self evaluation, immediate access to right or wrong answers and has obvious multiple benefits in a classroom environment.

There are 22 Edupeg workbooks, which are available in English, Afrikaans, isiXhosa, isiZulu, Sesotho & Pedi. They have no racial gender or cultural bias. The workbooks contain activities which promote and enhance the development of visual perception. They then incorporate Numeracy, Literacy, Language and Life Skills, which extend the development of the child, and increase mathematical awareness and proficiency, as well as promote and improve literacy and communication skills.

The realities in less advantaged classrooms

- Classes are often very large, and many teachers are without assistants, or have assistants who are not effectively used.
- Many classrooms are not big enough to accommodate a mat area, or lack a mat, and so small groups of pupils are not brought forward to work and interact directly with the teacher.
- Pupils have not been trained in routines which keep them actively involved at their tables.
- Classrooms lack sufficient resources, or resources are not used. Also methods of storing, distributing and caring for resources are not taught or practiced.
- There is no differentiation of pupils into ability groups.
- To maintain discipline in the large class, teachers resort to teaching from the front and require all the pupils to be attentive, and not to talk, except for the repetition in unison, required for rote learning.
- Pupils frequently spend long periods of time doing nothing.

The above severely disadvantages the learners because:

- Many learners enter school when they are not school-ready, having inadequately developed perceptual skills. They do not advance easily, until these skills have been developed.
- Educators often do not realise that the children learn skills through "doing", and the learners are not given sufficient opportunity to be actively engaged.
- Learners often have limited language. Without sufficient encouragements to talk and communicate, language does not develop, self-esteem is low, thinking skills are impeded, and social skills do not develop.
- Learners become very dependent, often copy each other, and do not develop creativity. Thinking skills and problem solving skills are also stunted.
- In many cases, learners adopt survival strategies, which are counterproductive, and difficult to "undo" later.
- In the educator's haste to "cover" the curriculum, many basic underlying skills are not given the time and attention needed, which then has a follow-on effect of the learners' battling as they progress, through their schooling.

Edupeg learning values

Edupeg understands that learners learn best when they:

- do things
- explore and discover things for themselves
- have fun
- communicate with each other
- are not afraid of failing
- feel good about themselves

Edupeg also helps learners to develop their skills and abilities. Edupeg develops learners who:

- can communicate
- can solve problems
- are confident
- can work with others
- have the skills they need to cope in life

A strong focus of Edupeg activities is on Numeracy.

Numeracy
Edupeg:

- draws on the child's intuitive and acquired knowledge in number as a stimulus for continued learning
- provides stimulation and enjoyment through the varied activities provided
- encourages confidence, understanding and creative individual thought
- consolidates basic number operations
- increases the ability to communicate mathematically
- builds on knowledge and experience and assists in the understanding of the above in the child's world
- encourages mathematical communication and the correct use of mathematical symbols and terminology
- promotes systematic and accurate written work, including all calculations
-

Language and Literacy play a crucial role in the acquisition of skills.

Language & Literacy
Edupeg:

- encourages familiarity and fluency of language
- supports vocabulary enrichment and extension
- provides exposure to accurate written language, spelling and reading
- creates opportunities for story-telling, including sequencing and ordering of events
- provides practice in and pride of mother tongue
- encourages appreciation of visual stimulation and discussion thereof
- supports interpretation, selection, understanding and processing of given information
- nurtures the development of thinking skills and problem solving techniques
- allows for opportunities to incorporate 2nd/3rd language usage, from the visual stimulation provided

Additionally,

Life Skills
Edupeg encourages and promotes:

- communication
- problem solving
- organisation and planning
- decision making
- comparison of methods and options
- discovery and exploring of relationships
- expanding the awareness of mathematics in it's relationship to human beings
- enjoyment and pleasure

Effective learning resources need to be well conceived and compiled.
Suitability of Edupeg materials

- Content written by experienced South African school educators
- All materials have proved to be durable and the majority, re-usable
- Activities are appealing to the target age group
- Approach and content of materials are curriculum appropriate
- The books contain no cultural, racial or gender bias

- Instructions are simple and clearly understood by learners
- The activities are able to satisfy the needs of learners with differing abilities
- Sufficient variety of activities for stimulation
- Adequate repetition of activities for revision and consolidation
- Simple provision for on-going and continual assessment
- Content, including pictures, diagrams and rubrics stimulate learners' interests and thinking skills, and expand their often limited world
- Activities are enjoyable, varied, challenging and promote independence and self-reliance
- The complexity and depth of activities promote concentration and the active involvement of the learners
- Material available in mother tongue
- Provided with a comprehensive Teacher Resource Book for additional guidance and further ideas

Education needs to be meaningful, and have positive outcomes, where accurate and appropriate teaching resources are utilised.

Benefits of Edupeg

- Learners are engaged in challenging tasks, appropriate for their capacity and level of skills
- There is an improvement in responsiveness of pupils – they are active
- The creation of an active, stimulating, learner centred environment which promotes pupil participation
- Methodical recording of set activities, including the method used to attain and determine answers – promotes concentration and thinking skills as well as neat, accurate recording, including spatial concepts
- Learners improve vocabulary, language, sentence construction and communication skills
- Experiences are provided for children to be active through the asking of open questions, where learners can be right. This builds confidence, self-esteem, fosters creativity and promotes individuality.
- Group work encourages and develops critical thinking skills, verbal communication, active involvement, listening skills, tolerance and problem solving strategies
- Gives learners themselves the opportunity to "find answers" utilising their own talents, and to figure it out rather than to "know" i.e. needing to rely on previous knowledge/rote learning
- Working independently promotes self-reliance and the ability of the learners to work without constant teacher intervention
- Many children fall down in assessment due to their lack of experience and exposure to the format of questioning. Edupeg provides such opportunities.
- Pictures offer experiences that children can own and learners can respond to these. They are not the "teacher's" pictures
- Diminishes the teacher's need to be prescriptive
- Instructions are clear, simple and easy to follow
- Fine motor control is developed
- Diminishes the need for loud/punitive classroom discipline
- Kinetic, Auditory, Tactile and Visual Learning takes place, NOT just Auditory.
- Utilisation of concrete and semi-concrete equipment to assist learners to develop a sound understanding of concepts
- Assists teachers to use and manage resources

A varied range of appropriate classroom practices need to be utilised, and educators need support to become more effective and proficient in the classroom.

Combatting Talk & Chalk!

- Educators are assisted to promote differentiated, learner-centred activities
- A balance can be created between exposition and learner activity
- Learners are actively engaged in tasks, appropriate for their developmental level
- Dialogue, thinking skills, concentration, vocabulary, communication, and tolerance, are all assisted and developed
- Learning takes on aspects of fun and enjoyment
- Learners become enthusiastic and less inert/passive
- Learners are absorbed in tasks and discipline problems diminish
- The pace and content of the lesson is not dictated by the slowest learners
- A handful of learners will not be responsible to "carry" the class, while a large percentage of the class remains mute/inert
- Educators are able to promote more individual questioning
- Teachers can diminish chanting in unison and rote learning
- Educators are able to fulfil continuous assessment, as they can **see** and **evaluate** what learners can **do**

Competent and prepared teachers promote productive learning environments. All teachers are supported by our sensitive, well qualified trainers.

Learn to plan

- Objectives are clearly defined and then the method to achieve these is decided upon. (The "how", "with what", "for how long", "what next"?)
- Suitable resources are selected to fulfil objectives
- Appropriate activities are selected, suitable for level of learner's ability (How many teachers **tell** us that they have a huge differentiation of learners in their class, but teach **all** the **same thing** at the **same time** for the **same length of time**)
- Concrete equipment can be included. (The lack of concrete teaching is a large contributor to the very low level of maths functioning seen in so many schools)
- Active learning enables students to gain skills and hone skills
- Provision is made for learners to have practice in reading instructions, not having instructions read
- Provision is made for learners to methodically and accurately fulfil written activities (NOT just filling in words/figures/completing sentences on photo-copied sheets)
- Written tasks promote concentration, consolidation of knowledge, logic, critical thinking skills, spatial concepts, pride and self-esteem
- Opportunities are created for the educators to assess the set tasks are created

Educators are assisted and encouraged as they seek to improve their skills and capacity.

Constructive classroom support

- Teachers are given practical support, not theoretical workshops
- Teachers who have a limited knowledge and experience of a subject/concept to be taught, as well as a lack of awareness of how best to achieve the desired outcome/ objective of a lesson, are assisted
- Even when teachers appear to understand the content of workshops/training courses, to implement this, **alone**, on their return to school, is **not easy**. Many lose heart and revert to old methods. (Poor system for integrated grade/phase support)
- Teachers become overwhelmed by the task of managing themselves and the learning environment. They need classroom support
- Training is internalised only when it is **used** in the classroom, in **context** and teachers need assistance with this. It is a lonely and difficult process and teachers can easily become discouraged

- Only in the classroom, can a real understanding of concepts and principles be reached
- Relationships of trust need to be established, where support and guidance are received without fear of criticism and censure
- Edupeg helps teachers to promote reasoning, observation and deduction
- Patience, persistence and perseverance are needed as trainers penetrate schools through sound educational practice

The HSRC Study shows that the development of children is hampered by poor teacher input. Assisting educators to become more proficient, more actively engaged in their teaching and more aware of teaching outcomes is crucial to improved learning.

Building confidence and self-esteem

- Assisting educators and learners to realise that if they **understand** concepts and **internalise** them, they can **build** on this knowledge, i.e. they do **not** need to memorise formulae/theories/recipes, etc
- Providing opportunities to "find"/"figure out"/"resolve" a problem using skills, knowledge, reasoning, talents, intelligence and **common sense!**
- Providing practical experiences to perform tasks that involve using concrete equipment, pictures, data, graphs, etc and allowing **time** for answers to be found
- Kinetic, auditory, tactile and visual learning must be encouraged, **not** just auditory
- Relating Maths (and other Learning Areas) to everyday life and real situations
- Reducing the learners' (and educators') anxiety and feelings of inadequacy through providing positive learning experiences. Success to be obtained through self-reliance
- Fostering creativity, language development, sentence construction, communication, thinking skills, opinions, self-expression, etc through adapting an "open" question policy, rather than a "closed", "Yes/No" approach. Children can be right! Educators to enter into dialogue with learners, and to probe the answers given
- Provide opportunities for children to read simple, clear instructions and then to complete the task. Many learners fall down on external assessments due to their lack of experience and exposure to this type of questioning

We all respond to recognition and acknowledgement and teachers are no different.

Addressing low morale

- Encourage teachers who have experienced intervention overload, not to respond with passive resistance
- To develop teachers, who presently feel inadequate
- We need to get teachers "doing", which will help to promote understanding
- Knowledge and awareness will diminish resistance (to change)
- We need to create opportunities for teamwork, experimentation, investigation and practice
- We need to assist teachers to try new methods, which will eliminate their needing to try to control and teach, large, often multi-grade classes from the front, which is **exhausting**
- We need to banish fear
- We need to create and promote hope

The above will be covered in an interactive workshop session, where participants will use the Edupeg resources in a simulated classroom environment.

What I hear, I forget. What I see, I remember. What I do, I understand. ***Confucius***

Teaching Mathematical Modelling to Tomorrow's Mathematicians or,

You too can make a million dollars predicting football results.

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Abstract:

One of the reasons for studying mathematics is to empower us with the tools to enable us to predict with some certainty what will happen in given scenarios. Meteorologists study weather patterns and gather data to produce mathematical models that allows them to forecast the weather, with various degrees of success. Car designers use complicated mathematical models to continually refine automobiles that are stronger, more efficient and more powerful. From sport to demographics to engineering to medicine to business, we are surrounded by mathematicians who are continually modelling the world around us.

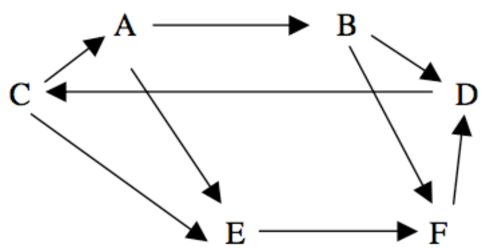
As educators of mathematics we spend the majority of our time on knowledge and content. Teaching the art of Problem Solving is a challenging endeavour indeed. Teaching Mathematical Modelling requires insight and planning to be effective.

This paper will explore an activity that I have devised to allow novice mathematicians to take on a modelling role. They create a model from data, refine the model based on new data, and finally evaluate the strength and weaknesses of their model.

Introduction:

In Queensland, Australia, teachers of years 11 and 12 design and write their own assessment. The assessment is required to be of two types, standard exam and extended response which most people would know as an assignment. There are three criteria all assessment items must address, I Knowledge and Procedures, II Modelling and Problem Solving and III Communication and Justification. I will not elaborate on the first and third of these Criteria, but address certain aspects of the second.

Firstly though, it would be timely to review some basic Dominance Theory using Matrices. This is a technique for ranking teams or players who are playing in a round robin competition and the competition is at the stage where every team has not played each other. Say there are 6 teams in a competition, and so far 3 of the 5 rounds have been completed. The results could be shown like this



Team A has had two wins, one over team B and one over team E but lost to team C. Suppose we were now required to rank the teams from 1 to 6. Clearly 3 teams, A, B and C have had 2 wins and a loss. Can we split these 3 teams to decide the top ranked team?

We could place these results in a matrix, called the Dominance matrix where a "1" represents a win, and a zero represents a loss, or did not play.

Team C vs	A	B	C	D	E	F
A	0	1	0	0	1	0
B	0	0	0	1	0	1
C	1	0	0	0	1	0
D	0	0	1	0	0	0
E	0	0	0	0	0	1
F	0	0	0	1	0	0

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} = \beta$$

First order dominance is simply a win. A defeated B so A has first order dominance over B. Second order dominance occurs for A over F as A defeated B and then B defeated F. We could then use this second order dominance to split the three teams on 2 wins. To calculate second order dominance we simply have to square the matrix above.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} = \beta^2$$

This indicates that team A had two second order dominances over team F as 1) A defeated B and B defeated F and 2) A defeated E and E defeated F.

Now to combine these two matrices, it is possible to weight the significance of second order dominance slightly less than first order by scalar multiplication, eg

$$\beta + 0.5\,\beta^2 = \begin{bmatrix} 0 & 1 & 0 & .5 & 1 & 1 \\ 0 & 0 & .5 & 1.5 & 0 & 1 \\ 1 & .5 & 0 & 0 & 1.5 & .5 \\ .5 & 0 & 1 & 0 & .50 & 0 \\ 0 & 0 & 0 & .5 & 0 & 1 \\ 0 & 0 & .5 & 1 & 0 & 0 \end{bmatrix} \text{(Sum rows)} \rightarrow$$

A	3.5
B	3
C	3.5
D	2
E	1.5
F	1.5

As A and C have 3.5 points, and C defeated A we could rank C above A. Likewise E and F have similar points, and E defeated F hence we could rank E above F.
This then allows us to theoretically rank our 6 teams after 3 of the 5 rounds and possibly use this ranking to predict subsequent rounds. The ranking would be C, A, B, D, E and F.
If A was drawn to play D in the next round we would expect A to win and so on. Obviously this does not always happen for a variety of reasons.