Results

Table 1 and Figure 1 show the distribution of the group of students. As we can see, the largest group is the Spanish one (94,7%), although there is a considerable number, in absolute terms, of members of the other groups.

GROUP	Frequency	Percentage
FOREING	71	0,9%
GUINEA	190	2,3%
PRISON	169	2,1%
SPAIN	7719	94,7%
Total	8149	100,0%

Table 1: Frequency Distribution of Groups

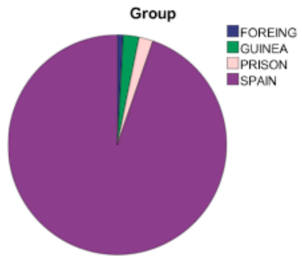


Figure 1: Sector Diagram of Groups

Figure 2 shows the distribution of standardized value of the different components of mathematical competence and global assessment of the mathematical competence for each of the groups considered.

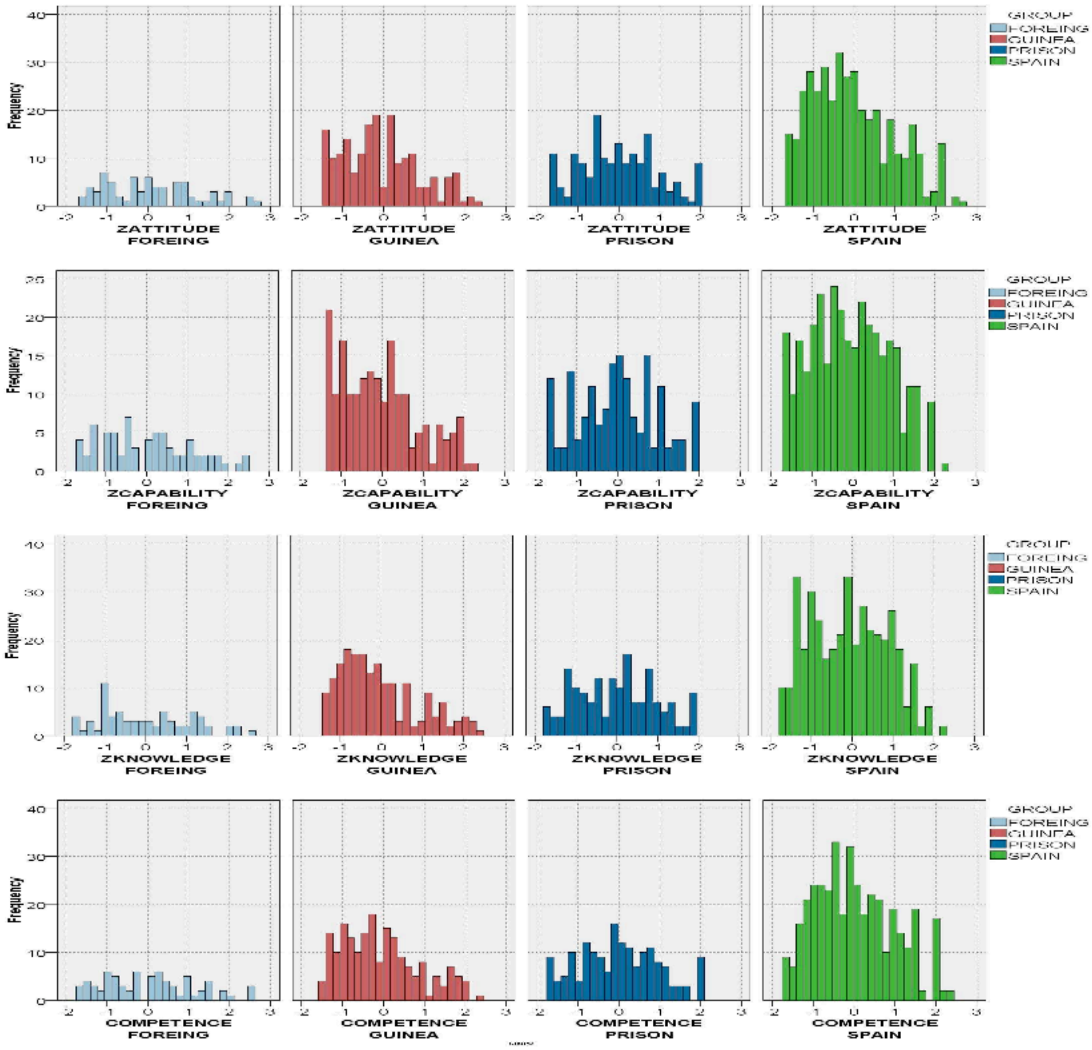
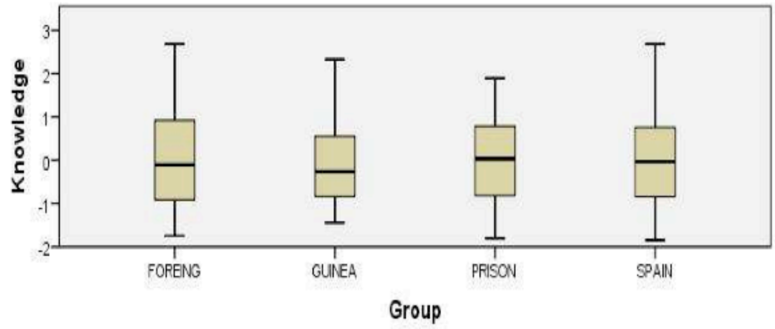
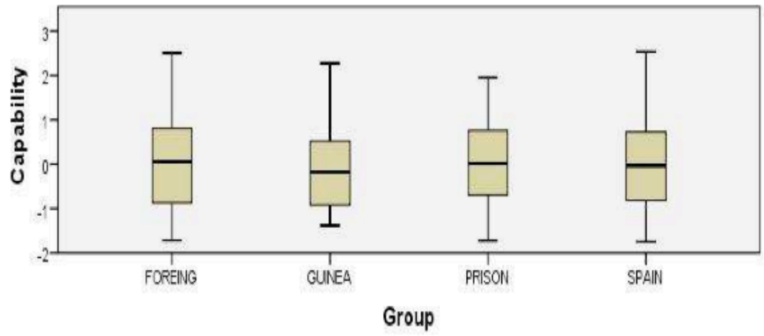


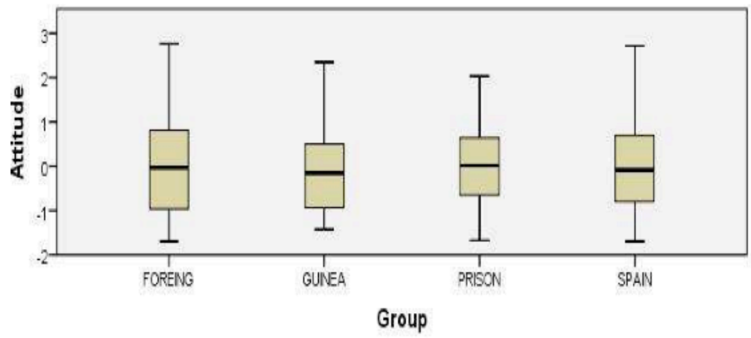
Figure 2: Distribution of the components of mathematical competence by group.



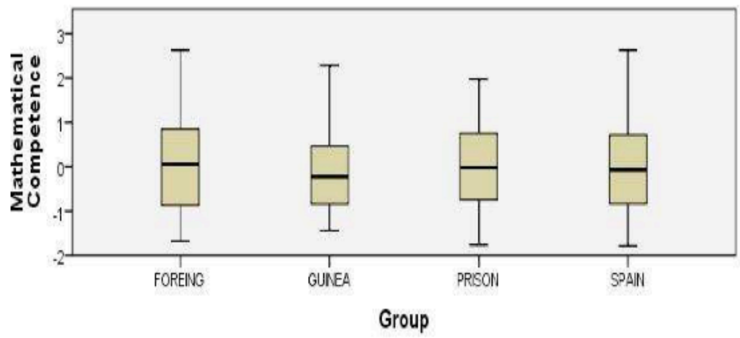
Statistic adjusted for ties	p-value
1.006	0.800



Statistic adjusted for ties	p-value
1.030	0.792



Statistic adjusted for ties	p-value
1.085	0.781



Statistic adjusted for ties	p-value
1.097	0.790

Figure 3: Kruskal Wallis test for comparison of levels of the components of the mathematical competence.

Figure 3 shows the results of Kruskal-Wallis test for comparison of the four groups, including statistical significance levels of the test. As it can be seen, all the comparisons are not significant, so we can accept the hypothesis of equal distributions of the components of mathematical competence in the differente groups.

Comparing the subcomponents of mathematical competence there were differences only in case of *Uncertainty*, see Figure 4. In this case, the statistics adjusted for ties is 55.496 and the p-value is 0.000.

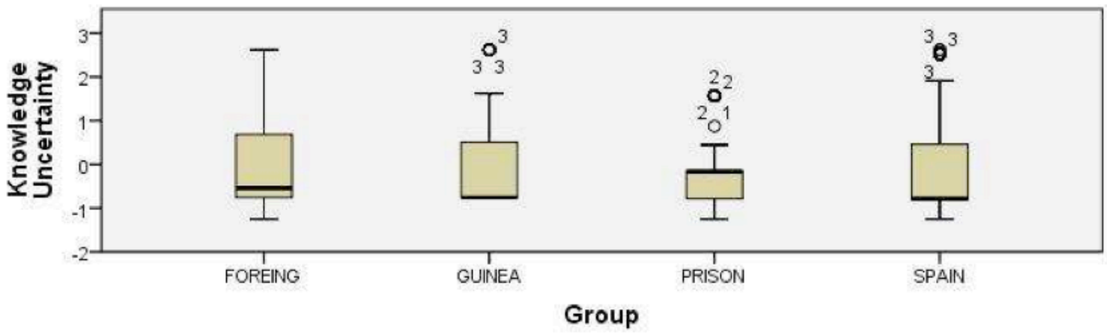


Figure 4 Kruskal Wallis test for comparison of levels of the subcomponent *Uncertainty*

Discussion

Assessment of the mathematical competence can be accomplished by adapting traditional assessment systems, such as the use of objective type tests, introducing aspects that allow them to quantify the level of each competence's component and subcomponent. Using this evaluation method is possible to obtain information about the student competence level in a more complete way than that obtained by traditional methods.

When comparing the level of mathematical competence among different groups of students attending to social, cultural and geographical factors, such as the origin of different countries, belonging to a developing country or the circumstances of these criminal serving time in a center prison, there are no significant differences in the level shown in the various components of mathematical competence. However, there are significant differences in the level of *Uncertainty* subcomponent. This subcomponent relates to knowledge concerning the study of phenomena involving chance and probability, as well as mathematics relating to statistical thinking. We have no clear explanation for this, although it may perhaps be due to cultural differences that may exist in the understanding of chance and statistics.

References

Leví, G. and E. Ramos (2011): Mathematical Competence Assessment of a Large Groups of Students in a Distance Education System, In: A. Rogerson (Ed.): *Turning Dreams in Reality: Transformations and Paradigm Shifts in Mathematics Education*, 11th International Conference of the Mathematics Education into the 21th Century Project, Grahamstown, South Africa.

Ramos, E; R. Vélez; V. Hernández; J. Navarro; E. Carmena and J.A. Carrillo (2009): *Sistemas inteligentes para el diseño de procedimientos equilibrados para la evaluación de competencias*, In M. Santamaría y A. Sánchez-Elvira (coord.): *La UNED ante el EESS. Redes de investigación en innovación docente 2006/2007*, Colección Estudios de la UNED, UNED, Madrid, pp.597-610.

Ramos, E; R. Vélez; V. Hernández; J. Navarro; E. Carmena and J.A. Carrillo (2010): *Competencias en Matemáticas Aplicadas a las Ciencias Sociales y su Evaluación Inteligente*, In M. Santamaría y A. Sánchez-Elvira (coord.): *La UNED ante el EESS. Redes de investigación en innovación docente 2007/2008*, Colección Estudios de la UNED, UNED, to be published.

PHANTOM GRAPHS.

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Abstract. While teaching "solutions of quadratics" and emphasising the idea that, in general, the solutions of $ax^2 + bx + c = 0$ are obviously where the graph of $y = ax^2 + bx + c$ crosses the x axis, I started to be troubled by the special case of parabolas that do not even cross the x axis. We say these equations have "complex solutions" but **physically, where are these solutions**?
With a little bit of lateral thinking, I realised that **we can physically find the actual positions of the complex solutions of any polynomial equation** and indeed many other common functions! The theory also shows clearly and pictorially, why the complex solutions of polynomial equations with real coefficients occur in conjugate pairs.

Fig 1: The big breakthrough is to change from an ***x* AXIS…..**

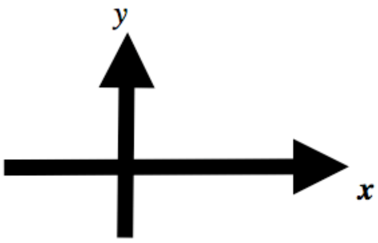
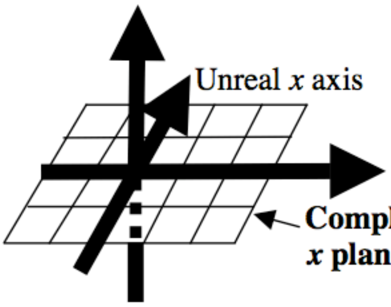


Fig 2

……..to a **complex *x* plane!**



Introduction. Consider the graph $y = x^2$.
We normally just find the positive y values such as: (±1, 1) , (±2, 4), (±3, 9) but we can also find **negative y values** even though the graph does not seem to exist under the x axis:

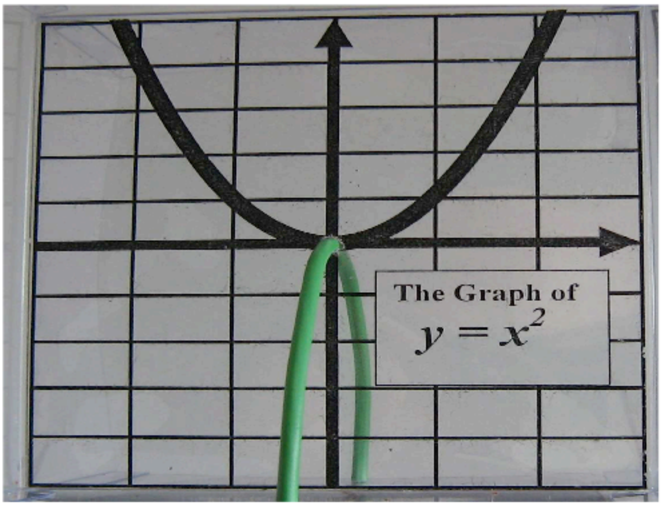
If $y = -1$ then $x^2 = -1$ and $x = \pm i$.
If $y = -4$ then $x^2 = -4$ and $x = \pm 2i$.
If $y = -9$ then $x^2 = -9$ and $x = \pm 3i$

Thinking very laterally, I thought that instead of just having a ***y axis*** and an ***x AXIS*** (as shown in **Fig 1**) we should have a ***y axis*** but a **complex *x* PLANE!** (as shown on **Fig 2**)

This means that the usual form of the parabola $y = x^2$ exists in the normal ***x, y*** plane **but another part of the parabola exists at right angles to the usual graph.**

Fig 3 is a Perspex model of $y = x^2$ and its "phantom" hanging at right angles to it.

Fig 3



<u>"PHANTOM GRAPHS".</u> Now let us consider the graph $y = (x - 1)^2 + 1 = x^2 - 2x + 2$

The minimum real y value is normally thought to be $y = 1$
but now we can have any real y values!

If $y = 0$ then $(x - 1)^2 + 1 = 0$

so that $(x - 1)^2 = -1$

producing $x - 1 = \pm i$

therefore $x = 1 + i$ ***and*** $x = 1 - i$

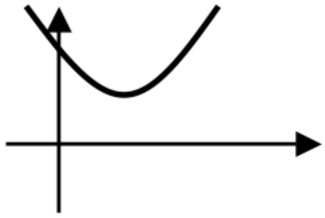
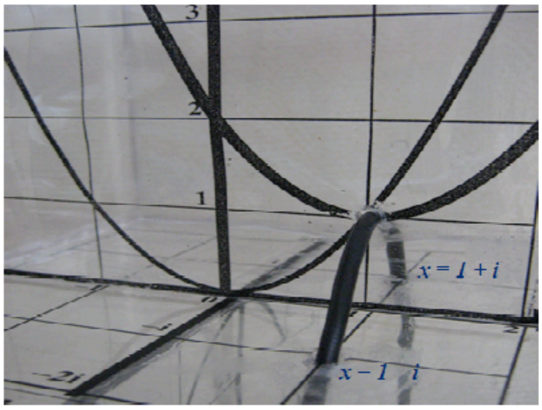


Fig 4



If $y = -3$ then $(x - 1)^2 + 1 = -3$

so that $(x - 1)^2 = -4$

therefore $x = 1 + 2i$ ***and*** $x = 1 - 2i$

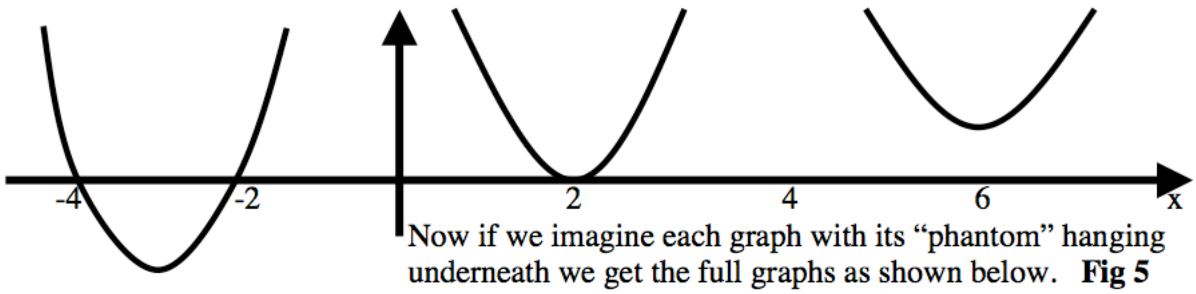
Similarly if $y = -8$ then $(x - 1)^2 + 1 = -8$

so that $(x - 1)^2 = -9$

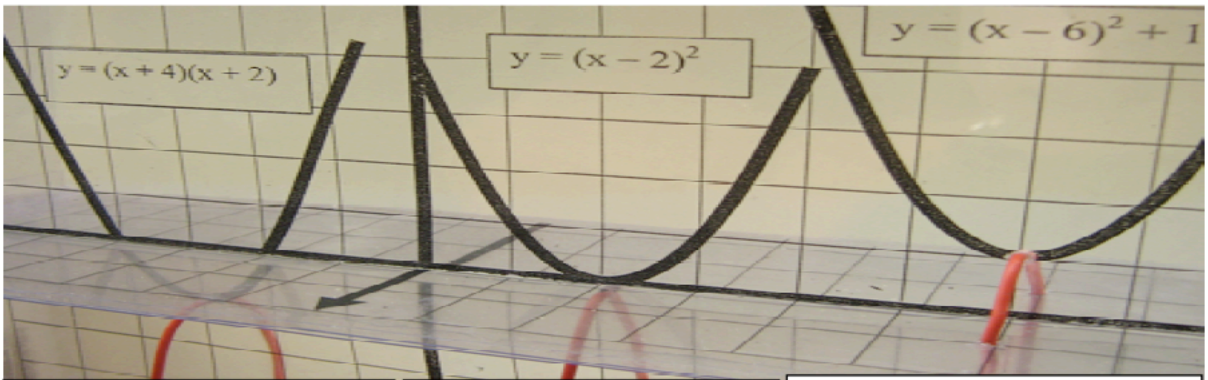
therefore $x = 1 + 3i$ ***and*** $x = 1 - 3i$

The result is another "phantom" parabola which is "hanging" from the normal graph $y = x^2 - 2x + 2$ and the exciting and fascinating part is that the **solutions of** $x^2 - 2x + 2 = 0$ are $1 + i$ and $1 - i$ which are **where the graph crosses the x plane**!
See **Fig 4**

In fact ALL parabolas have these "phantom" parts hanging from their lowest points and at right angles to the normal x, y plane.
It is interesting to consider the 3 types of solutions of quadratics.
Consider these cases: $y = (x + 4)(x + 2)$; $y = (x - 2)^2$; $y = (x - 6)^2 + 1$



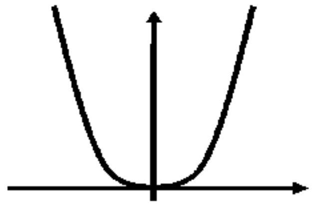
Now if we imagine each graph with its "phantom" hanging underneath we get the full graphs as shown below. **Fig 5**



Here the "phantom" has no effect on the solutions $x = -4$ *and* $x = -2$.	Notice that the curve goes through the point $x = 2$ twice! (a double solution)	The solutions are where the graph crosses the ***x plane*** at $x = 6 \pm i$ **(a conjugate pair)**

Now consider the graph of $y = x^4$

We normally think of this as just a U shaped curve as shown.
This consists of points (0, 0), (±1, 1), (±2, 16), (±3, 81) etc
The fundamental theorem of algebra tells us that equations of the form $x^4 = c$ should have 4 solutions not just 2 solutions.

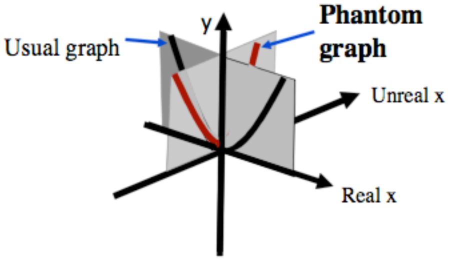


If $y = 1$, $x^4 = 1$ so using De Moivre's Theorem: $r^4 cis\ 4\theta = 1 cis\ (360n)$
$r = 1$ and $4\theta = 360n$ therefore $\theta = 0, 90, 180, 270$ producing the 4 solutions :
$x_1 = 1\ cis\ 0 = 1,\ x_2 = 1\ cis\ 90 = i,\ x_3 = 1\ cis\ 180 = -1$ and $x_4 = 1\ cis\ 270 = -i$

If $y = 16$, $x^4 = 16$ so using De Moivre's Theorem: $r^4 cis\ 4\theta = 16 cis\ (360n)$
$r = 2$ and $4\theta = 360n$ therefore $\theta = 0, 90, 180, 270$ producing the 4 solutions :
$x_1 = 2\ cis\ 0 = 2, x_2 = 2\ cis\ 90 = 2i, x_3 = 2\ cis\ 180 = -2, x_4 = 2\ cis\ 270 = -2i$

Fig 6

This means $y = x^4$ has another **phantom** part at right angles to the usual graph.



The points (1, 1), (–1, 1), (2, 16), (–2, 16) will produce the ordinary graph but the points (i, 1), (–i, 1), (2i, 16), (–2i, 16) will produce a similar curve at right angles to the ordinary graph. **Fig 6**

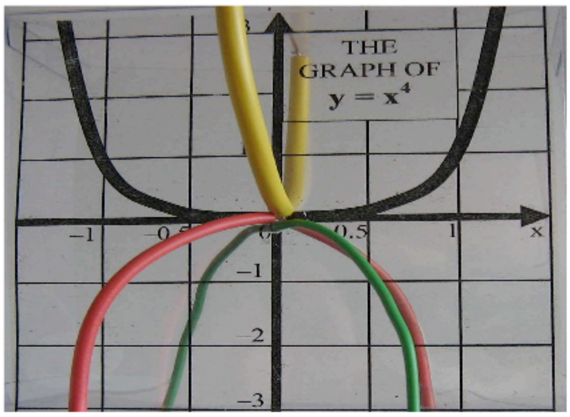
But this is not all!

We now consider **negative** real y values!

Consider $y = -1$ so $x^4 = -1$
Using De Moivre's Theorem:
$r^4 cis\ 4\theta = 1 cis\ (180 + 360n)$
$r = 1$ and $4\theta = 180 + 360n$ so $\theta = 45 + 90n$
$x_1 = 1\ cis\ 45,\quad x_2 = 1\ cis\ 135,$
$x_3 = 1\ cis\ 225,\ x_4 = 1\ cis\ 315$

Similarly, if $y = -16$, $x^4 = -16$
Using De Moivre's Theorem:
$r^4 cis\ 4\theta = 16 cis\ (180 + 360n)$
$r = 2$ and $4\theta = 180 + 360n$ so $\theta = 45 + 90n$
$x_1 = 2\ cis\ 45,\ x_2 = 2\ cis\ 135,$
$x_3 = 2\ cis\ 225, x_4 = 2\ cis\ 315$

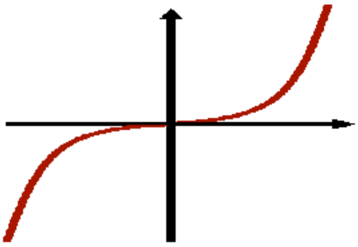
Fig 7 (photo of Perspex model)



The points corresponding to negative y values produce two curves identical in shape to the two curves for positive y values but they are rotated 45 degrees as shown on **Fig 7.**

NOTE: Any horizontal plane crosses the curve in 4 places because all equations of the form $x^4 = \pm c$ have **4** solutions and it is clear from the photo that the solutions are conjugate pairs!

<u>Consider the basic cubic curve $y = x^3$.</u>



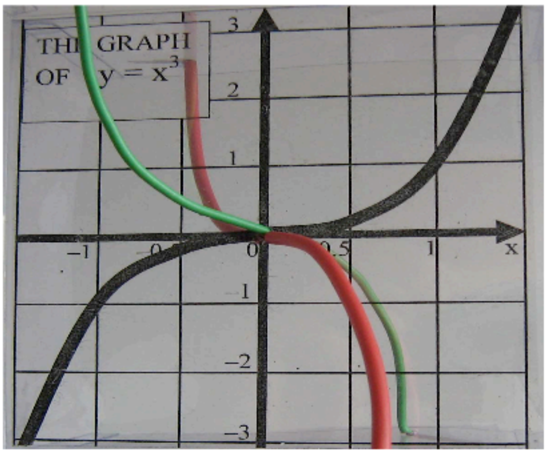
Equations with x^3 have 3 solutions.
If $y = 1$ then $x^3 = 1$
so $r^3 cis\ 3\theta = 1 cis\ (360n)$
$r = 1$ and $\theta = 120n = 0, 120, 240$
$x_1 = 1\ cis\ 0,\ x_2 = 1\ cis\ 120,\ \ x_3 = 1\ cis\ 240$

Similarly if $y = 8$ then $x^3 = 8$
so $r^3 cis\ 3\theta = 8 cis\ (360n)$
$r = 2$ and $\theta = 120n = 0, 120, 240$
$x_1 = 2\ cis\ 0,\ x_2 = 2\ cis\ 120,\ \ x_3 = 2\ cis\ 240$

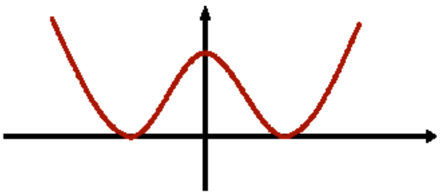
Also y can be negative. If $y = -1,\ \ x^3 = -1$
so $r^3 cis\ 3\theta = 1 cis\ (180 + 360n)$
$r = 1$ and $3\theta = 180 + 360n$ so $\theta = 60 + 120n$
$x_1 = 1\ cis\ 60,\ x_2 = 1\ cis\ 180, x_3 = 1\ cis\ 300$

The result is THREE identical curves situated at 120 degrees to each other!
(See Fig 8)

Fig 8 (photo of Perspex model)



<u>Now consider the graph $y = (x + 1)^2(x - 1)^2 = (x^2 - 1)(x^2 - 1) = x^4 - 2x^2 + 1$</u>



Any horizontal line (or plane) should cross this graph at 4 places because any equation of the form $x^4 - 2x^2 + 1 = C$ (where C is a constant) has 4 solutions.

If $x = \pm 2$ then $y = 9$
so solving $\quad x^4 - 2x^2 + 1 = 9$
we get : $\quad x^4 - 2x^2 - 8 = 0$
so $\quad (x + 2)(x - 2)(x^2 + 2) = 0$
giving **x = ±2 and ±√2 *i***

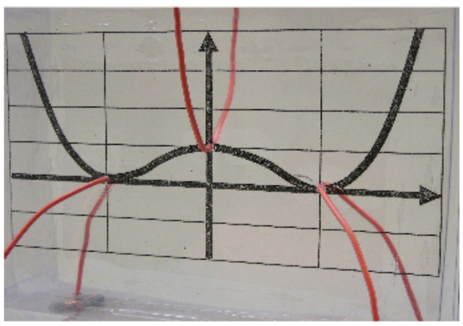
Similarly if $x = \pm 3$ then $y = 64$
so solving $\quad x^4 - 2x^2 + 1 = 64$
we get $\quad x^4 - 2x^2 - 63 = 0$
so $\quad (x + 3)(x - 3)(x^2 + 7) = 0$
giving **x = ±3 and ±√7 *i***

The complex solutions are all of the form **0** ± n***i***. This means that a **phantom curve**, **<u>at right angles</u>** to the basic curve, stretches upwards from the maximum point.

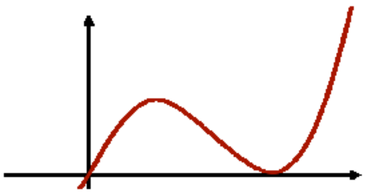
If $y = -1,\ \ x = -1.1 \pm 0.46i$, $1.1 \pm 0.46i$
If $y = -2,\ \ x = -1.2 \pm 0.6i$, $1.2 \pm 0.6i$
If $y = -4,\ \ x = -1.3 \pm 0.78i$, $1.3 \pm 0.78i$

Notice that the real parts of the x values vary. This means that the **phantom** curves hanging off from the two minimum points are not in a vertical plane as they were for the parabola. See **Fig 9. Clearly all complex solutions to $x^4 - 2x^2 + 1 = C$ are conjugate pairs.**

Fig 9 (photo of Perspex model)

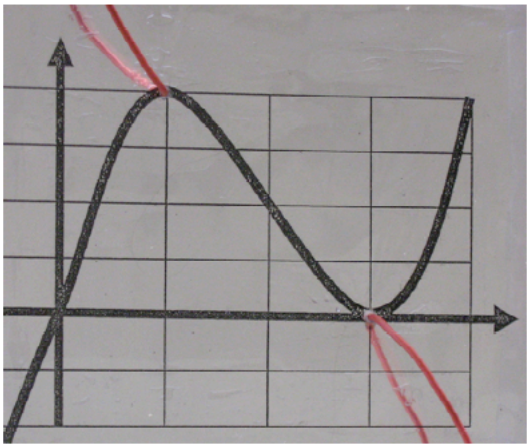


Consider the cubic curve $y = x(x-3)^2$



As before, any horizontal line (or plane) should cross this graph at 3 places because any equation of the form: $x^3 - 6x^2 + 9x = $ "*a constant*", has 3 solutions.

Fig 10 (photo of Perspex model)

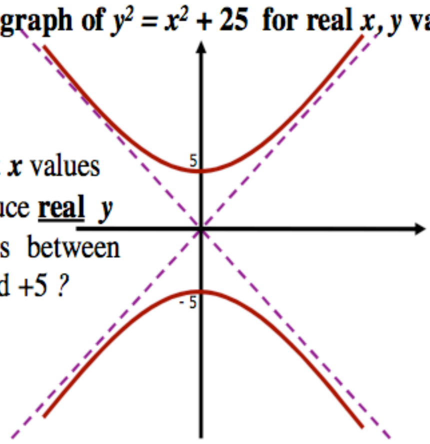


If $x^3 - 6x^2 + 9x = 5$ *then* $x = 4.1$ and $\underline{\mathbf{0.95}} \pm 0.6i$
If $x^3 - 6x^2 + 9x = 6$ *then* $x = 4.2$ and $\underline{\mathbf{0.90}} \pm 0.8i$
If $x^3 - 6x^2 + 9x = 7$ *then* $x = 4.3$ and $\underline{\mathbf{0.86}} \pm 0.9i$
So the left hand phantom is leaning to the left from the maximum point (1, 4).

If $x^3 - 6x^2 + 9x = -1$ *then* $x = -0.1$ and $\underline{\mathbf{3.05} \pm 0.6i}$
If $x^3 - 6x^2 + 9x = -2$ *then* $x = -0.2$ and $\underline{\mathbf{3.1} \pm 0.8i}$
If $x^3 - 6x^2 + 9x = -3$ *then* $x = -0.3$ and $\underline{\mathbf{3.14} \pm 0.9i}$
So the right hand phantom is leaning to the right from the minimum point (3, 0). See Fig 10

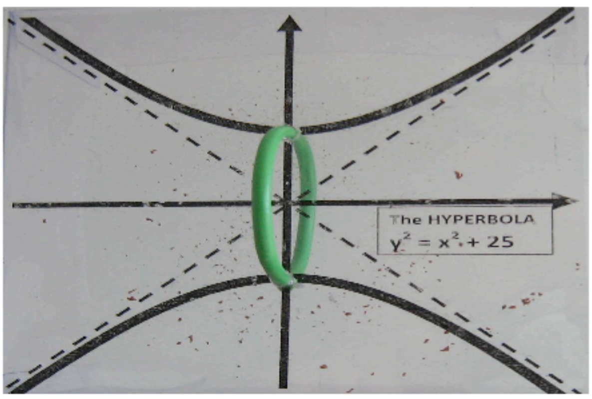
The HYPERBOLA $y^2 = x^2 + 25$. This was the most surprising and absolutely delightful Phantom Graph that I found whilst researching this concept.

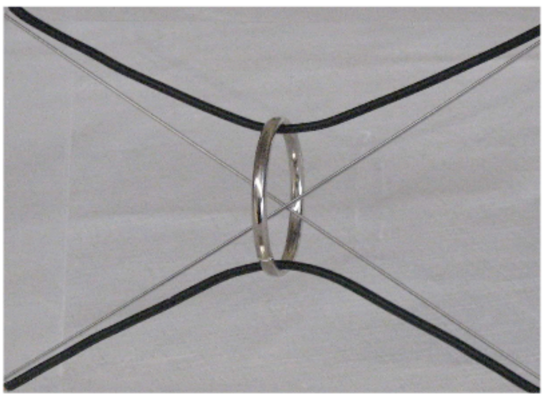
The graph of $y^2 = x^2 + 25$ for real x, y values.



What x values produce **real** y values between -5 and +5 ?

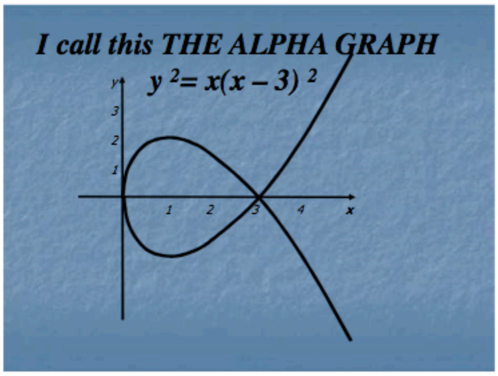
If $y = 4$ then $16 = x^2 + 25$
and $-9 = x^2$
so $x = \pm 3i$
Similarly if $y = 3$ then $9 = x^2 + 25$
so $x = \pm 4i$
And if $y = 0$ then $0 = x^2 + 25$
so $x = \pm 5i$
These are points on a circle of radius 5 units.
$(0, 5)$ $(\pm 3i, 4)$ $(\pm 4i, 3)$ $(\pm 5i, 0)$
The circle has complex x values but real y values.
This circle is in the plane at right angles to the hyperbola and joining its two halves!
See photos below of the Perspex models.

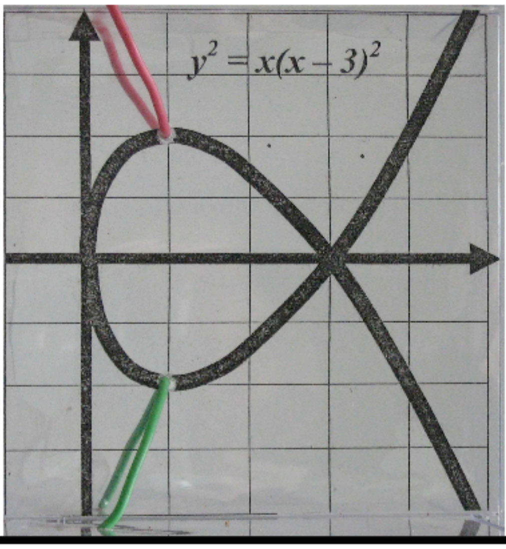




AFTERMATH!!!

I recently started to think about other curves and thought it worthwhile to include them.





Using a technique from previous graphs:
I choose an x value such as $x = 5$,
calculate the y^2 value, ie $y^2 = 20$ and $y \approx 4.5$
then solve the equation $x(x-3)^2 = 20$
already knowing one factor is $(x-5)$

$$\text{ie} \quad x(x-3)^2 = 20$$
$$x^3 - 6x^2 + 9x - 20 = 0$$
$$(x-5)(x^2 - x + 4) = 0$$
$$x = 5 \text{ or } \tfrac{1}{2} \pm 1.9i$$

This means (5, 4.5) is an "ordinary" point on the graph but two "phantom" points are ($\tfrac{1}{2} \pm 1.9i$, 4.5)

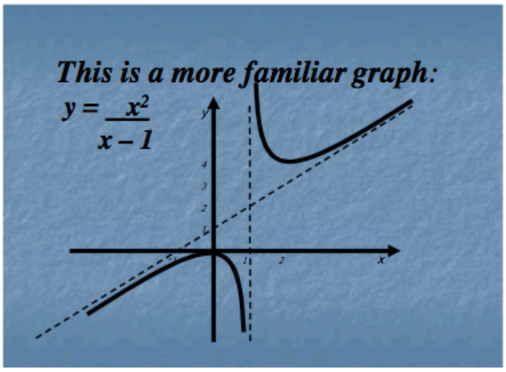
Similarly:
If $x = 6$, $y^2 = 54$ and $y = \pm 7.3$ so $x(x-3)^2 = 54$

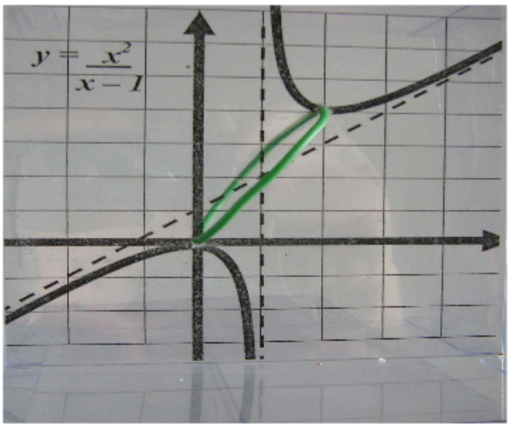
$$x^3 - 6x^2 + 9x - 54 = 0$$
$$(x-6)(x^2+9) = 0$$
$$x = 6 \text{ or } \pm 3i$$

And
If $x = 7$, $y^2 = 112$ and $y = \pm 10.6$ so $x(x-3)^2 = 112$

$$x^3 - 6x^2 + 9x - 112 = 0$$
$$(x-7)(x^2 + x + 16) = 0$$
$$x = 7 \text{ or } -\tfrac{1}{2} \pm 4i$$

Hence we get the two phantom graphs as shown.





$y = \dfrac{x^2}{x-1}$ **Here we need to find complex x values which produce real y values from 0 to 4.**

If $y = 0$ $\quad x = 0$

If $y = 1$ $\quad x = \dfrac{1}{2} \pm \dfrac{\sqrt{3}i}{2}$

If $y = 2$ $\quad x = 1 \pm i$

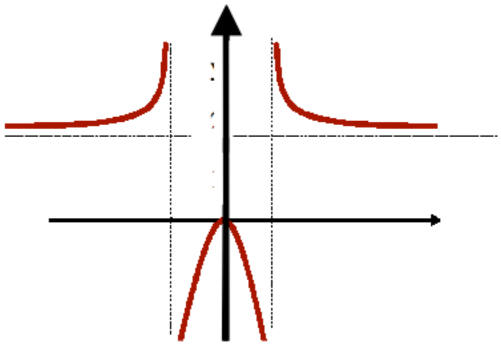
If $y = 3$ $\quad x = \dfrac{3}{2} \pm \dfrac{\sqrt{3}i}{2}$

If $y = 4$ $\quad x = 2$

These points produce the phantom "oval" shape as shown in the picture on the left.

Consider the graph $y = \dfrac{2x^2}{x^2 - 1} = 2 + \dfrac{2}{x^2 - 1}$

This has a horizontal asymptote $y = 2$
and two vertical asymptotes $x = \pm 1$



If $y = 1$ ***then*** $\dfrac{2x^2}{x^2 - 1} = 1$

so $2x^2 = x^2 - 1$

and $x^2 = -1$

producing $x = \pm i$

If $y = 1.999$ ***then*** $\dfrac{2x^2}{x^2 - 1} = 1.999$

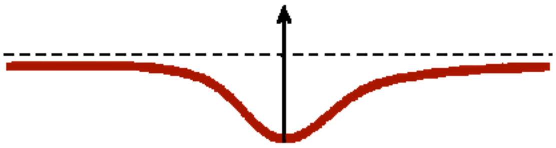
so $2x^2 = 1.999x^2 - 1.999$

and $0.001x^2 = -1.999$

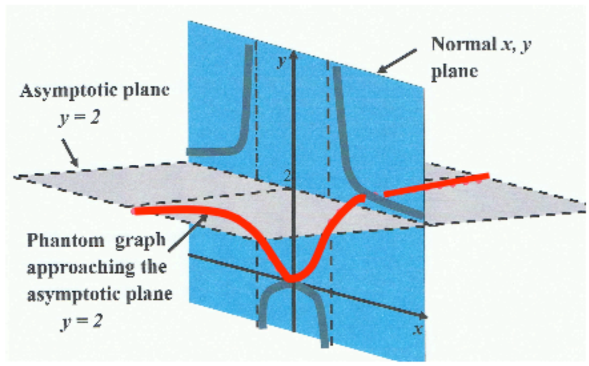
Producing $x^2 = -1999$

$x \approx \pm 45\, i$

This implies there is a "phantom graph" which approaches the horizontal asymptotic plane y = 2 and is at right angles to the x, y plane, resembling an upside down normal distribution curve.

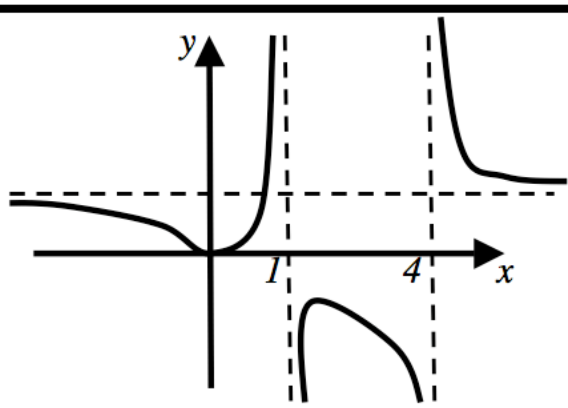


Side view of "phantom" approaching y = 2



Consider an apparently "similar" equation but with a completely different "Phantom".

$y = \dfrac{x^2}{(x-1)(x-4)} = \dfrac{x^2}{x^2 - 5x + 4}$



The minimum point is (0, 0)
The maximum point is (1.6, –1.8)

If $y = -0.1$, $x = 0.2 \pm 0.56i$

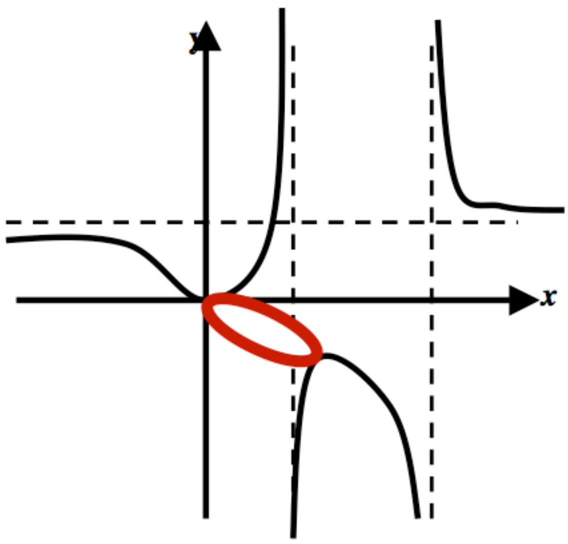
If $y = -0.2$, $x = 0.4 \pm 0.7i$

If $y = -0.5$, $x = 0.8 \pm 0.8i$

If $y = -1$, $x = 1.25 \pm 0.66i$

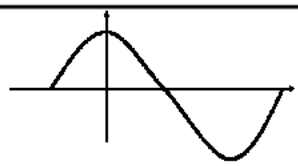
If $y = -1.5$, $x = 1.5 \pm 0.4i$

If $y = -1.7$, $x = 1.6 \pm 0.2i$



These results imply that a "phantom" ***oval*** *shape joins the minimum point (0, 0) to the maximum point (1.6, –1.78).*

The final two graphs I have included in this paper involve some theory too advanced for secondary students but I found them absolutely fascinating!



If $y = \cos(x)$ what about y values > 1 and < - 1 ?

Using $\cos(\boldsymbol{x}) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots\ldots\ldots$

Let's find $\cos(\pm\, \boldsymbol{i}) = 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \frac{1}{8!} \ldots\ldots\ldots$

$\approx$ **1.54 (ie > 1)**

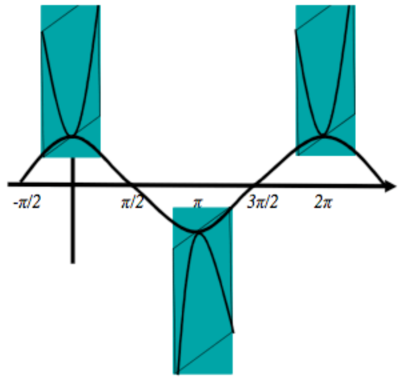
Similarly $\cos(\pm\, 2\boldsymbol{i}) = 1 + \frac{4}{2!} + \frac{16}{4!} + \frac{64}{6!} + \ldots$

$\approx$ **3.8**

Also find $\cos(\boldsymbol{\pi + i}) = \cos(\pi)\cos(\mathrm{i}) - \sin(\pi)\sin(\mathrm{i})$

$= -1 \times \cos(\mathrm{i}) - 0$

$\approx$ **– 1.54 (ie < –1)**



These results imply that the cosine graph also has its own "**phantoms**" in vertical planes at right angles to the usual ***x, y*** graph, emanating from each max/min point.

Finally consider the exponential function $y = e^x$. How can we find x if $e^x = -1$?

Using the expansion for $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots\ldots$

We can find $e^{xi} = 1 + xi + \frac{(xi)^2}{2!} + \frac{(xi)^3}{3!} + \frac{(xi)^4}{4!} + \frac{(xi)^5}{5!} + \ldots\ldots$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \ldots\right) + i\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\right)$$

$$= \quad (\cos x) \quad + \quad i\ (\sin x)$$

If we are to get **REAL y values** then using $e^{xi} = \cos x + i \sin x$, we see that **sin *x*** must be zero.

This only occurs when $x = 0, \pi, 2\pi, 3\pi, \ldots$ (or generally $n\pi$)

$e^{\pi i} = \cos\pi + i\sin\pi = {}^{-}1 + 0i$, $e^{2\pi i} = \cos 2\pi + i\sin 2\pi = +1 + 0i$

$e^{3\pi i} = \cos 3\pi + i\sin 3\pi = {}^{-}1 + 0i$, $e^{4\pi i} = \cos 4\pi + i\sin 4\pi = +1 + 0i$

Now consider $y = e^X$ where $X = x + 2n\pi i$ (ie even numbers of π)

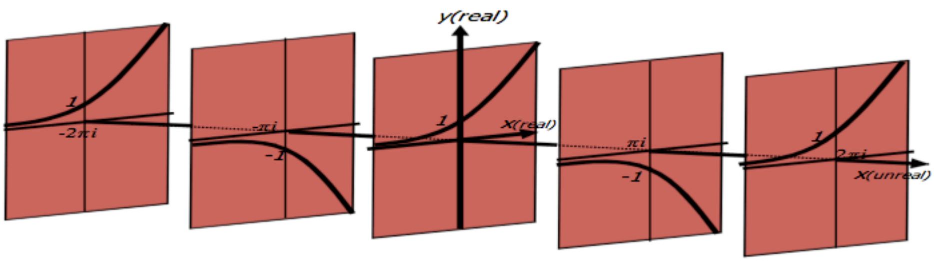
ie $y = e^{x + 2n\pi i} = e^x \times e^{2n\pi i} = e^x \times 1 = e^x$

Also consider $y = e^X$ where $X = x + (2n+1)\pi i$ (ie odd numbers of π)

ie $y = e^{x + (2n+1)\pi i} = e^x \times e^{(2n+1)\pi i} = e^x \times {}^{-}1 = -e^x$

This means that the graph of $y = e^X$ consists of **parallel identical curves** if $X = x + 2n\pi i$
$= x +$ even N^{os} of **πi**

and, **upside down parallel identical curves** occurring at $X = x + (2n + 1)\pi i = x +$ odd N^{os} of **πi**



Graph of $y = e^X$ where $X = x + n\pi i$

Workshop: Error Analysis of Mathematics Test Items

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The workshop will help mathematics teachers develop an understanding of learner errors and misconceptions and how these errors and misconceptions can be embraced in the teaching and learning of mathematics. The workshop will draw on an activity developed by the Data Informed Practice Improvement Project (DIPIP). Some examples of learner errors from an international test will be given. Participants will be given the opportunity to discuss and identify the mathematical content needed to answer the question, analyse the correct answer and then analyse the incorrect answers, identifying the reasons for learner errors and the misconceptions that produce these errors.

Background:

The Data Informed Practice Improvement Project (DIPIP) is an innovative professional development project aiming to develop sustainable professional learning communities amongst mathematics teachers. Teachers engage with data from their classrooms and work towards understanding learners' errors better. This is done by trying to understand why learners are making errors and how teachers should respond to and work with the errors.

We have developed a number of activities in the project to engage teachers with their learners' thinking. One of these activities is error analysis, where we look at test items and have conversations about the errors that learners make, and the reasons behind these errors. We work with teachers to support them in understanding and engaging with learner errors in the classroom, and to view errors as reasonable (Nesher, 1987; Smith, diSessa & Roschelle, 1993; Drews, 2005).

Purpose of the workshop:

To share ideas on how teachers can be assisted in working with errors in the teaching and learning of mathematics by

- Engaging with learner errors,
- Making meaning of the learners' thinking behind the errors
- Recognising that errors can show valid learner thinking.

Theoretical ideas on error analysis:

Error analysis forms part of the key activities in the DIPIP project. Errors are a result of a "consistent conceptual framework based on earlier acquired knowledge", called misconceptions (Nesher, 1987: 33) and make sense to learners in terms of their current thinking. Learners do not just make errors - these errors make sense to them as a result of the conceptual links that they make to knowledge they acquired previously. It should be noted that misconceptions may lead to correct answers, even when the mathematical thinking producing these answers is partly or entirely incorrect (Nesher, 1987). A classic example of this is the case of Benny as discussed in Erlwanger (1973).

Borasi (1987) views errors as "springboards for enquiry". In other words, errors can raise important issues for further exploration of the mathematics. Because errors make sense to those who make them, it's important that errors be embraced in the teaching and learning of mathematics, and not be ignored, or merely corrected. It is also important to know how errors and misconceptions can inform our instructional practices. Errors "provide evidence that the expected result has not been reached, and that something else has to be done" (Borasi, 1987: 4) and errors can therefore be used to "investigate the nature of fundamental mathematical notions" (Borasi, 1987: 5). While we agree with Borasi (1987) that errors can be springboards for enquiry we need to acknowledge that errors can be an indication to teachers of misconceptions that exist and need to be addressed. Teachers would normally not teach errors or misconceptions. For learners to develop misconceptions is a normal part of learning. All learners develop misconceptions at some point, even those who have good teaching. When teachers teach in ways that do not embrace errors, there is a likelihood that misconceptions might develop or be perpetuated. It is for this reason that teachers need to understand the type of errors that learners make and also understand why they make those errors. Understanding learners' errors can help teachers develop teaching strategies that can engage with learners' misconceptions. Teachers can never entirely prevent errors, so it is important that they can deal with them as they come up.

<u>Assessment tasks on which the workshop is based:</u>

The workshop activity is based on results of a standardised, international test that learners of participating schools have written, with focus on Algebra. This test was developed as part of the research programme "Concepts in Secondary Mathematics and Science" (CSMS) (Hart, 1981), and has been extended by "Increasing Competence and Confidence in Algebra and Multiplicative Structures" (ICCAMS). According to Hart (1981), there are six different ways of interpreting and using letters: letter evaluated, letter not used, letter used as an object, letter used as a specific unknown, letter used as a generalised number and letter used as a variable (p.4). Hart (1981) discusses these categories as follows:

Letter Evaluated – when the response suggests that the letter is given a numerical value instead of being treated as an unknown or generalised number.

Letter Not Used – letter is acknowledged without being given a meaning or it is simply ignored

Letter as Object – letter used to denote an object

Letter as Specific Unknown – letter thought of as a particular but unknown number

Letter as Generalised Number – letter seen as being able to take several values

Letter as Variable – letter seen as representing a range of values, which is more of a dynamic view

The above categories are used as guidelines for analysis of errors. When teachers use these categories to analyse learner responses, they will identify the type of errors that learners have made and discuss these errors in relation to the teaching and learning of mathematics. Templates that we have developed through DIPIP are used to analyse errors that learners make.

Items chosen:

For this workshop we have chosen six items to look at and will use the answers of different learners to work on the error analysis activity with participants. We will identify the errors that learners made and workshop participants will discuss why we think learners made those errors. The items were chosen because of the richness in the errors that the learners made.

The workshop will provide participants with opportunities to look at classroom data in different ways. Data from classroom can help teachers develop methods of looking deeply into learner needs and see how these learner needs can inform teachers' learning needs (Earl & Katz, 2006)

Boudett, City, & Murnane (2006) argue that teachers can deepen their understanding of learners' strengths and misconceptions by looking collaboratively at their learners' work and talking about it in meaningful ways. In looking at their learners' work teachers can get opportunities to understand their learners' thinking, especially when learners make errors.

Analyses:

In analysing the items participants in the workshop will use a template that will be given to them to look at the following:

- Test Item

  Here we will look at the question and provide participants the opportunity to answer the question in as many different ways as possible.

- Content Description

  Participants will need to identify the mathematical content needed to answer the question.

- Analyses of Correct Response

  Analysing what knowledge, skills and procedures are required to get the correct response for this question

- Analysis of Errors

  This is the stage in the analysis that normally brings about the most discussion. Looking at the answers of the learners we look at the actual errors made by learners. Not all errors are discussed in depth, but the ones which are common. The aim is to understand the thinking processes of the learners and to identify any misconceptions that could have contributed to this particular error. All possible reasons for the error should be discussed.

- Critical Concept(s)

  The final stage, having done the error analysis, is to identify the critical concept(s) needed by the learner. The questions asked are what is missing between the error and the correct response. This can be seen as the link between the error and the correct response. These critical concepts become the learning needs of the learners and in the DIPIP project lessons would be developed around this concept.

Conclusion:

It is important for teachers to understand the role that errors play in the teaching and learning of mathematics. The workshop will try and raise an awareness of how errors can be embraced in mathematics classrooms instead of being ignored, or merely corrected. The error analysis activity can provide participants with skills to handle errors that learners make in class as well as the errors they make when they write their homework.

We will provide participants with an opportunity at the end of the workshop to give an evaluation of how the workshop went; what they gained; what works well for them and what does not; and also make input on any matter they deem necessary in working with learner errors and misconceptions.

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Isomorphic Visualization and Understanding of the Commutativity of Multiplication: from multiplication of whole numbers to multiplication of fractions

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In 2010, I wrote a paper on the developments of visualization, functional materials and actions in teaching mathematics. This paper was a historical survey, but in the recent paper we put emphases on practical issues, taking from the case of the commutativity of multiplication a model for the isomorphic type of visualization we have developed.

From historical view to practical issues on the development of using visualization

In a previous paper of 2010, a discussion was given on the type of visualization used in Finnish mathematics textbooks in the 19^{th} and 20^{th} centuries (Malaty 2010). In the recent work, we are moving to put emphasis on practical issues and present an example, through which we can explore some of the main roles visualization can have in teaching mathematics. Developing mathematics teaching approaches has been always one of my main interests, and working in teacher education and mathematical clubs, in Finland, has given me a chance to develop and test my teaching approaches. One of the elements used in these approaches is visualization (Malaty 2006a, Malaty 2006b, Malaty 2006c, Malaty 1996). Visualization has been for us a facilitator to give students a chance to understand and discover mathematical concepts and relations, and as well a tool to demonstrate, solve and pose mathematical problems. The example we are going to present is related to the teaching of the commutativity of multiplication.

Common mistakes in the visualization of the commutativity of multiplication

Fig.1 represents one of our textbooks' visualization (Haapaniemi et al. 2002, 126). At first, we can notice that the figure contains unneeded elements, and this makes it far from the simplicity of iconic visualization. Such mistake is common in textbooks, but the main problem, we see, is the use of visualization to make ungrounded generalizations. This is as well the case in our example. The textbook uses the given figure (Fig.1) to present two statements $5 \cdot 2 = 10$ and $2 \cdot 5 = 10$. But these statements are not justified by the figures presented, but by the multiplication tables. Visualization role here is a modest one, and near to be only decorative. This is still the case, when textbooks provide more iconic visualization. In Fig.2, of the same textbook, and in the same page, two statements, $3 \cdot 2 = 6$ and $2 \cdot 3 = 6$, are introduced and justified by multiplication tables and not upon visualization.

Fig. 1. Complicated visualization

Fig. 2. Iconic visualization

Textbooks' effect is rematkable on what happanes in the classroom, including the use of textbooks' visualization. But, other types of mistakes have been observed in classroom teaching. For the commutativity of multiplication, not rare to see on the boards a type of mechanical performance like the one of Fig.3. While the numbers included in the statement $2 \cdot 3 = 3 \cdot 2$ are visualized (Fig.3), no attention is given to the meaning of the multiplication operation. Numbers are visualized, as if for summands of a sum and not for multiplier and multiplicand of a product.

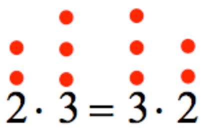

Fig.3. Mechanical performance

Iconic, dynamic 2-dimensional visualization

In building visualization for mathematical relations, we have developed iconic dynamic types of visualization to get isomorphism with the relation we visualize. For the commutativity of multiplication, our types of visualization are 2-dimentional ones. At the beginning, and for young children, we use perpendicular dimensions, but at relevant stage we ask students to investigate the possibility of using oblique dimensions.

Visualization of the commutativity of multiplication of whole numbers

This I start by writing on the board the expression $2\cdot 3$. After that, I draw only one row of three circles, as in Fig. 4. Then I ask children:

Fig.4. One row of three circles

How many times three red circles I have drawn? – Only one time. After getting this answer, I draw an arrow as in Fig. 5, and then continue: *Right. But, here above is written* $2\cdot 3$ *, what I have to do?* – Draw another row. *Here?* I add a new arrow as in Fig.6, and then I draw the new row of three red circles as in Fig.7

Fig.5. Adding an arrow to mean one time of three circles

Fig.6. Adding another arrow

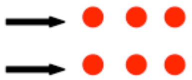

Fig.7. Drawing the other row of three circles

The 2-dimentional iconic visualization (Fig.7) has been built in a dynamic way to get a figure isomorphic to the expression $2\cdot 3$. After that, we can continue our work with children in different ways, taking in mind the level of the group. A challenging and interesting one is to continue as follows: *But, this figure* (Fig.7) *is also representing the expression* $3\cdot 2$ (where I write numeral 2 in red and numeral 3 in other color like green), *who can tell me how this is possible?* If my last question does not help, I can use another approach like the one of the next dialog. *How many red circles you can see in one column?* (I draw in green vertical arrow as in Fig. 8, and then also in green a vertical segment as in Fig.9) – Two. *How many times such 'two circles' we have?* - Three. Yes, *who can draw more arrows to show that we have'3 times 2 circles'?* When a student gets a figure like that of Fig.10,

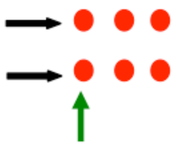

Fig.8. Adding in green vertical arrow

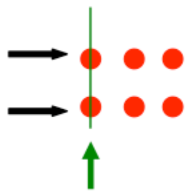

Fig.9. Adding in green vertical segment

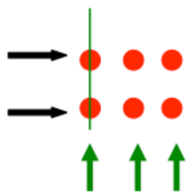

Fig.10. Drawing more vertical arrows

we continue: Great. *'3 times 2'*, this expression I want to write here; *which color is relevant for writing numeral 3?* – Green. Right. *What about the color relevant for numeral 2?* – Red. *Why not green or other color?* – Because, here, 2 is the number of red circles. At this moment, I write the expression $3\cdot 2$ in the same row of the expression $2\cdot 3$, which has been already written on the top of the board, and leave a space between the two expressions to draw a box in a form of a rectangle, as in Fig.11. In addition, I put three large cards over the open sentence, two for inequality relations and the third for the relation of equality, as in Fig.12. Then, I ask children to have a time to think about the relevant card to change the open sentence into a true statement? *Take your time, and raise your hand when you are ready to tell me, which card is relevant and where to put it?*

Fig.11. Open sentence

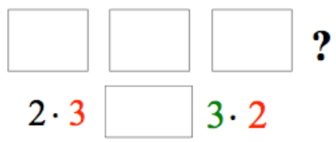

Fig.12. Adding three large cards of relations

Two meanings of dynamic visualization, and isomorphism

Getting Fig.10 for the commutativity of multiplication was an outcome of a process and not given as such, at once. For this reason we call such

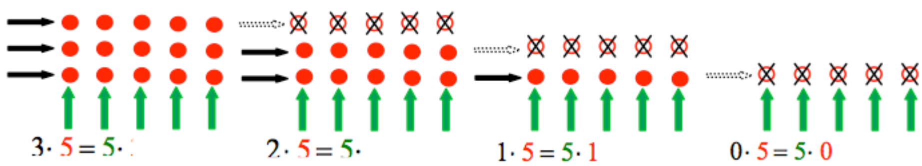

Fig.13. From $3\cdot 5=5\cdot 3$ **, to** $2\cdot 5=5\cdot 2$ **then** $1\cdot 5=5\cdot 1$ **and Finally** $0\cdot 5=5\cdot 0$

visualization a dynamic visualization. Nevertheless, the main reason of giving this name is the possibility of modifying such figure to present other cases, including the general case. For instance, Fig.10 can be used by students to show that the commutativity of multiplication does exist for any two whole numbers. Among others, students can modify Fig.10 to show that, $2\cdot 5=5\cdot 2$, $3\cdot 5=5\cdot 3$, $3\cdot 7=7\cdot 3$, $4\cdot 7=7\cdot 4$, $5\cdot 7=7\cdot 5$, $5\cdot 10=10\cdot 5$, $5\cdot 100=100\cdot 5$, $5\cdot 104=104\cdot 5$ and $7\cdot 104=104\cdot 7$. Not only such cases can be discussed, but students can visualize a statement like $17\cdot 104=104\cdot 17$ before they learn how to perform in any of this statement's sides. This is an evidence of the isomorphism of our type of visualization with the relation it visualizes. In addition, we can modify this 2-dimensional iconic dynamic visualization to allow us to visualize special cases, in particular $0\cdot 5=5\cdot 0$ and $0\cdot 0=0\cdot 0$, as Fig.13 and Fig. 14 show.

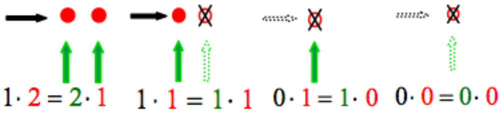

Fig.14. From $1\cdot 2=2\cdot 1$, to $0\cdot 0=0\cdot 0$

Isomorphic visualization and generalization

The most important is that, our visualization can be used for the general case of the commutativity of multiplication for whole numbers. The general case can be presented by the statement; $n\,a = a\,n$, where n and a are whole numbers. In Fig.15, because the number of red circles in a row is a and we have n rows, the total number of circles is na. As we have n rows, in a column we must have n circles. But, the number of such columns is a, because in a row we have a circles. Thus, the total number of circles is an. Now, as Fig.15 gives us the chance to visualize the general case, this figure can be modified to visualize the special cases $1\cdot 5=5\cdot 1$ and $0\cdot 5=5\cdot 0$, presented above (Fig.13), in the general form; $1\cdot a=a\cdot 1$, $0\cdot a=a\cdot 0$.

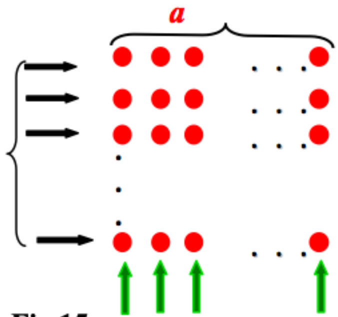

Fig.15.

Visualization's isomorphism with the algebraic proof and the need for visualization in writing proofs

$$
\begin{aligned}
n\,a &= n\,(1+1+1+\ldots+1 \quad [a\ terms]) \\
&= n+n+n+\ldots+n \quad [a\ terms] \\
&= a\,n
\end{aligned}
$$

Fig.16. A proof of distributivity

In Fig.16 we give a simple proof for the commutativity of multiplication for whole numbers. This proof is built on the distributivity of multiplication over addition for whole numbers. And, this distributivity can be proved easily, upon the associativity of addition for whole numbers. For this chain of proofs, we need to have this associativity as an axiom. In addition, both associativity and distributivity properties can be easily visualized. Regards the isomorphism of algebraic proof (Fig.16) with the visualization of the general case (Fig.15), we can notice that ones in the first line of the proof (Fig.16) match the red circles in the first row of Fig.15. In addition, n in the first line of the proof (Fig.16) corresponds the rows' number in Fig. 15. In the second line of the proof (Fig.16), n corresponds the number of circles in a column (Fig.15), while in the algebraic proof it is the result of multiplying 1's by n according to the distributivity of multiplication over addition. Without using distributivity, we can rely on the concept of multiplication to prove that:

$$
\begin{aligned}
n\,(1+1+1+\ldots+1 \quad [a\ terms]) \\
= \quad 1+1+1+\ldots+1 \quad [a\ terms]+ \\
1+1+1+\ldots+1 \quad [a\ terms]+ \\
1+1+1+\ldots+1 \quad [a\ terms]+ \\
\vdots \\
1+1+1+\ldots+1 \quad [a\ terms] \\
= \quad n+n+n+\ldots+n \quad [a\ terms].
\end{aligned}
$$

Fig.17. Proof based on the concept of multiplication

$n\,(1+1+1+\ldots+1 \quad [a\ terms]) = n+n+n+\ldots+n\,[a\ terms]$ (Fig.17). This last proof (Fig.17) shows how our visualization for the general case of the commutativity of multiplication for whole numbers (Fig.15) is not different than this proof. The *n* series in the proof (Fig.17) correspond the *n* rows in our visualization (Fig.15), and the ones in the proof (Fig.17) correspond the red circles (Fig.15).

This shows the need to develop visualization to become as much as possible isomorphic to the mathematical entity visualized. For young children, this gives them a chance to think mathematically and be ready to move to more abstract level, like the algebraic level. When we look at the last proof, we can notice that, bringing *n*'s at the last line (Fig.17) is done visually from the ones of the columns over these *n*'s, and this what we are doing in an isomorphic way in our visualization (Fig.15). Getting these *n*'s of the proof (Fig.17) algebraically is possible, and this can make the algebraic proof rather closed to our visualization of the general case (Fig.15). For space reason, we leave this proof outside this paper. But, we have to mention that such proof can show how visualization is still needed in writing this proof. This means that visualization is not only needed for young children, but as well in studying mathematics and making mathematics.

Isomorphic visualization, multiplication by a fraction and unit fraction

The isomorphism between our 2-dimensional dynamic iconic visualization and the commutativity of multiplication for whole numbers is clearly strong, and therefore we can use it in a modified form to visualize the commutativity of multiplication of fractions. Fractions here are proper fractions, those less than one, and improper fractions, those equal or greater than one. Thus, whole numbers are also fractions. When we start to work with multiplication by fractions, different than whole numbers, we need to develop our reading of a product. For instance reading $\frac{1}{3}\cdot 6$ as "one third times 6" is difficult to understand, and therefore we need to modify our reading style as we in fact making enlargement for the concept of multiplication. Thus, instead of saying three times six, two times six, one times six and "one third 'times' six"; it is now a time to say three sixes, or three of sixes; two sixes, or two of sixes; one six; one third six or "one third 'of' six". Reading "one third of six" brings to light the relation between multiplication by a fraction and division by the reciprocal of this fraction. With this reading style, we can easy get the value of the product. For instance, one third of six is two as it is the quotient we get in dividing 6 by 3. Here to remember that in algebra we do not say 2 times *a*, but 2*a*. One thing more, related to fractions and fractions multiplication, we have to here mention. This is the concept of unit fraction, which is a fraction with 1 as nominator. This concept wasn't known in Finnish schools, but our work has made a positive change towards understanding and using of it. For instance, in multiplying $\frac{3}{8}$ by 2 we underline the fact that we need to multiply 3 by 2, exactly as multiplying 3 ones by 2. The only difference is the unit of the product. In multiplying three eighths by two the product is not 6 ones, but 6 eighths, as the unit of the multiplicand is not of ones, but of eighths. The idea of "unit" has a wider role in our work. Regards arithmetic and decimal system, place value has a special meaning, as each digit's value depends on the unit of the place. Our experience shows that understanding of the unit idea brings meaning to children for their performance of skills and makes this performance easier. And, both achievements bring motivation for the study of mathematics.

Commutativity and visualization of the multiplication of a whole number by a unit fraction

Multiplying a whole number by $\frac{1}{1}$, as improper unit fraction, is possible to discuss. But, this is an easy and trivial case, and therefore we move to discuss a more substantial case, and take the expression $\frac{1}{5}\cdot 3$, where 3 is multiplied by a unit fraction $\frac{1}{5}$.

Fig.18. Drawing a rectangle

To get isomorphic visualization to the expression $\frac{1}{5}\cdot 3$, we start by writing this expression on the board, where I use a color, like red here, in writing the multiplicand.

Then I ask students: *What is the multiplicand we have in this expression?* – Three. *We now shall try to represent the multiplicand 3 by rectangles, are you ready?* –Yes. Which color, I have to use in drawing the rectangles? – Red. Then I draw a rectangle as in Fig.18, and ask students: *How many rectangles I have drawn?* – One. Yes, *how many of such rectangles I need to draw more?* – Two. Right, then I draw two more rectangles as in Fig.19. *Who can read the expression, written on the top of the board?* – "One fifth of three". Great, *who can use blue color and draw a straight line to find and present to us one fifth of three?* Such approach or a modified one has to lead to get a figure like the one of Fig.20. Then we again give students a challenge by saying: but, this figure in fact shows that $\frac{1}{5}\cdot 3=3\cdot\frac{1}{5}$. *Who can show us that this statement is true?* This discussion has to lead us to a figure with three arrows as in Fig.21.

Fig.19. Visualization of 3

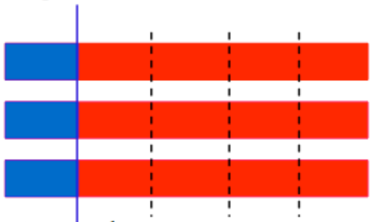

Fig.20. $\frac{1}{5}\cdot 3$

The color used in writing the fraction $\frac{1}{5}$ is blue, and this is because we have agreed with students to color the part of the three rectangles, which represents one fifth of three, by the same color we give to the line we use in cutting the rectangles. We can make a story about the effect of the color of the line used. Speaking about using of stories in teaching mathematics, in multiplying 3 by one fifth we can use a story like 5 children in a party sharing equally three cakes of different taste. We can use also a type of functional materials to help those in need of working on hands, where they can cut by themselves strips of red colored papers.

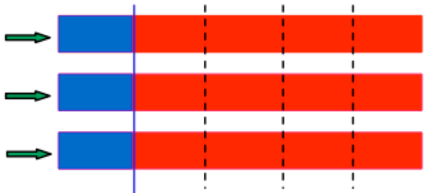

Fig.21. $\frac{1}{5}\cdot 3=3\cdot\frac{1}{5}$

Commutativity and visualization of the multiplication of a whole number by a fraction different than unit fraction

Let us here start with a proper fraction $\frac{2}{5}$ as multiplier and use the same multiplicand of the last case, i.e. 3. After dealing with the case of $\frac{1}{5}\cdot 3=3\cdot\frac{1}{5}$, it is not difficult to visualize $\frac{2}{5}\cdot 3=3\cdot\frac{2}{5}$, and this can be given to students as a problem to solve. Fig.22 presents such visualization. Here, we do not use colors in writing the statement. As we need to free students from using such colors. About such coloring use, on one hand, we can go back to use such type of coloring, when we find it necessary to recall ideas, and on the other hand colors can work in an imaginary form for both students and teachers in their discussions. In our case, when we do not use colors in writing statements, we can visualize the expression 'two fifth of three' using two colors, instead of one. In Fig.22 we have colored "one fifth of three" in blue and the other "one fifth of three" in yellow. The other choice is to color both in one color, like blue. Here to notice that, Fig.22 is modified from Fig.21, and this show that we can continue this process, to go from Fig.21 to any case of the multiplication of a whole number by a fraction, proper or improper. In addition, we can modify Fig.21 to visualize the general case of multiplying a whole number by a fraction, i.e. we can use a visualization to show that $\frac{m}{n}\cdot a=a\cdot\frac{m}{n}$, where *m, n* and a are whole numbers and $n\neq 0$. We can also investigate the isomorphism of the algebraic proof of this statement and the figure visualizes it.

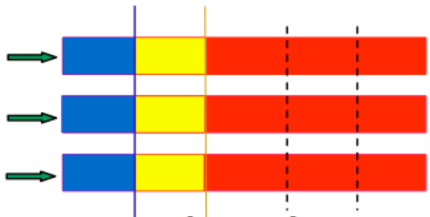

Fig.22. $\frac{2}{5}\cdot 3=3\cdot\frac{2}{5}$

Commutativity and visualization of the multiplication of a fraction by a fraction

Let us start with the visualization of the commutativity of multiplication of a unit fraction by a unit fraction, like $\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{3}\cdot\frac{1}{2}$. At first we visualize a whole by a rectangle (Fig.23), and then draw two segments to divide the rectangle into 3 congruent rectangles and color one in blue to visualize $\frac{1}{3}$ (Fig.24). To get 'half of one third' we draw a horizontal segment, in red to divide the blue rectangle into two congruent rectangles and color one in red, here the lowest. This 'half of one third' (Fig.25) is obviously 'one third of a half', as we can imagine, or here draw in a similar way a horizontal segment in the whole rectangle to divide it into two congruent rectangles and color the lowest in red to visualize half (Fig.26), then draw a vertical segment in blue to get 'one third of this half' (Fig.27).

Fig.23. Visualization of a whole

Fig.24. Visualization of $\frac{1}{3}$

Fig.25. $\frac{1}{2}\cdot\frac{1}{3}$

Fig.26. Visualization of a $\frac{1}{2}$

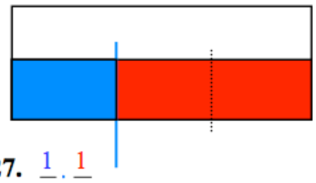

Fig.27. $\frac{1}{3}\cdot\frac{1}{2}$

Reflections

Last discussed visualization is for the commutativity of the multiplication of two unit fractions, but we can easy modify it for use for any two fractions and for the general case, where improper fractions are included. Algebraic proof can be provided to the statement of general case, and this proof can show how our iconic dynamic visualization is isomorphic to mathematics. Visualization can bring understanding of mathematics, but on the other hand making visualization needs understanding of mathematics. All our figures in this paper are 2-dimensional ones. Using of rectangles is possible to the case of whole numbers, and this use has a closed relation with the proof of rectangle's area statement. In our visualization above, parallelograms can replace rectangles. This is similar to the case of the 2-dimensional visualization for the multiplication of a whole number by another whole number, where we can use oblique dimensions.

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Assessing the teaching efficacy beliefs of teacher trainees

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Abstract

Research shows that self-efficacy is an important concept which influences a teacher's ability to teach and the effectiveness with which the teaching is done. Each teacher trainee has a sense of efficacy with regards to teaching which is influenced by many factors. This study aimed to determine the teaching self-efficacy of third-year teacher education students in three categories: student engagement, instructional strategies and classroom management. A questionnaire was administered as the survey instrument and provided data which the researchers analysed and interpreted. It was found that at this stage of the student teachers' careers, that is, at the end of their third year of study, the student teachers responded with overwhelming positive self-efficacy beliefs with regard to their future occupation.

Introduction

A growing number of educational researchers are interested in relationships between teacher efficacy and other educational variables. For example, teachers' efficacy judgements have been correlated with decreased burnout (Brouwers & Tomic 2000), increased job satisfaction (Caprara, Barbaranelli, Borgogni, & Steca 2003), and commitment to teaching (Coladarci 1992). Ross (1998) reviewed 88 teacher efficacy studies and suggested that teachers with higher levels of efficacy are more likely to (1) learn and use new approaches and strategies for teaching, (2) use management techniques that enhance student autonomy and diminish student control, (3) provide special assistance to low achieving students, (4) build students' self-perceptions of their academic skills, (5) set attainable goals, and (6) persist in the face of student failure. This shows that there is a relationship between teaching efficacy and student academic performance.

Self-efficacy refers to the beliefs about one's capabilities to learn or perform behaviours at designated levels (Bandura 1997) and is said to have a measure of control over individual's thoughts, feelings and actions. The beliefs that individuals hold about their abilities and outcome of their efforts influence in great ways how they will behave. It is the realization of this relationship between individual beliefs and subsequent behaviours that prompted researchers' interest in self-efficacy. Self-efficacy has been applied in educational settings. The influence that self-efficacy has on motivation, learning and academic achievement has been investigated and reported (Pajares 1996; Schunk 1995). Self-efficacy has also been reported for individual subjects such as mathematics (Pajares & Miller 1994).

Furthermore, researchers have shown increasing interest in the teaching efficacy of prospective teachers. Student teaching or teaching practice is generally considered the most beneficial component of preparation by prospective and practising teachers as well as teacher educators (Borko & Mayfield 1995). It is during teaching practice that students develop a positive or a negative attitude towards teaching as a career, indicating that teaching practice can have both positive and negative influences. For example, poorly chosen placements result in feelings of inadequacy, low teacher efficacy and an unfavourable attitude towards teaching (Fallin & Royse 2000) whereas extensive and well-planned field experiences can help prospective teachers develop confidence, self-esteem and an enhanced awareness of the profession.

Theoretical framework

Self-efficacy is explained in the theoretical framework of social cognitive theory espoused by Bandura (1986, 1997) which states that human achievement depends on interactions between one's behaviours, personal factors and environmental conditions. The behaviour of an individual depends largely on early experiences at home. The home environment that stimulates curiosity will help build self-efficacy by displaying more of that curiosity, and exploring activities would invite active and positive reciprocity. This stimulation enhances the cognitive and affective structures of the individual which include his ability to sympathise, learn from others, plan alternative strategies and regulate his own behavior and engage in self-reflection (self-efficacy) (Mahyuddin, Elias, Cheong, Muhamad, Noordin & Abdullah 2006).

The self-regulating system referred to, affords an individual the capacity to alter his environment, and influences his subsequent performance. Therefore, the beliefs he has of himself are the key elements in exercising control and personal efficacy. This affects behaviour in two ways: either he engages in tasks he feels competent and confident in or he simply avoids those that he feels incompetent in. Self-efficacy helps to determine how much effort, perseverance and resilience are being put into a task. The higher an individual's sense of efficacy, the greater the effort, persistence and resilience that will be put into a task. Efficacy beliefs also trigger emotional reactions. Individuals with low self-efficacy and who believe that a task is tough can develop stress, depression and a narrow vision on how to solve the problems. On the other hand, those with high efficacy would be more relaxed in solving difficult tasks. Therefore, these influences are strong determinants of the individual's level of achievement.

The development of self-efficacy of prospective teachers is influenced by many factors such as mastery learning and vicarious experience. The positive and negative influences of self-efficacy on prospective teachers have been found to be context specific. It is against this background that this study sought to determine the self-efficacy beliefs of prospective teachers studying in the School of Teacher Education at a University of Technology with the aim of also determining the predictors of their teaching efficacy.

Method

This study used a descriptive survey research design. Efficacy beliefs of pre-service teachers were examined through a survey instrument administered at the end of the first semester of the third year of the programme. The participants in this study were third year B.Ed (FET) students in the School of Teacher Education at a University of Technology in South Africa. Students in all five B.Ed (FET) programmes offered by the School of Teacher Education participated in the study.

A questionnaire was used as the instrument to collect data from the respondents. The TSES included 24 items on a 5-point scale yielding three subscales: Efficacy for Classroom Management, Efficacy for Instructional Strategies, and Efficacy for Student Engagement. 136 students answered and returned the questionnaire. The questionnaires were issued to the students during class time to optimize participation. It was emphasized, however, that participation was not compulsory. The data obtained from the questionnaires was analysed by the researchers themselves. The information was then presented in tables from which written interpretations were made.

Results

Descriptive statistics provided a sample profile and summarized variables.

Table 1: Gender of respondents (N=136)

Gender	Frequency	%
Male	70	51.47
Female	66	48.53
Total	136	100

A good distribution of male and female respondents was obtained with close to 50% making up each group.

Table 2: Distribution of respondents per programme

Programme	Male	Female	Total	%
Natural Sciences (NS)	36	25	61	45
Economic and Management Sciences (EMS)	15	24	39	29
Technology	13	1	14	10
Languages	1	11	12	9
Computer Science	5	5	10	7
Total	70	66	136	100

The distribution of respondents per programme was not even with nearly half of the respondents in the Natural Sciences programme and nearly a third of the respondents in the Economic and Management Sciences programme, while the Technology, Languages and Computer Sciences programmes together made up the other 26% of respondents. This distribution is due to the number of students that registered and which can be accommodated in the different programmes in the School of Teacher Education. Fortunately, it does not influence the validity of the data as no comparison between the respondents in the different programmes was made.

Table 3: Teaching Efficacy Beliefs of the respondents (N=136)

Questions	Mean	SD
Q1	3.881	0.993
Q2	4.326	0.854
Q3	4.378	0.854
Q4	4.474	0.905
Q5	4.170	0.966
Q6	4.459	0.689
Q7	3.970	0.810
Q8	4.000	0.914
Q9	4.341	0.774
Q10	4.230	0.897
Q11	4.133	0.991
Q12	3.793	1.037
Q13	4.296	0.856
Q14	4.459	0.655
Q15	4.170	0.877
Q16	4.022	0.902

Q17	4.015	1.007
Q18	4.096	0.800
Q19	4.193	0.877
Q20	4.400	0.794
Q21	3.911	0.966
Q22	3.815	1.038
Q23	4.126	0.814
Q24	4.222	0.870
Average	4.162	0.881

Respondents were requested to indicate their opinion about what they would do to deal with teaching situations presented to the respondents as questionnaire statements and indicated in Table 3 as Q1 to Q24. A scale from 1 to 5 was presented with 1 representing "Nothing", 2 representing "Very Little", 3 representing "Some Influence", 4 representing "Quite a Bit" and 5 representing "A Great Deal".

On the whole it seems as if the teacher trainees felt that they could have a very big influence on the learners' learning as the average of the means of all the questions is 4.162 out of a possible 5. They are therefore 83.24% percent certain that they can have a positive influence on their learners. The question in which the respondents scored highest was Q4: How much can you do to motivate learners who show low interest in school work? A mean of 4.474 was achieved in this question. The teacher trainees are therefore 89.48% certain that they will be able to motivate their learners to work harder once they start to teach full-time. The question in which the respondents scored lowest was Q12: How much can you do to foster learner creativity? A mean of 3.793 was achieved in this question. The teacher trainees are therefore 75.86% certain that they can develop their learners' creativity.

The lowest standard deviation of 0.655 was obtained in Q14: How much can you do to improve the content understanding of a learner who is failing, with a mean of 4.459, while the greatest standard deviation of 1.038 was obtained in Q22: How much can you assist families in helping their children do well in school, with a mean of 3.815. The high mean of 4.459 and relatively low standard deviation of 0.655 in Q14, mean that more student trainees feel that they can improve the content understanding of a learner than the number of trainees who feel that they can assist families in helping their children do well in school, as Q22 has a lower mean of 3.815 and a relatively higher standard deviation of 1.038.

Table 4: Teaching Efficacy Beliefs regarding student engagement

Questions	**Mean**	**SD**
Q1	3.881	0.993
Q2	4.326	0.854
Q4	4.474	0.905
Q6	4.459	0.689
Q9	4.341	0.774
Q12	3.793	1.037
Q14	4.459	0.655
Q22	3.815	1.038
Average	4.194	0.868

The average of the means and standard deviations determined with regard to questions on student engagement produced scores of 4.194 and 0.868 respectively.

Table 5: Teaching Efficacy beliefs regarding instructional strategies

Questions	Mean	SD
Q7	3.970	0.810
Q10	4.230	0.897
Q11	4.133	0.991
Q17	4.015	1.007
Q18	4.096	0.800
Q20	4.400	0.794
Q23	4.126	0.814
Q24	4.222	0.870
Average	4.149	0.873

The average of the means and standard deviations determined with regard to questions on instructional strategies produced scores of 4.149 and 0.873 respectively.

Table 6: Teaching Efficacy beliefs regarding classroom management

Questions	Mean	SD
Q3	4.378	0.854
Q5	4.170	0.966
Q8	4.000	0.914
Q13	4.296	0.856
Q15	4.170	0.877
Q16	4.022	0.902
Q19	4.193	0.877
Q21	3.911	0.966
Average	4.143	0.901

The average of the means and standard deviations determined with regard to questions on classroom management produced scores of 4.143 and 0.901 respectively.

Herewith a summary of the information provided in Tables 4, 5 and 6.

Table 7: Summary Table of Teaching Efficacy Beliefs

Category	Mean	SD
Student engagement	4.194	0.868
Instructional strategies	4.149	0.873
Classroom management	4.143	0.901
Overall Teaching Efficacy	4.162	0.881

Clearly, the teacher trainees' efficacy beliefs with regard to the three sub-scales do not differ much. The difference in mean scores between Instructional strategies and Classroom management is a mere 0.006, the difference in mean scores between Instructional strategies and Student engagement is 0.045 and the difference in mean scores between Student engagement and Classroom management is 0.051. The greatest difference in means between the sub-scales is therefore 1.216%, while the smallest difference in means between the sub-scales is a mere 0.145%. The standard deviations of the three sub-scales show a similar pattern.

Conclusion

This paper reported on the first half of a study into the self-efficacy beliefs of third-year teacher trainees studying in the School of Teacher Education at the Central University of Technology, in the Free State Province of South Africa. A questionnaire was administered to the respondents to determine their self-efficacy beliefs with regard to teaching at this stage of their careers. The students responded with overwhelming positive self-efficacy beliefs with regard to their future occupation. A follow-up questionnaire will be administered to these same students after they have completed a six-month work-integrated learning experience during the first six months of 2011. Their self-efficacy beliefs will then once again be determined and changes in their beliefs are predicted.

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On Economic Interpretation of Lagrange Multipliers

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Abstract

Lagrange multipliers play a standard role in constraint extrema problems of functions of more variables. In teaching of engineering mathematics they are readily presented as quantities of formal type in the algorithm for finding of constraint extrema. The paper points to important interpretations Lagrange multipliers in optimization tasks in economics

Introduction

The *Lagrange multipliers method* is one of methods for solving constrained extrema problems. Instead of rigorous presentation we point to the rationale of this method. Recall that for a function f of n variables the necessary condition for local extrema is that at the point of extrema all partial derivatives (supposing they exist) must be zero. There are therefore n equations in n unknowns (the $x's$), that may be solved to find the potential extrema point (called *critical* point). When the $x's$ are constrained, there is (at least one) additional equation (constraimt) but no additional variables, so that the set of equations is overdetermined. Hence the method introduces an additional variable (the Lagrange multiplier), that enables to solve the problem. More specifically (we may restrict to finding of maxima), suppose we wish to find values $x_1,...,x_n$ maximizing

$$y = f(x_1,...,x_n)$$

subject to a constraint that permits only some values of the $x's$. That constraint is expressed in the form

$$g(x_1,...,x_n).$$

The Lagrange multipliers method is based on setting up the new function (the *Lagrange function*)

$$L(x_1,...,x_n,\lambda) = f(x_1,...,x_n) + \lambda g(x_1,...,x_n), \tag{1}$$

where λ is an additional variable called the *Lagrange multiplier*. From (1) the conditions for a critical point are

$$\begin{aligned} L'_{x_1} &= f'_{x_1} + \lambda g'_{x_1} \\ &\;\;\vdots \\ L'_{x_n} &= f'_{x_n} + \lambda g'_{x_n} \\ L'_{\lambda} &= g(x_1,...,x_n), \end{aligned} \tag{2}$$

where the symbols L', g' are to denote partial derivatives with respect to the variables listed in the indices. Of course, equations (2) are only necessary conditions for a local maximum. To confirm that the calculated result is indeed a local maximum second order conditions must be verified. Practically, in all current economic problems there is on economic grounds only a single local maximum.

In a standard course of engineering mathematics the Lagrange multiplier is usually presented as a clever mathematical tool („trick“) to reach the wanted solution. There is no large spectrum of sensible examples (mostly a limited number of simple “well-tried” school examples) to show convincingly the power of the method. Economic interpretation of the Lagrangian multiplier provides a strong stimulus to strengthen its importance. This will be central to our next considerations.

Economic interpretations

In the sequel we will examine two useful interpretations $1^0, 2^0$ of the Lagrange multipliers.

1^0 Rearrange the first n equations in (2) as

$$\frac{f'_{x_1}}{-g'_{x_1}} = ... = \frac{f'_{x_n}}{-g_{x_n}} = \lambda. \qquad (3)$$

Equations(3) say that at maximum point the ratio of f'_{x_i} to g'_{x_i} is the same for every x_i and moreover it equals λ. The numerators f'_{x_i} give the **marginal contribution** (or **benefit**) of each x_i to the function f to be maximized, in other words they give the approximate change in f due to a one unit change in x_i. Similarly, the denominators have a marginal cost interpretation, namely, $-g'_{x_i}$ gives the **marginal cost** of using x_i (or marginal "taking" from g), in other words the approximate change in g due to a unit change in x_i. In the light of this we may summarize, that λ is the *common benefit-cost ratio* for all the $x's$, ie.

$$\lambda = \frac{marginal \;\; benefit \;\; of \;\; x_i}{marginal \;\; cost \;\; of \;\; y_i} = \frac{f'_{xi}}{-g'_{x_i}}. \qquad (4)$$

<u>Example</u> A farmer has a given length of fence, F, and wishes to enclose the largest possible rectangular area. The question is about the shape of this area. To solve it, let x, y be lengths of sides of the rectangle. The problem is to find x and y maximizing the area $S(x, y) = xy$ of the field, subject to the condition (constraint) that the perimeter is fixed at $F = 2x + 2y$. This is obviously a problem in constraint maximization. We put $f(x, y) = S(x, y)$, $g(x, y) = F - 2x - 2y$ and set up the Lagrange function (1)

$$L(x, y, \lambda) = xy + \lambda(F - 2x - 2y). \qquad (5)$$

Conditions (2) are $L'_x = y - 2\lambda, L'_y = x - 2\lambda, L'_\lambda = F - 2x - 2y$. These three equations must be solved. The first two equations give $x = y = 2\lambda$, i.e. x must be equal to y and due to (5) they should be chosen so that the ratio of marginal benefits to marginal cost is the same for both variables. The benefit (in terms of area) of one more unit of x is due to (4) given by $S'_x = y$, (area is increased by y), and the marginal cost (in terms of perimeter) is $-g'_x = 2$ (from the available perimeter is taken 2 for both variables are increased by the same length 1). As mentioned above, the conditions (4) state that this ratio must be equal for each of the variables. Completing the solution we get $x = y = \frac{F}{4}, \lambda = \frac{F}{8}$. Now let us discuss the interpretation of λ. If the farmer wants to know, how much more field could be enclosed by adding an extra unit of the length of fence, the Lagrange multiplier provides the answer $\frac{F}{8}$ (approximately), i.e. the present perimeter should be divided by 8. For instance, let 400 be a current perimeter of the fence. With a view to our solution, the optimal field will be a square with sides of lengths $\frac{F}{4} = 100$ and the enclosed area will be 10 000 square units. Now if perimeter were enlarged by one unit, the value $\lambda = \frac{F}{8} = \frac{400}{8} = 50$ estimates the increase of the total area. Calculating the "exact" increase of the total area, we get: the perimeter is now 401, each side of the square will be $\frac{401}{4}$, the total area of the field is $(\frac{401}{4})^2 = 10050{,}06$

square units. Hence, the prediction of 50 square units given by the Lagrange multiplier proves to be sufficiently close.

2^0 Rewrite the condition $g(x,y)=0$ in (1) as $c(x,y)=k$, where k is a parameter. Then for the partial derivative of the Lagrange function with respect to k we get $L'_k=-\lambda$. From the interpretation of a partial derivative we conclude, that the value $-\lambda$ states the approximate change in L (and also f) due to a unit change of k. Hence the value $-\lambda$ of the multiplier showss the approximate change that occurs in f at the point of its maxima in response to the change of k by one in the condition $c(x,y)=k$. Since usually $c(x,y)=k$ means economic restrictions imposed (budget, cost, production limitation), the value of multiplier indicates so called the **oportunity cost** (of this constraint). If we could reduce the restriction (i.e. increase k) then the extra cost is $-\lambda$. If we are able to realize an extra unit of output under the cost less than $-\lambda$, then it represents the benefit due to the increase of the value at the point of maxima. Clearly to the economic decision maker such information on opportunity costs is of considerable importance.

Example The profit for some firm is given by $PR(x,y)=-100+80x-0{,}1x^2+100y-0{,}2y^2$, where x,y represent the levels of output of two products produced by the firm. Let us further assume that the firm knows its maximum combined feasible production to be 325. It represents the constraint $x+y=325$. Putting $g(x,y)=x+y-325=0$ we set up the Lagrange function $L(x,y,\lambda)=-100+80x-0{,}1x^2+100y-0{,}2y^2+\lambda(x+y-325)$. Applying Lagrange multipliers method we get the solution $x=183{,}335, y=141{,}667, \lambda=-43{,}333$ with the corresponding value of the profit $PR(183{,}335;141{,}667)=21358{,}420$. Now we reduce the restriction altering the constraint equation to $x+y=326$. Finding the new solution as before we have $x=184, y=142, PR(184{,}142)=21401{,}6$. We see that the increase in profit brought about by increasing the constraint restriction by 1 unit has been $43{,}18$ - approximately the same as the value $-\lambda$ that we derived in the original formulation. It indicates that the additional increase of labour and capital in order to increase the production has the opportunity cost approximately $43\ ,3$.

Conclusion

In instructing of engineering mathematics the Lagrange multipliers method is mostly applied in cases when the constraint condition $g(x,y)=0$ can not be uniquely expressed explicitly as the function $y=f(x)$ or $x=h(y)$. When solving constraint extrema problems in economics the bulk of constraint conditions may be expressed explicitly, so the reason to use the Lagrange multipliers method would seem to be too sophisticated regardless of its theoretical aspects. With a view to the crucial importance of the economic interpretations of Lagrange multipliers is the use of the method primarily preferred. Concrete applications of the presented interpretation principle may be developed in many economic processes. More deeper study on the role of the Langrange multipliers in optimization tasks may be found in Rockafellar (1993).

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Dreams, Paradigm Shifts and Reforms in Mathematics Education: Classification and Plan of Action

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The present paper is concerned with the classification of possible areas of change in mathematics education into dreams, paradigm shifts and reform in order to set up an appropriate plan of action to deal with each. Dreams include integration and non-formal teaching. Paradigm shifts include: developing creativity, interests of students, self education, changing methods of teaching and evaluation, considering complexity and the role of "teacher", while reforms include paying more attention to application of mathematics, the use of technology, and educational activities, and concentrating on mathematical concepts and the "points of departure" in mathematics. A special section is devoted for the suggested plan of action in each of the classified areas of change. However, changing the mentality of the concerned people - especially teachers - is needed in all cases as well as consistency among all components of the educational system and curricula. The studied changes are applicable in almost all countries, may be in different manner and different time.

Introduction

The present paper is concerned with the classification of possible areas of change in mathematics education into dreams, paradigm shifts and reforms (1) in order to set up an appropriate plan of action to deal with each. The major criterion of classification is the degree of applicability of these changes. Dreams seems to be in one stream - not applicable at the moment, while paradigm shifts are applicable under certain conditions and reforms - in the other stream- are ready to be applied after taking necessary decisions and procedures. However, the position of those changes is changeable according to many factors, with time and availability of encouraging environment to come first as major factors.

Classification of possible areas of change (2)

Dreams

1- Teaching mathematics in integrated contexts to the extend to which it could be said that there is no mathematics education as such (3).

2- School materials will be presented in the form of integrated activities through knowledgeable projects.

3- There will be no "traditional" formal teaching of mathematics. Students will "theorize" for themselves.

4- Higher education, especially the study for the first degree, will follow the same pattern, with more direction to the field of study and research assignments (instead of projects).

Paradigm shifts

1- The major aims of teaching mathematics are to develop creativity, to make the study enjoyable and to prepare students to deal with future changes (both in knowledge and jobs).

2- The whole school system will be based upon multiple intelligences theory (4).

3- Adopting problem-solving and "research problems" as dominating methods of teaching mathematics.

4- Complexity is considered in all educational activities[5]. Emphasis must be given to "commons" among different systems to state assumptions underlying different formulas and the existence of different possible solutions.

5- Evaluation of students is mainly based on continuous and non-formal evaluation. Great attention will be given to self evaluation and discussion of student's reports and "research work".

6- The major job of a teacher is as facilitator.

Reforms

1- Studying applications of mathematics in other disciplines and in life, as an essential part of school mathematics.

2- Concentrating on conceptual bases with very little attention to computations, with the use of calculators and computers.

3- Intensive use of technology, with emphasis on data collection, building knowledge and self-learning.

4- Studying the history of mathematics, with particular emphasis on the "departure points" (the cultural historical approach).

5- Paying more attention to school activities relevant to mathematics education.

Suggested plan of action

1- Starting with reforms and attempting some "paradigm shifts".

2- As for reforms:

a) Plan to implement reforms, e.g. prepare necessary equipments, conducting in-service teacher education programmes, changing current pre-service teacher education programmes- when needed … etc.

b) Studying past experiences and considering lessons from them. For instance, some good applications of mathematics are included in some text books[6], but ignored by teachers, may be because they are not included in examinations.

c) Participation of teachers in all procedures leading to reform.

3- As for adopting paradigm shifts, the following procedures can be taken:

a) Convincing teachers and the public opinion - in general, with the value of these paradigm shifts and explaining the way to implement them.

b) Consistency of all components of the whole educational system as well as those of mathematics curricula including aims, content, methods of teaching, using technology, educational activities and evaluation.

c) Priorities for change are for: text-books, teaching methods, means and tools of evaluation and school activities.

4- As regards dreams, many steps can be taken, such as:

a) Paving the way to change the mentality of planners, text-book writers, administrators, teachers, parents and students for specific changes, particularly integration.

b) Changing the whole system of both pre-service and in-service teacher education.

c) Encouraging attempts to integrate branches within a subject and among some subjects.

A Final Word

The author would like to confirm the following:

a) The previous classification is flexible and some of its items are interacted and interrelated.

b) Many of these previous elements- especially reforms- are applied in some countries.

c) The mentioned areas of change are applicable in almost all countries, may be in different manner and different time.

Notes

(1) Although the author can not deal with all the terms involved within the available space, he would like to explain that he means by paradigm shift "a revolution due to a fundamental change in our world view which changes even the way reality is perceived and understand".

See: Kuhn, Thomas S. (1972). **The Structure of Scientific Revolutions** London: Phoenix.

Quoted from: Rugerson, Alan (2010). "The DQME Project as Part of a World-Wide Paradigm Shift In Mathematics Education", **A Background Paper Presented to the DQME3 Meeting**, June 29 - July 2, 2010, Ciechocinek (Poland).

(2) For sources of these areas of change, the writer reviewed his papers which were presented to ME21 Conferences (1999, 2000, 2003, 2004 and 2005), in addition to the following paper:

Mina, Fayez M. (2010). "Some Suggested Alternatives to Activate Some New Trends in Mathematics Education", **A Paper Presented to the Conference of the Egyptian Society of Mathematics Education**, Cairo, 3rd August, 2010. (In Arabic).

(3) It seems that the only attempt to integrate mathematics with life is done by MISP in rather primary education. See:

Gamble, Andy and Rogerson, Alan (1997). **Making Links**. New Zealand: User Friendly Resource Enterprises.

Rogerson, Alan (1995). **Playing Cards**, and **Building Bridges**. (The same publisher).

Rogerson, Alan (1998). **Divine Designs;** and **Patterns & Tiling** (The same publisher).

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(4) See:

Gardner, Howard (1983). **Frames of Mind: The Theory of Multiple Intelligences**. New York: Basic Books.

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(5) Complexity can be described in terms of the following terms:

1- There is no more simple and absolute laws controlling motion and globe.
2- Unity of human knowledge.
3- Research is no more neutral.
4- Thought is no more controlled by logic, and knowledge is no more certain.
5- It is suggested that the main goal of science is to understand realty with the intention to influence and change it.
6- Cohesion of knowledge and its technological applications.
7- The development in technologies of communication, measurement and its units and scientific calculations.

See:

Mina, Fayez M. (2003). **Issues in Curricula of Education**. Cairo the Anglo-Egyptian Bookshop, pp. 23 - 26. (In Arabic).

(6) For Example: See the series of mathematics text-books MATH POWER™ from 7th to 12th Grades, published by McGraw-Hill Ryerson Limited in Canada.

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An Initial Examination of Effect Sizes for Virtual Manipulatives and Other Instructional Treatments

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Abstract
This paper is a meta-analysis comparing the use of virtual manipulatives with other instructional treatments. Comparisons were made using Cohen *d* effect size scores, scores which report treatment effect magnitude but are independent of sample size. Findings from 29 research reports yielded 79 effect size scores. Effect size scores were grouped and averaged to determine overall effects comparing use of virtual manipulatives alone, and in combination with physical manipulatives, to other instructional treatments. Results yielded moderate effects when virtual manipulatives were compared to all other instructional methods combined, large effects when compared to traditional instruction with textbooks only, and small effects when compared to instruction using physical manipulatives only. Combining physical and virtual manipulatives and comparing this treatment with other instructional methods resulted in moderate effect sizes for all comparisons.

Virtual Manipulatives and Mathematics Learning

Virtual manipulatives are "computer-based renditions of common mathematics manipulatives and tools" (Dorward, 2002, p.330). Moyer, Bolyard and Spikell (2002) define them as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). Virtual objects such as pattern blocks, base -10 blocks, tangrams, and geoboards can be found as internet applets or as small stand-alone application programs. Many virtual manipulatives include features designed to focus the attention of learners by highlighting and enforcing mathematical concepts which support children's integrated-concrete knowledge (Dorward & Heal, 1999; Sarama & Clements, 2009). In the past two decades since the emergence of virtual manipulatives, there have been a number of research studies documenting the effects of virtual manipulatives as a mathematics instructional treatment. This meta-analysis synthesizes the research on this treatment by calculating averaged effects of virtual manipulatives on student achievement when compared with other instructional treatments.

Learning Mathematics with Virtual Manipulatives

Several constructs support the use of virtual manipulatives for learning mathematics concepts. *Representational fluency* is defined as a student's ability to transfer ideas easily from one representation to another, a skill which some researchers suggest can be strengthened through the use of technology by providing students with greater access to multiple and dynamic mathematical representations (Zbiek, Heid, Blume, & Dick, 2007). Virtual manipulatives develop representational fluency by linking symbolic, pictorial and concrete representations (e.g., placing a "90°" beside a picture of a right angle); and by linking different types of representational models (e.g., a number line model showing ½ and a region model showing ½). By interacting with dynamic objects, students using virtual manipulatives learn to define, solve and prove mathematical problems by observing connections between their actions and the virtual objects (Durmus & Karakirik, 2006). When virtual manipulatives focus on linking representations, this can influence students' selection of problem solving methods. Manches, O'Malley and Benford (2010) observed that, during partitioning activities, students using virtual manipulatives used more compensation strategies while students using physical manipulatives used more commutative strategies.

Another construct which plays an important role in student learning is the *fidelity* of the technology tools (Zbiek et al., 2007). The degree of alignment between a tool and the mathematical properties is a measure of a tools' *mathematical fidelity*. The degree to which the tools reflect the users' thought processes is defined as *cognitive fidelity*. Seeing visually the consequences of their actions on virtual objects provides students with visual feedback as they test and prove new understandings. When using technology, cognitive fidelity can be even further enhanced as the user's actions are both represented and constrained, making the mathematical properties and relationships even more explicit for learners (Durmus & Karakirik, 2006). There are large collections of virtual manipulatives on the internet with resources linked to national mathematics standards (e. g., National Library of Virtual Manipulatives, http://nlvm.usu.edu; National Council of Teachers of Mathematics Illuminations http://illuminations.nctm.org; and Shodor Curriculum Materials http://shodor.com/curriculum/).

Research Questions

The purpose of this meta-analysis was to conduct an initial examination of effect sizes for virtual manipulatives when compared with other instructional treatments. Two research questions guided the analysis: 1) What are the effects of virtual manipulatives as an instructional treatment in mathematics on gains in student achievement? 2) What are the effects of virtual manipulatives as an instructional treatment in studies of differing durations?

Methods

The study used quantitative methods for a meta-analysis examining the effect sizes of multiple studies. Effect size scores were calculated and used to answer the research questions.

Data Sources

Following the search procedures and standard criteria outlined by Boote and Beile (2005), we conducted a comprehensive search of databases. These included electronic and manual library searches in educational and international databases such as ERIC, PsycInfo, Dissertation Abstracts, Web of Science, Google Scholar, and Social Sciences Index using search terms such as: virtual manipulatives, dynamic manipulatives, computer manipulatives, virtual tools, mathematics manipulatives, mathematics tools, technology tools, computer tools, mathematics applets, and computer applets. In addition to uncovering research on virtual manipulatives, the search located a large body of research focusing specifically on commercially developed dynamic geometry software (e.g., Geometer's Sketchpad, Cabri, and GeoGebra), which is a separate line of inquiry and beyond the scope of this analysis.

Criteria for Inclusion in the Meta-Analysis

From a collection of 135 publications discussing the use of virtual manipulatives, 74 articles and dissertations were identified as empirical research studies, 66 of which had been peer reviewed. The other 61 articles were papers expressing opinions, developing theories, or suggestions for instruction. To build a comprehensive base of studies, only three criteria were used to remove studies from the empirical pool originally identified. Sixteen studies were excluded because of study design, type of applet, and threats to validity such as history, mortality, instrumentation, testing, selection, regression, and maturation (Gall, Gall & Borg, 2003). A final total of 58 studies met our criteria for further review, and 29 of these studies contained effect sizes.

Analysis

Within the 29 studies, there were 79 effect score cases comparing virtual manipulatives with other instructional treatments. Effect size scores are used to report the magnitude of treatment effects and are independent of sample sizes, thus making the comparison of studies across multiple settings possible. Effect size scores were computed using gain scores, differences between post test scores, and F values used to calculate Cohen's d scores. The 79 effect score cases were grouped to obtain averaged effect size scores.

Results

The following results report comparisons between virtual manipulatives as an instructional treatment, a) with all other instructional treatments, b) with instruction using physical manipulatives only, and c) with traditional instruction using textbooks only. The effect sizes reported for each of the comparisons of the analysis are averages of 79 case effect sizes yielded from the 29 studies. Descriptions of effect sizes are based on the suggestions of Urdan (2010) that an effect size of less than 0.20 be considered small, effect sizes in the range of .25 to .75 are considered moderate, and those over .80 are considered large.

Effects of Virtual Manipulatives as an Instructional Treatment

The first research question focused on the effects of virtual manipulatives as an instructional treatment in mathematics on gains in student achievement. The following comparisons are presented in Table 1: a) all instruction using virtual manipulatives compared with all other methods of instruction; b) instruction in which only virtual manipulatives were used compared with all other methods of instruction, with instruction using physical manipulatives, and with traditional instruction using textbooks; and, c) instruction in which virtual manipulatives and physical manipulatives were combined as a treatment compared with all other methods of instruction, with virtual manipulatives used alone, with physical manipulatives used alone, and with traditional instruction using textbooks.

Table 1
Effect Size Scores for Virtual Manipulatives Compared with Other Treatments

Comparisons	Number of Comparisons	Effect Size
Virtual Manipulatives Used		
& Other Instructional Treatments	70	0.37 (0.44)*
Virtual Manipulatives Used Alone		
& Other Instructional Treatments (combined)	53	0.37 (0.46)*
Physical Manipulatives	35	0.18 (0.32)*
Traditional Instruction (textbook)	18	0.73
Virtual and Physical Manipulatives Used Together		
& Other Instructional Treatments (combined)	26	0.33
Virtual Manipulatives	9	0.26
Physical Manipulatives	11	0.20
Traditional Instruction (textbook)	6	0.69

Note: *Effect size with one outlier.

The comparison of all studies using virtual manipulatives for instruction with all other instructional treatments yielded a moderate averaged effect score (0.37; 0.44, with one outlier). The analysis of the 53 cases comparing instruction using only virtual manipulatives with other instructional treatments also yielded a moderate effect (0.37/0.46 with one outlier). An analysis of the 35 cases comparing instruction using only virtual manipulatives to instruction using physical manipulatives yielded a small/moderate effect (0.18/0.32 with one outlier); and the analysis of 18 cases comparing instruction using only virtual manipulatives to classroom instruction using textbooks yielded a moderate effect (0.73).

In the analysis of studies where virtual manipulatives were combined with physical manipulatives (VM/PM combined) for instruction and compared with other instructional methods, 26 cases yielded a moderate effect (0.33). In the comparison of VM/PM combined with the use of virtual manipulatives alone, nine scores yielded a moderate effect (0.26). VM/PM combined compared with physical manipulatives alone in 11 scores produced a small effect (0.20). Finally when VM/PM combined was compared with classroom instruction using textbooks this produced a moderate effect (0.69). In summary, the largest averaged effect scores for the virtual manipulatives were produced when comparisons were made between virtual manipulatives and classroom instruction using textbooks. Other comparisons produced moderate or small averaged effects. Overall the effect size results demonstrated that virtual manipulatives produced positive averaged effects on student achievement when they were used as an instructional treatment for mathematics teaching.

Effects of Virtual Manipulatives Based on Treatment Duration

The second research question focused on the effects of virtual manipulatives as an instructional treatment in studies of differing durations. This analysis examined the length of the instructional treatments when virtual manipulatives were used for instruction. Length of treatment categories were aggregated by days and number of effect size scores per category. The five categories for the analysis were 1 day, 2 days, 3-5 days, 6-10 days, and more than 10 days. These results are reported in Table 2.

Table 2
Effect Size Scores for Virtual Manipulatives by Length of Treatment

Length of Treatment	Number of Comparisons	Effect Sizes
1 day	10	0.13
2 days	5	0.36
3-5 days	12	0.21
6-10 days	10	0.48
More than 10 days	31	0.47 (0.62)*

Note: *Effect size with one outlier.

In approximately half of the comparisons students participated in instruction involving virtual manipulatives for durations longer than ten days. The shortest length of treatment (1 day) yielded the smallest averaged effect size score (0.13) when comparing instruction using virtual manipulatives to other methods of instruction. Treatments of 2, 6-10, and more than 10 days of virtual manipulative treatment, all yielded moderate average effect size scores (0.36, 0.48, and 0.47/0.62, respectively). The results of the comparisons indicate that studies of longer durations tend to report larger effect sizes while studies in which virtual manipulatives are used for shorter durations tend to report smaller effect sizes.

Discussion & Conclusions

The purpose of this study was to use a meta-analysis to synthesize the quantitative results from research on virtual manipulatives. From the meta-analysis, there are several patterns that emerged. Overall, the virtual manipulatives are an effective instructional treatment for teaching mathematics when compared with other instructional methods. It is also interesting to note that the average effect size scores for virtual manipulatives compared with traditional instruction using textbooks are larger than when comparing virtual manipulatives with physical manipulatives. However, combining virtual and physical manipulatives as a treatment compared with traditional instruction using textbooks resulted in some of the largest effects produced in this study. These results suggest that the virtual manipulatives have unique affordances that have a positive impact on student achievement in the learning of mathematics. The results also suggest that combining virtual and physical manipulatives for instruction provides students with representations available in each manipulative type that are a visual support for students and promote students' representational fluency.

Results of the meta-analysis also suggest that the length of treatment for virtual manipulatives influences the average effect size scores. This result is similar to other studies on instructional treatments showing that longer treatment durations provide more opportunity for the effects of a treatment to be determined through research. This result makes sense, particularly since there are various factors in the virtual manipulative environment which may be new to students, such as finding webpages or manipulating dynamic objects, and these activities take time for students to learn so that they can interact effectively with the virtual manipulatives.

Although averaged effect sizes indicate that virtual manipulatives are as effective, and may even be more effective tools of instruction than other methods, little is known about how learner characteristics, applet features or instructional methods affect student learning while using virtual manipulatives. Additional research is needed to determine if the use of virtual manipulatives as an instructional tool is more effective for some students than others. Although it has been suggested that virtual manipulatives could be successfully used in both gifted and intervention instruction, to date, there are limited research studies investigating differences in virtual manipulative use as related to student abilities. There is also great variability in applet features, structures and the amount of guidance they provide. To further enhance the use of virtual manipulative applets, research is needed which compares the effects of different applet characteristics on student learning and compares which applets are most effective for teaching which specific concepts within each mathematical domain. For example, Haistings (2009) indentified variations in learning when students used the same virtual manipulative applet with and without symbolic linking; and, Bolyard (2006) compared the effects of two different virtual manipulative applets for integer instruction. These types of investigations may help researchers to identify relationships between applet features and impacts on student learning and achievement.

This meta-analysis found that virtual manipulatives have a moderate average effect on student achievement when compared with other methods of instruction, and that larger effect size scores are produced when studies have longer treatment durations. While these results confirm the effectiveness of virtual manipulatives for mathematics instruction, they do not reveal *why* virtual manipulatives are effective. Further research on specific affordances that promote learning, effects for different mathematical domains, and implications of virtual manipulative use for different students will significantly contribute to our understanding of the features that make virtual manipulatives effective and will answer the question of why virtual manipulatives impact student achievement during mathematics instruction.

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New and Emerging Applications of Tablet Computers such as iPad in Mathematics and Science Education.

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Abstract

This research paves the way for work and development in adopting the latest technologies in tablet computing in learning and teaching mathematics and science related topics. In particular, the more human-like interface features, offered by Apple's iPad and other touch devices is being investigated for educational development. Preliminary studies by the author have demonstrated that students have a preference for using a device such as iPad in helping them with their studies. They have reported the unique touch interface, portability and easy eBook reading abilities as some of the significant features of iPad. This study has also identified the teaching capabilities of iPad from the teacher's point of view. As part of this component of the research, the iPad was used to actively involve students in discussing and undertaking a series of specially developed case studies in classroom.

A number of useful and relevant apps for learning and teaching mathematics and science have also been identified as part of this research project.

Key words: Technology, interface, Education

Computers' Application in Education – An Evolution

When back in the 1980s, Commodore 64 (C64) entered the homers of several hundred thousands of people in different parts of the world; it revolutionized how one should work with and use computers. That is whenever and to a certain extent wherever one wishes. In other words, computers were not restricted to only computer labs of learning and industrial organizations. The other significant contribution to computer aided learning was the multimedia features of these relatively inexpensive home computers.

Numerous software packages for educational purposes were designed and developed for personal computers such as Commodore 64. They included programs with abilities to teach with text, sound, images and relevant graphs. A significant approach to reinforcing learning was the use of multimedia quizzes with sound and colour for learning enhancement. See Schembri, T., & Boisseau, O. (2001) for details on Commodore 64.

The progress in the technology, its capabilities and educational applications have continued and enhanced exponentially over the recent years. The latest developments focus on the web based learning systems for the purposes of better understanding. For further information, see Chau (2007). Although the Internet based learning plays a major role in education and its delivery, the latest hardware features promise exciting developments. These features have enabled the application developers create interesting and educationally effective apps.

The iPad's Potential Use in Education

When Apple introduced iPad in 2010, its potential and applications in education were realized and considered by many academics around the world. In one university alone, with which the author is familiar, there are around 400 of these devices. Academics and others involved in education are eager to find ways of using these sleek devices. With the release of iPad in 2010, the Touch technology has had even more serious implications for education. Just before the formal launch of iPad in the US, Fry S (2010) had the following comment after interviewing Steve Jobs (Apple's CEO) and reviewing the product for Time Magazine:

"When I eventually got my hands on one, I discovered that one doesn't relate to it as "tool"; the experience is closer to one's relationship with a person or an animal."

According to Fry (2010), Tracy Futhey, of Duke University, was quite optimistic about iPad's potential in education and commented that:

"The iPad is going to herald a revolution in mashing up text, video, course materials, students input ...We are very excited."

Putting the speculations aside, the burning question, at the moment, is: What makes iPad superior to conventional notebook type computers?

iPad offers a different and more natural way of interface. For instance Apple's tap, pinch and draw capabilities using fingers on iPad, iPhone or even iPod are good examples.

The experience through the Apple's Touch technology does certainly create a more natural interface between the user and the machine. To demonstrate this capability, applications which utilize the touch features intensively may be referenced here. For instance, the painting and drawing apps for iPhone enable a user to experiment with painting in a totally innovative fashion. The painter uses iPhone screen as a canvas and the fingers as brushes. The colours are selected by tapping and touching a colour wheel. The chosen colour is placed on the user's palette and the index finger then starts drawing and painting on the screen. iPhone is extremely responsive to strokes and the tiniest detail as desired by the painter are depicted on the canvas. The pinch and zoom feature is used to draw and paint the fine details. The following image (Figure 1) was painted by the author using Brushes app on the iPhone. The painting experience does certainly create a much closer relationship between the painter and the subject. Hence, this experiment demonstrates the special technological advantages offered by iPad/iPhone.

Figure 1 – Diamond Head and a Waterfall Scene Painted by Finger on iPhone

Let us do not forget that progress in notebooks has also been taking place. Many modern notebooks are quite technologically advanced and have interesting and useful features. Hence, the hardware features of iPad cannot be the main reasons and basis for its popularity of use in fields such as education.

One of the main contributory factors towards the popularity of iPad and the desire to explore its potential in learning and teaching is the availability of the apps. These apps are basically pieces of software programs (in traditional terms) which run on devices such as iPad and iPhone. They cover numerous fields such as languages, arts, music, science, mathematics and statistics. The list is continually growing. These apps are readily available at reasonably modest prices on the app store and are accessible via Apple's iTunes. According to the iTune's app store prices, the majority of the educational apps cost under the $10 mark. A very large proportion of these sell for an amount less than five dollars (AUD and USD are almost the same at the time of writing). The apps have several distinct features and advantages over the conventional programs. Firstly, they are inexpensive. For a fraction of the price of a traditional PC software package, one can purchase an app. In terms of features, they are not too far behind their older cousins either. For instance, the author has recently investigated the suitability of an app for teaching the fundamentals of planning, execution and control in a Project Management. The outcome of this investigation was a pleasant surprise compared with the large-scale packages such as MS Project. This app (Project Planner) priced only $3.99 satisfies the needs of teaching the basics of Plan, Execute, Control and Report quite adequately.

Another innovative technology which certainly has a place in the modern approaches to learning is the Amazon Kindle. Kindle is a specially developed hardware and software packaged into a very compact and attractive tablet. Kindle has free international electronic book, magazine or document download capabilities via 3G and wireless connection. Kindle with its whispenet synchronization between the user's different devices, is a very good example of seamless technology for learning. For further information on seamless learning, see Looi (2010). Hence the user can download numerous items of interest from the Amazon's Kindle Store. In addition to its very useful features such as an active dictionary and free 3G access to Wikipedia and the Internet, it is equipped with an experimental text to speech function. When switched on, this function allows the reader to listen to the text on the page. The author has experimented with this feature for the purposes of speed reading training. This experiment was carried out by setting the speech pace to fast and the text on the screen was scanned at the same speed by the author. It was observed that the need for sub-vocalization was removed from the process. Although sub-vocalization is an important factor in comprehension, it is also an inhibitor in achieving higher speeds. The author has comfortably achieved speeds above 250 words per minute with a close to full comprehension outcome.

One of the main features of the Amazon Kindle is its ability to access several hundreds of thousands of books from the Kindle Store. There are also several major international magazines, periodicals and newspapers available on the site. The book collection includes a comprehensive coverage of topics in mathematics and science. The list of these books, in size and coverage, is growing all the time. It must be mentioned that Amazon has also developed and provided a Kindle eReader app for iPad and iPhone which is free to download and use. The app contains most of the features and functionalities of the very successful Kindle eReader. The eReader capabilities of iPad, in general, are certainly a preferred feature amongst the learners. According to the feedback collected from the author's students in three different classes, this feature is definitely amongst the top three. Although text is a traditional learning mode, it continues to remain one of the most effective forms of media. With the technological advancement in the tablet computers such as iPad, text can become even more powerful in terms of learning and also an appealing way of fully immersing in the topic. The features such as quick access to books, reasonable prices of eBooks, portability of a large collection and text to speech capabilities will certainly help this popularity.

These and similar technologies are very likely to become readily available on other tablet computers. They have a great potential for education in many fields. They can even build on the immersive and real-time engagement as in Virtual Reality in online courses. For challenges of using virtual reality in online courses, see Stewart et al (2010). In order to test the technology's acceptance and perceptions about its suitability and effectiveness, a series of surveys were conducted by the author in the past two years. As a challenge to determine these technologies' serious uses in education, the author set himself the task of undertaking the research and writing this paper utilizing several apps on an iPhone. Some examples included apps on communication (text and voice mail), data collection and Statistics, MS Word, document scanning and PDF converter and image cropping. The statistical analysis was also carried out using an iPhone app. The next sections presents the methodology and results for this ongoing work.

Learners' Perceptions and Preferences

In 2010, the author conducted an investigation into the educational applications of the latest developments in modern computing. The main purpose was to determine the learners' needs and preferences in terms of the latest developments. The participants of this investigation were people who were either directly involved in some form of learning for themselves or closely related to others such as their children or spouses. Adults of both genders from totally different walks of life and backgrounds were selected and contacted for the survey and data collection in this study. These people included college and university students, professionals such as nurses, dentists, technicians and teachers. The study included respondents with varying cultural, linguistic and geographical characteristics too. An aspect of this investigation was to study and compare the levels of interaction-enjoyment for both computer and human teacher. The respondents were asked to rate their perception of the level of enjoyment on a 1 to 5 scale. An initial analysis of the responses determined that the interaction in terms of enjoyment for human teacher has a much larger mean (4.1) than computer teacher (2.8). As Normal Distribution curves in Figure 2 show, the standard deviation for human teacher is also smaller than computer teacher and the respondents appear to have preferences very close to 4 (3 and 5). A t-test even at 1% level of significance indicated that indeed the null hypothesis of identical population means (for computer and human teachers) ought to be rejected. Therefore, it can be concluded that learners, in general, perceive that the learning process with an actual (human) teacher is more enjoyable than a virtual (computer) teacher.

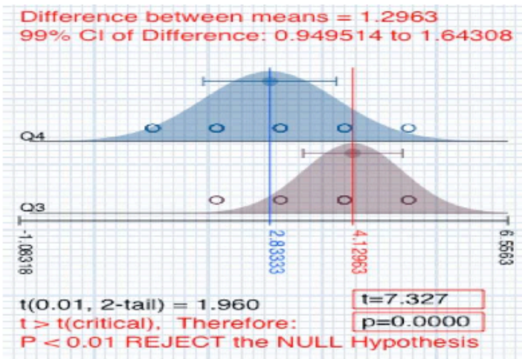

Figure 2 – Difference between the Means of the Responses – Computer and Human Teachers

This finding is rather interesting because the respondents would perceive a computer teacher to have a place in the future education. Hence, the innovative approaches offered by iPad and the available apps is, without a doubt, embraced by the users but education, in its traditional format, is definitely preferred. The respondents' very positive response (Figure 3) to the following question (Q7) is demonstrative of their belief in future technologies for learning and teaching.

Please rate the effectiveness of the following scenario which may take place in the future:

You buy/borrow a book on a topic of your choice, take it home and open it. You then ask the book in your language of choice some questions. The book starts talking and explaining to you by showing you 3 dimensional images. It then invites you to physically (but virtually) interact with them. So, it helps you to learn your topic (e.g. a craft or a skill) by letting you experiment; and it gives you feedback all the time! 1 (Low) 2 3 4 5 (High)

	Q7
Mean	3.7037
Std Dev	1.22289
Sxbar	0.166414
95% CI	3.3742 to 4.0332
Sum	200.0
Median	4.000
Min.	1.000
Max.	5.000
Range	4.000
n	54

Figure 3 – Future Technologies

Conclusions

The main purpose of this paper was to establishment the main reasons and motives for iPad's popularity. The availability of numerous apps related to mathematics and science learning is also a very positive feature. App developers are continuously producing new applications and updating the existing ones. It was concluded that iPad's sleek and contemporary design should not be the sole contributing factor to a desire to explore the educational applications for it. There are other reasons which should be considered. For instance, due to its special features such as portability, software (app) cost effectiveness and easy access, iPad does offer a great deal to learning and teaching in general.

Although there seems to be a positive belief in the future of iPad-like devices in learning and teaching, the learners still have a preference for having a human teacher. The outcome of the investigation conducted by the author, regarding the effectiveness and level of enjoyment with an actual teacher support this finding. The respondents, however, are certainly in favour of an advanced and intelligent system which can respond to learners' needs. Hence, tablet computers such as iPad will have a place in mathematics and science education.

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Science, Technology, Engineering, and Mathematics (STEM) Development: Pathways for Universities to Promote Success

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Abstract

The focus of this paper is on polcies used in university communities to promote Science, Technology, Engineering, and Mathematics (STEM) development. The paper outlines how STEM can be expanded to engage stakeholders including colleges and universities, students and faculty, and academic and industry leaders. The paper emphasizes proven practices that engage and sustain educators and students in STEM experiences. Most needed to facilitate STEM efforts is a strong commitment from institutional leadership and clear measures to scale-up efforts throughout the campus. Key partners in this effort include university department heads, university deans, community leaders, business and industry leaders (e.g., Chamber of Commerce), departments of education, higher education committees, and the governor's office. Successful STEM efforts attract multiple partners who collectively share risks and rewards of the STEM efforts.

Introduction

In recent years, there has been an increased focus on the critical importance of educating individuals in the fields of science, technology, engineering, and mathematics (STEM). This focus has resulted in wide-spread support and funding to develop and encourage numerous initiatives that place an emphasis on STEM. A variety of reports, including *Rising Above the Gathering Storm* (National Academy of Sciences, 2007) and *Before It's Too Late* (National Commission on Mathematics and Science Teaching for the 21st Century, 2000), echo the purpose of a high quality mathematics and science education and its importance in preparing citizens for competition in a global society with STEM-based careers and professions. These reports emphasize the need for better preparation of K-20 teachers and students in all areas of STEM. To address this need, many STEM faculties in disciplinary departments at universities are working in collaboration with faculty in colleges of education. These partnerships are often formed in an effort to support mathematics and science teaching and learning in partnerships with school systems. Many of these partnerships are funded by outside agencies as a way to encourage the partnership and collaboration (for example, the National Science Foundation has funded these types of collaborations in the Math and Science Partnership Program throughout the United States). This paper outlines three proven strategies for engaging and sustaining STEM disciplinary faculty, College of Education faculty, and K-20 students in STEM experiences. These strategies will be outlined throughout the remainder of the paper and include: 1) Expanding the STEM pathway for student success, 2) Professional development and learning for partners in the STEM effort, and 3) Using technology to facilitate STEM efforts.

Expanding the STEM Pathway for Student Success

A STEM Pathway includes all of the experiences that students have from K-12 school to a career in a STEM-based field (for example, experiences in elementary school, middle school, high school, undergraduate education, internships, and graduate education). One of the first proven strategies for engaging and sustaining STEM disciplinary faculty, College of Education faculty, and K-20 students in STEM experiences in the STEM Pathway is to develop structures on the university campus to increase student interest, participation, and achievement in STEM fields, especially among underrepresented students. Student diversity and a commitment to preparing all students must be a fundamental component of the work of our universities. It is imperative to engage faculty members who are experts in the science, mathematics, engineering and technology disciplines with students to provide experiences for the students that reveal the types of activities in which STEM faculty are engaged as part of their work. These scientists can also expose students to employers who work in the STEM fields to demonstrate how the college degree in STEM transfers into the world of work. Leadership for these experiences and structures that facilitate this type of interaction are provided by college and university presidents, provosts, and chancellors who have a vision for STEM education on their campuses. In order for the STEM pathways to be transformative and seamless, leadership should include federal agencies, STEM disciplinary societies, faculty and administrators employed in a wide variety of academic institutions, and employers in private industry.

One essential goal in expanding the STEM pathway should be on encouraging, recruiting, and mentoring students as they enroll in more rigorous mathematics, science, technology and engineering courses, both as high school students and when they matriculate to college as undergraduate students. However, attaining this goal is no small feat. First, university faculty who work with STEM students should agree that quality matters substantially more than quantity, and that a focal point should be on how STEM faculty teach rather than how much they teach. This is particularly true in the introductory science, mathematics, engineering and technology courses at the university where STEM students experience their first interaction with the activities of their future careers. These STEM courses must be well-connected and aligned to current issues and societal challenges so that students can easily recognize the application of the STEM content they are studying to real world problems. Indicators from STEM industries can be used to assist university faculty in this alignment and to emphasize the connections between bodies of knowledge and STEM work. It is also important to connect and integrate research and education and use research findings to drive and inform education. The best practices must be assembled and become tools and the knowledge basis for learning. This practice will improve the quality and productivity of undergraduate intellectual experiences. However, there seems to be de-emphasis on the importance of the STEM Pathway in some countries. For example, in the United States, federal support of basic research in engineering and physical sciences has experienced modest to no growth over the last thirty years. In fact, as a percentage of GDP, funding for physical science research has been in a thirty year decline. In contrast, at no time in history has the possession of knowledge been so strong an indicator of economic wealth.

In the midst of these economic constraints, one approach to strengthening the STEM Pathway depends on broad visions for collaboration among various stakeholders. These broad initiatives include the development of learning opportunities to improve undergraduate education and engage the university in wide reform approaches that analyze teaching and learning. Universities need to understand exemplary teaching and provide ample opportunities for colleagues to witness these practices so that faculty can transfer innovation from one department to another and one university to another. To do this will require the university to invest in building intellectual communities that share knowledge and synthesize shared data, thereby forging partnerships that stimulate research and teaching innovation on the university campus. Universities can then expand their partnerships to include business, industry, chambers of commerce, professional societies, National Science Foundation and other funding sources. Ultimately, and probably the most difficult undertaking, is for universities to inform and encourage the public to understand how critical the STEM pathway enterprise is to their welfare.

Professional Development and Learning for Partners in the STEM Effort

A second strategy, that is absolutely necessary for engaging and sustaining STEM disciplinary faculty, College of Education faculty, and K-20 students in STEM experiences, is professional development and learning opportunities for each of the partners in the effort. For these opportunities to be institutionalized, the leadership of the institution must facilitate the development structure, put into place mechanisms for scaling up the number of STEM faculty participants, and sustaining the interdisciplinary STEM learning environments. One example of a leadership question to ask might be: How can STEM faculty be supported and rewarded for their work in STEM education? There are currently many more science, technology, engineering, and mathematics (STEM) faculty and other professionals working with school systems in partnerships to support mathematics and science teaching and learning as a result of funding incentives. One unique feature of some of these partnerships is that K-12 school teachers work together with mathematicians and scientists in the field. This type of interaction leads to collaborations between teachers and scientists doing field work and can result in improvements in teachers' classroom instruction. Researchers report that there are benefits for everyone involved, including teachers, scientists, and school students, in these types of STEM partnerships (Siegel, Mlynarczyk-Evans, Brenner, & Nielsen, 2005). For example, as a result of a teacher and scientist collaboration in a STEM partnership, there is teacher learning and scientist learning which then impacts the teacher's school classroom teaching and the scientist's university teaching (Canton, Brewer, & Brown, 2000; Dresner, 2002; Dresner & Starvel, 2004). Another outcome of these types of interactions is that research scientists and mathematicians who work with schools and teachers begin to better understand mathematics and science teaching in school systems and how their own work and expertise connects with school teaching and learning for students (McCombs, Ufnar, & Shepherd, 2007).

The poor rankings for students in the United States on measures of mathematics and science, when compared with their international peers, are one indicator that the need for STEM professional development at the K-20 levels is a growing concern. In *Rising above the Gathering Storm* (2007), the National Academies concluded that "the scientific and technological building blocks critical to our economic leadership are eroding at a time when many other nations are gathering strength." At the core of effective science teaching is "student-centered and inquired based opportunities for students to actively be engaged in learning." One of the most important ways to initiate institution-wide reform is to begin with an examination of the pedagogy being used to teach STEM courses at the K-20 levels. Institutions should ask important questions such as: Does this pedagogy meet the needs of 21st Century learners and of the STEM discipline and its careers? These critical examinations of pedagogy can spur the need for training and support as faculty adopt new cutting-edge pedagogies that meet the needs of learners and the discipline. Some changes will focus on intellectual communities and situated cognition that integrate new social interaction theories and new technologies that facilitate technologically advanced collaborations. The universities' colleges of education are likely experts in pedagogy and can assist in modeling some of these instructional innovations. Faculty from academic units across the university campus can also learn about diverse learning styles, ways to apply instructional technology to sustain learning, and particularly how social media plays an important role in the current generation's models for learning.

The preparation and development of teachers has come under fire and some leaders question the value and necessity of teacher preparation programs, implying that they do not take teacher education seriously. There are some very good teacher education models, yet many are expensive. Recently the Association of Public Land Grant Universities (APLU) designed an Analytical Framework for STEM Teacher Education to be used in the development and assessment of STEM teacher preparation programs. This framework describes an effective teacher preparation program for science and mathematics teachers, and provides some very specific examples of effective criteria and processes that should be used in teacher preparation programs. These criteria serve as a model and a guide for improving the preparation of STEM teachers. Once teachers are in the classroom it is important to continue their development as lifelong learners throughout their profession. But as reports show, career development for teachers in the United States is not systematic and cohesive, as it is in some other countries (Stigler & Hiebert, 1999). The National Staff Development Council's (Wei, Darling-Hammond, Andree, Richardson, & Orphanos, 2009) report about professional development trends in education reported that teachers experience much less in-depth professional development and spend less time in professional development than they did four years ago. The decline in professional development, in a system where teachers are already lacking professional development opportunities in STEM, will have significant impacts in teachers' instruction and student learning in STEM for years to come.

Utilizing Technology to Facilitate STEM Efforts

A third proven strategies for engaging and sustaining STEM disciplinary faculty, College of Education faculty, and K-20 students in STEM experiences is to use current technologies to enhance learning and interaction. Advancing technologies have minimized academic institutions as a depository of knowledge. Using knowledge networks is a much more vital and instrumental way of shaping new knowledge today, rather than having students recite what is already known. It is important for universities to identify the kinds of technology that are available and to remain current on those technologies to facilitate and engage more learner-centered teaching in STEM courses. Connecting with students in the K-12 STEM Pathway means that institutions should find ways to effectively use social and interactive asynchronous and synchronous media such as twitter, wikis, podcasts, e-books, blogs, social networking, gaming, videos, internet2, simulations, Second Life, or other interactive programs. These tools can be used effectively to promote student interest and achievement in STEM, in general education, and in STEM majors. One example could be the use of robots to teach students engineering principles and design. In fact, one Korean Education Policy is planning to use robots to teach thousands of kindergarteners by 2013.

Many universities are beginning to offer coursework and entire programs of study through the use of technology. While digital education is designed currently for adult learners, in time it is likely that the formats of on-line instruction will adapt to provide technological options for all levels of education and learners. These advances have the potential to make learning more individualized, interactive, and self-directed and could be used in a variety of STEM learning environments with students. For example, animal dissection using computer models has already replaced the dissection of real animals in many biology classrooms.

Both universities and public schools will need to keep current with the technologies that are already infused in students' daily lives. Unfortunately rapid shifts toward the use of social media in society have not been followed by equally rapid shifts to these technologies by schools and universities. Institutions must become more agile in responding to technology changes so that the technologies students have learned to use in their daily lives can be applied to problem solving in their school and university experiences, particularly as those technologies apply to the solution of STEM problems. Unfortunately in many places, it is easy for students to see more technology at the gas station in their public school or university classrooms. An important way to change this trend it to ensure that instructors have opportunities to experience these technologies as part of their daily work so that they can integrate the technologies into their classrooms and interactions with students. Some college campuses were early adopters of podcast technologies, which provide their incoming students with tools to acclimate and navigate around campus. For these campuses it is common to see students walking around the campus with ear buds listening to campus map information, campus event information, or a STEM lecture for one of their classes. There are also public schools where students are using their IPod technology to listen to podcasts from their teach or another source on topics such as the process of photosynthesis or the difference between equilateral and isosceles triangles. This technology individualizes learning and allows students to have anytime access to the STEM content they are studying in their courses.

Closing Thoughts

The purpose of this paper was to outline three proven strategies for engaging and sustaining STEM disciplinary faculty, College of Education faculty, and K-20 students in STEM experiences. These strategies focused on the importance of enhancing the STEM pathway for student success, the critical need for professional development and learning for partners in STEM efforts, and impact of new technologies on facilitating STEM efforts. Without the advances in knowledge and understanding that are brought about by study in STEM fields, our future generations will experience a great deficiency of understanding. In the book, *The Demon Haunted World*, Carl Sagan implies that this deficiency of understanding foreshadows an alarming future. He writes: "Finding the occasional straw of truth awash in a great ocean of confusion and bamboozle requires vigilance, dedication, and courage. But if we don't practice these tough habits of thought ... we risk becoming a nation of suckers, a world of suckers, up for grabs by the next charlatan who saunters along." Learning in the sciences, technology, engineering and mathematics are essential to the type of understanding that Sagan describes. Yet at many institutions of higher education only a small portion of the student undergraduate population completes a mathematics or science course after their freshman or sophomore year of college. This unfortunate pattern signals a lack of understanding about the importance and applicability of STEM to their daily lives and future careers and contributes to a citizenry that is less and less knowledgeable about science, technology, engineering and mathematics. The challenges of renewable energy, clean water, and global climate change will be the problems faced by the current generation of students studying STEM in K-20 education. Will this generation be less informed about science, technology, engineering and mathematics and make their state and national decisions based on what their Facebook friends value? Or can we count on this generation to use science, technology, engineering and mathematics to solve the world's problems and advance STEM for the entire planet? Only an ongoing commitment to STEM pathways, instructional development, and integrated technologies will produce a favorable result for all.

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The basics of set theory – some new possibilities with ClassPad

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Abstract:

The basics of set theory consists in sets, elements, lists, set-builder notation, subsets, equal sets, the empty set, union, intersection, difference, symmetric difference and Cartesian product.
Sets are one of the most fundamental concepts in mathematics but we can not calculate with set operations and set relations on the ClassPad. On the other hand we can write in the text mode with special symbols of the set theory in the ClassPad, e.g. $\in$, $\notin$, $\cup$, $\cap$, \, $\subset$, $\subseteq$, $\neq$, …

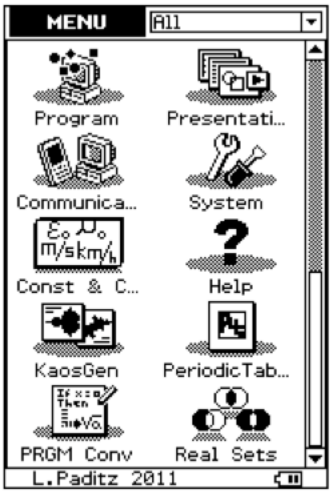
Thus some students of informatics tried to introduce the set theory in the operating system version 3.05 (published 2010). They followed two ways of solution:
1. Create a so called Add-In for ClassPad to calculate with sets of real numbers.
2. Create a Basic-program for ClassPad to calculate with finite sets of numbers or words.

In the first case we use the set-builder notation {x|…}, e.g. {x|a<x<b} or {x|x≥c} or {x|2,3,5}. In the other case we use the notation "{2,3,5}" or "{x1,x2,x3}" or "{(a,b),(a,c),(2,c),(d,4)}" or"{Anna, Alan, Max, Marc, Tanja}" to fulfil set theory operations. Here we work with strings. Thus we get no problems with system variables x1, x2, x3 or Max.

During the workshop the participants could download the new programs to their own calculators or notebooks and check the new possibilities or they follow a demonstration with the ClassPad Manager software.

1. The Add-In "Real Sets"

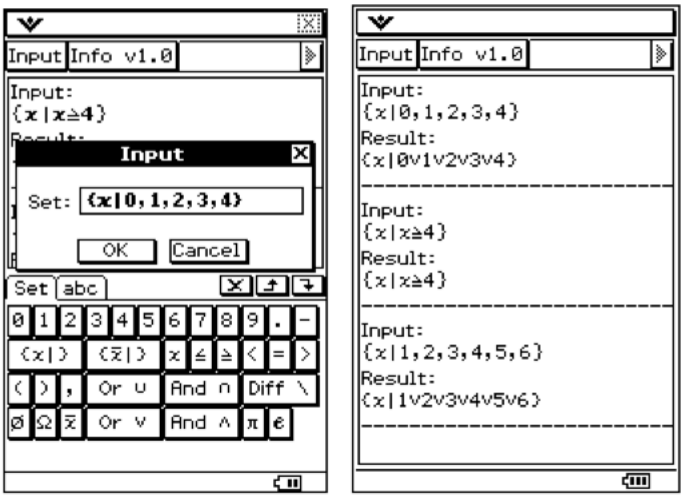
The students wrote a program in C^{++} and used than the CASIO-SDK (software development kid) to compile the source program into a ClassPad Add-In. The Add-In can be installed in a handheld calculator but not in the PC-manager-software. Thus the students compiled their program additional in an exe-file, which runs on a Windows-PC (**Real_Sets.exe**). Here we need additional a special library, the **ClassPadDLLgcc.dll**, in the exe-file folder.



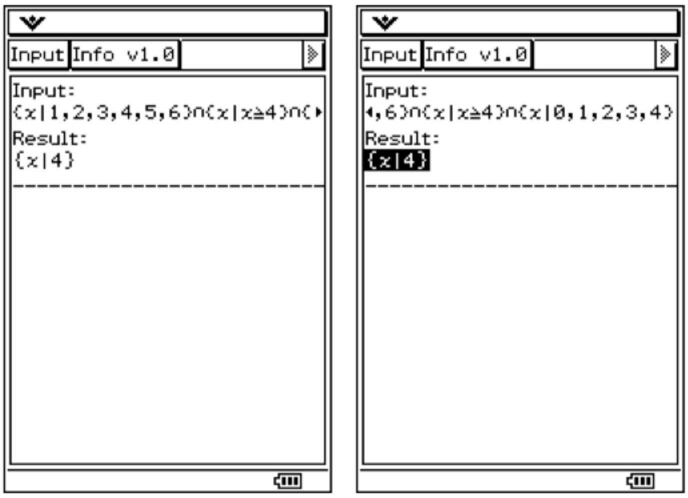
The new **Real Sets**-Icon in the menu

The students created a special keyboard for the set theory!

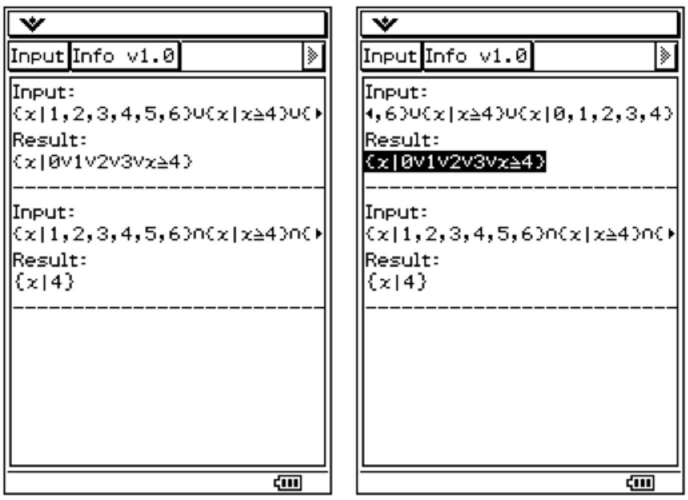
Now let us calculate some examples:
Let be A = {x| 1,2,3,4,5,6}, B = {x| x≥4} and C = {x| 0,1,2,3,4}



The output-window shows 3 results

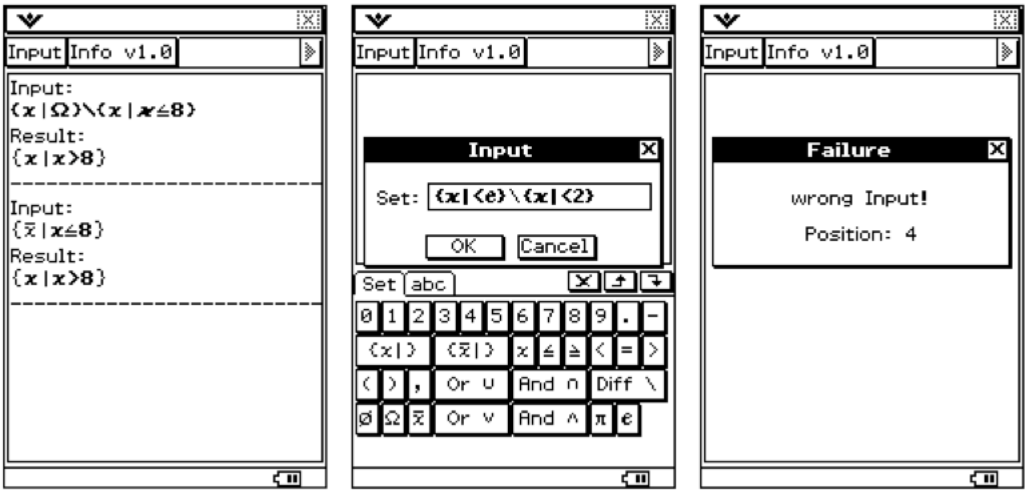


A∩B∩C

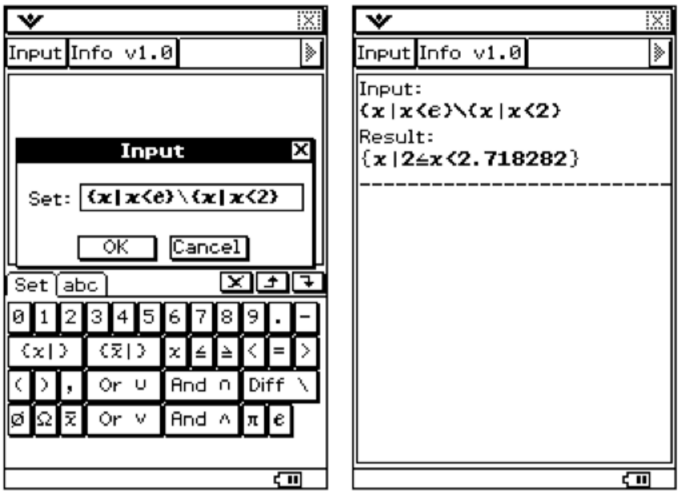


A∪B∪C

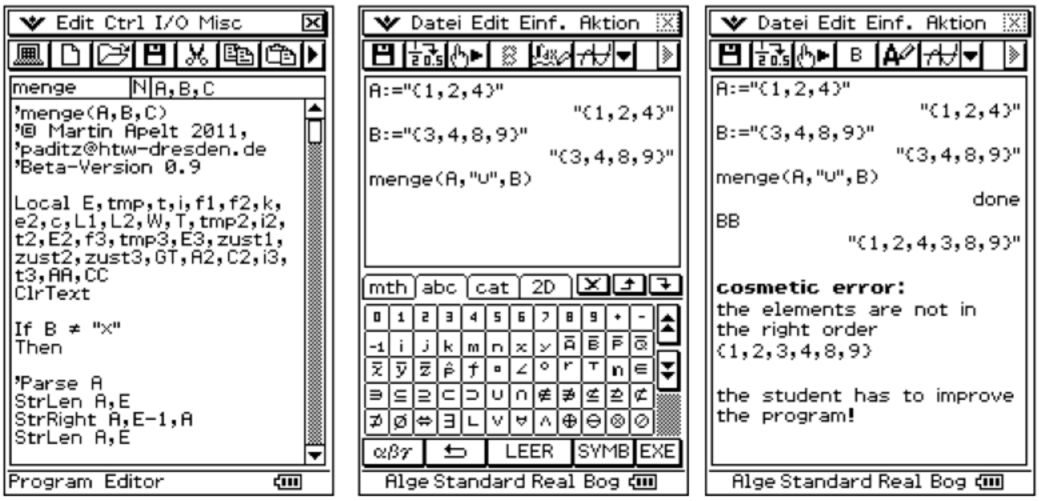
For the empty set here we have the symbol Ø and the full set (all real numbers) is Ω.
Additionally we can calculate the complementary set in Ω by the help of the x-bar notation.
We can use brackets (…) for multiple operations, e.g. A∩(B∪C).

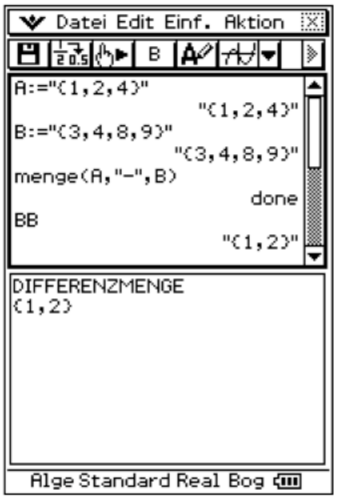


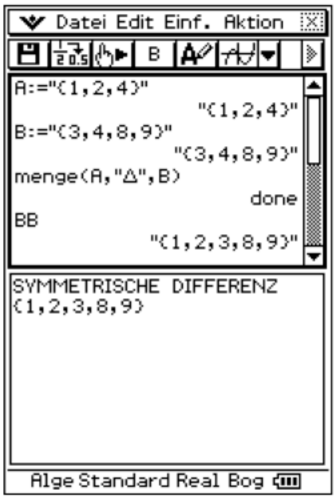
Sometimes we get an error message (we forgot x after |).



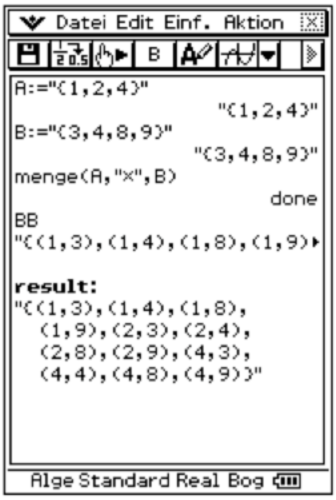
2. The program "menge" (see SA2011-workshop-Paditz.vcp)
After download the program in the library-folder of the ClassPad we can use it e.g. in the eActivity-menu. The function menge has the syntax **menge("A","operation","B")** or **menge("A","relation","B")**. The result is stored in the variable BB and appears in a separate window. At first have a look in the source text (Beta version 0.9).





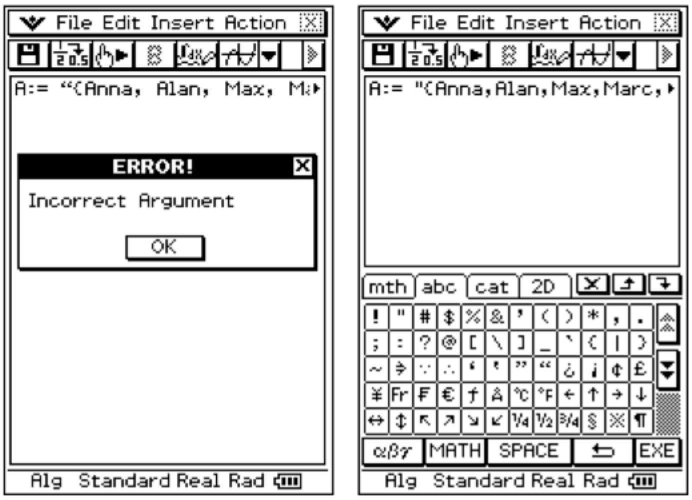


the result window is not optimal

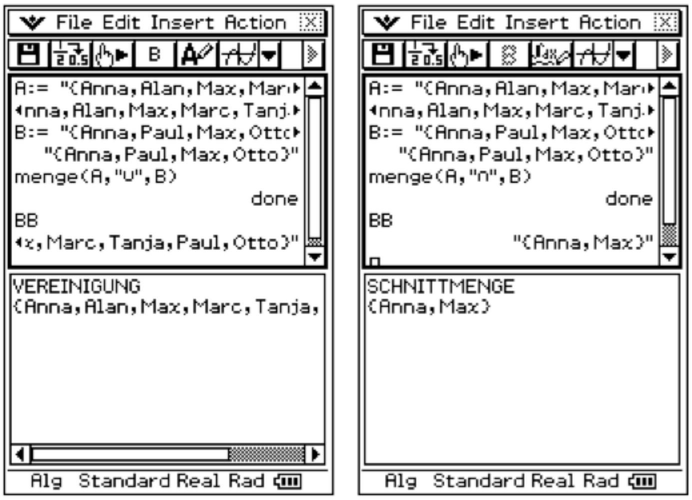


Now we can work with names:

Let be A:= "{Anna, Alan, Max, Marc, Tanja}" and B:= "{Anna, Paul, Max, Otto}"

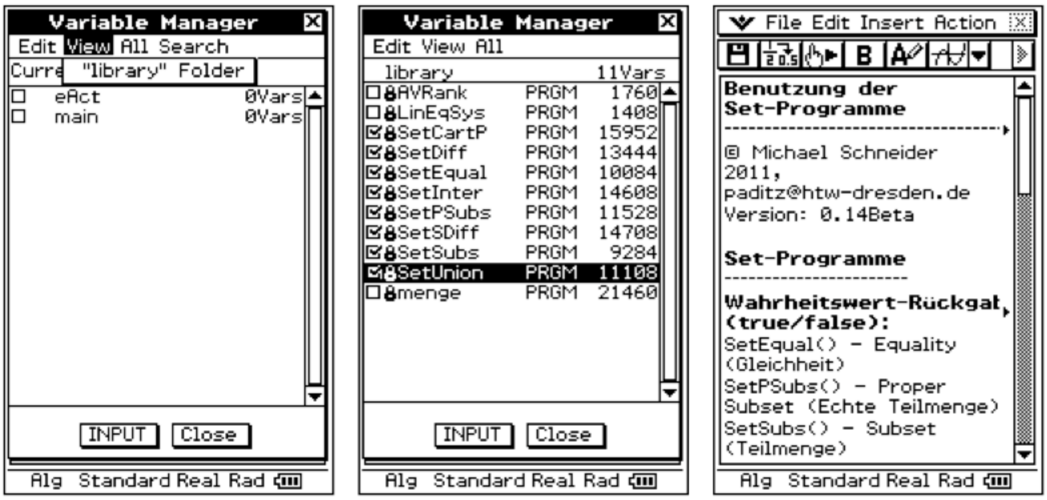


use the correct strings "…"



and so on.

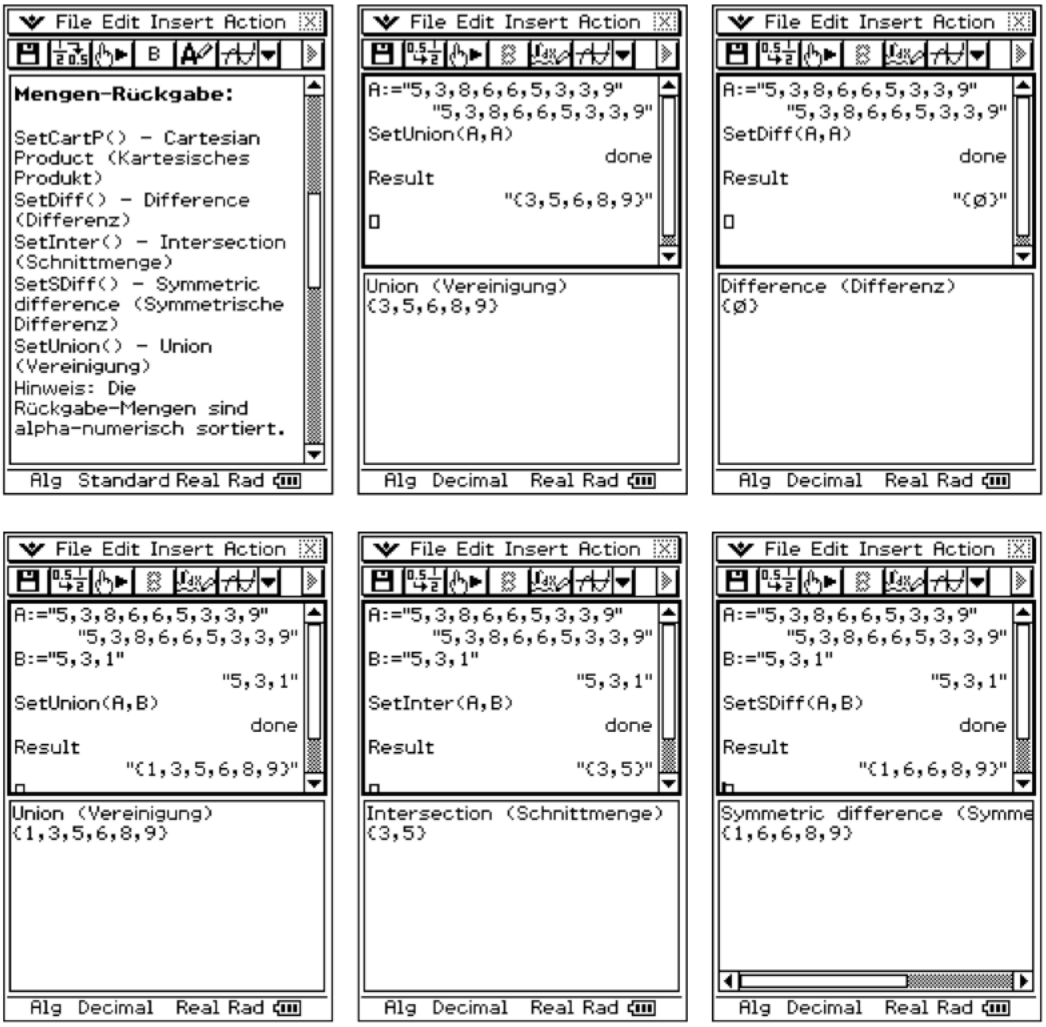
3. The individual programs for one task (see SA2011-workshop-Paditz.vcp)
Another student has created 8 several single programs for one operation or relation.



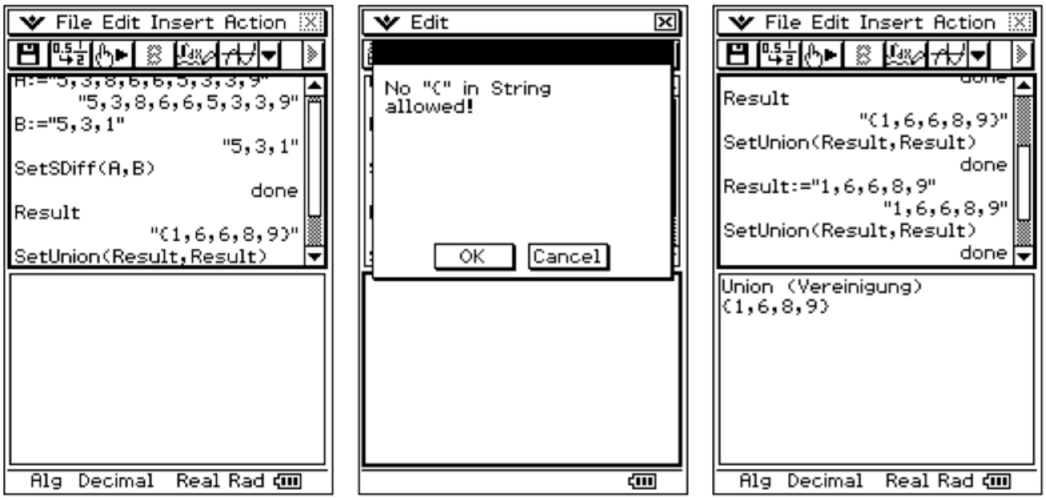
The programs have English names: SetUnion (for $\cup$), SetInter (for $\cap$), SetDiff (for \ ), SetSDiff (for Δ), SetCartP (for $\times$), SetSubs (for $\subseteq$), SetPSubs (for $\subset$), SetEqual(for =).

Let us check these programs. We idn't need any operation or relation symbol. The syntax is as follows: Set…(A,B), where the sets are given in strings " " but without brackets { and } .

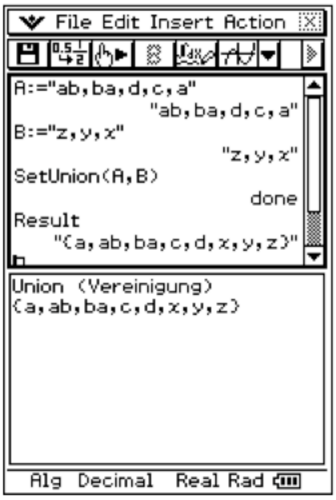
The result is stored in the variable **Result**, the elements are in alpha-numerical order (sorted by their ASCII-codes)



In the last result is a cosmetic error: double 6, better {1,6,8,9}. The student has to improve his program. To simplify the result we use SetUnion(Result,Result) = Result. At first we get an error message.



Here we remark, that the input is without brackets { and }, but the result variable has brackets!



4. Final remarks

During the learning process our students have good ideas to solve several problems in development new programs. The final step is to check the new programs and improve the errors, which appear during the first check of the new created programs. Here we have the Add-In in the final version 1.0 but the other programs in a Beta version 0.9 and 0.14 respectively. Later we will publish some updates.

For the HP 50g calculator you can find set programs here (by Clemens Heuson)
http://www.heuson-software.de/heusoneng.htm

In the internet we have a nice symbolic and numeric calculator:
http://www.tusanga.com/
Here we can do set theory calculations too.

A next step could be to create a program for drawing Venn diagrams, cp.
http://en.wikipedia.org/wiki/Set_%28mathematics%29

Download:
http://www.informatik.htw-dresden.de/~paditz/Set-Theory-SA2011.zip

The Set-Theory-SA2011.zip contains following parts:

- the **Real_Sets.exe** together with **ClassPadDLLgcc.dll** for a Windows-PC
- the CASIO ClassPad Add-in application: **Real Sets.cpa**
- the CASIO ClassPad Manager Virtual ClassPad File: **SA2011-workshop-Paditz.vcp**

CHALLENGES AND POSSIBILITIES IN EMERGENCY EDUCATION: INSIGHTS FOR MATHS TEACHING AND LEARNING AT A JOHANNESBURG REFUGEE SCHOOL.

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Abstract

Zimbabwean refugees and economic migrants at the Central Methodist Church (CMC) Refugee House, in central Johannesburg have successfully established a combined school-St Albert Street Refugee School. This paper comes out of research carried over a period of five months which employed the ethnographic approach and gathered data through classroom non-participant observation, interviewing and document collection. Using the framework of the Direct Instructional Model (DIM), an approach recommended in emergency education, the paper highlights the challenges facing teaching and learning at the Refugee School and explores possible alternatives to some of the challenges. The paper then reflects on how these challenges affect the effective teaching and learning of maths and looks for feasible pathways for maths classroom practices in refugee situations. The possibilities discussed for the refugee maths classrooms are informed by the literature on emergency education and acceptable maths practices.

Introduction: Context of the Study

Zimbabweans have migrated to South Africa mainly because of the country's economic crisis which started slowly in the late 1990s. Political violence and intimidation also led many Zimbabweans to flee their country. It is estimated that about half of Zimbabwe's adult population have either migrated or fled to South Africa.

Some Zimbabwean economic migrants and political refugees have been given refuge and provided with shelter at the Central Methodist Church (CMC), in central Johannesburg. Between 2004 and 2005 a few Zimbabweans trickled to the church to seek accommodation, basic provisions and financial assistance from the generous Bishop Paul Verryn.

In 2007 more than 2 500 refugees were staying at the Refugee House, sleeping on the bare floors, corridors, steps and halls in the five-storey building. At the peak of the Zimbabwe crisis in 2008 the church housed close to 4 500 political refugees and economic migrants.

It is within this context that the need for education was identified. Thus the refugees at the church house started and established a primary and a secondary school (St Albert Street Refugee School) with a combined enrolment of about 500 learners.

This paper comes out of my Master of Education research report and draws from the research report, extracts and instances of maths teaching and learning observed in and across primary and secondary classes. The paper highlights challenges facing teaching and learning in an emergency education context at the Refugee School and provides possible ways forward to some of these challenges. It then reflects on how these challenges affect maths teaching and learning and looks for feasible pathways for maths classroom practices in refugee situations. Investigation of the challenges facing emergency education, and how these generally affect maths teaching and learning is done through the theoretical framework of the United Nations' High Commission of Refugees' recommended teaching and learning approach called the

"Direct Instructional Model" (UNHCR, 2003).

St Albert Street Refugee School and its curriculum

The St Albert Street Refugee School is about a kilometre away from CMC Refugee Centre and was opened in July 2008 by four volunteer refugee teachers, after they found out that there was an increase in the number of children at the centre who were not attending school. When I left the research site on November 15 2009 the school had a total of 21 teachers, 534 learners of which 421 were accompanied students and 113 unaccompanied students. The latter term denotes learners who came from their country of origin without parents or guardians and stay at the CMC Refugee House, most of these are Zimbabwean children. The combined school provides tuition from Grade 0 (R) to Form 6 (equivalent to Grade 12). The school follows the Cambridge Curriculum. The decision to follow this British originating curriculum was necessitated by the fact that the refugee learners could not register for the South African Matric as they did not have identity documents. The local Cambridge examination centre run by the British Council did not require the refugee exam-writing candidates to have identification cards, birth certificates or refugee papers. It is on these grounds that the refugees collectively decided to adopt the Cambridge curriculum.

Research Methodology

In carrying out my research I used the ethnographic methodology. I employed three strategies for gathering data that is non-participant observation, interviewing and document collection, over a period of five months from mid-June up to mid-November 2009. Daily observations of the teaching and learning interactions were done for one week in Grade 1, 7, Form 2 (equivalent of Grade 9) and Form 3 (equivalent of Grade 10) classes at the St Albert Street Refugee School. During these non-participant observations of classrooms teaching and learning interactions, field notes were compiled. I used standardised open-ended interview schedules to solicit for information from key knowledgeable members of the refugee community. I also managed to collect school principal reports, the school timetable, curriculum guides and school pamphlets.

Challenges facing the St Albert Street Refugee School and their implications for the teaching and learning of mathematics.

The CMC education system, like any other refugee education initiatives, was regularly in endless financial crises. Lack of funding is the main reason cited for poor quality refugee education (Sinclair 2001). Inadequate finance implies limited supplies in teaching and learning materials, textbooks and furniture and this inhibits learners from concentrating on learning (Williams 2001). There is need for make shift writing pads for the primary school learners who use two large open church halls which have long fixed benches (pews), used by the church congregants. The school lacks relevant current textbooks, teaching and learning materials, stationery for learners and this hinders teaching and learning. In the observed Grade 7, Form 2 and 3 maths lessons only the teachers had Mathematics textbooks, from which they copied examples that were worked in the classes (Fieldwork observation notes, 2 & 3 October, 2009). The non-availability of textbooks in maths classes at the Refugee School inhibits effective and independent maths learning and closes opportunities for students to experience and encounter authentic maths practices.

The deteriorating physical conditions at the CMC Refugee School were at par or even worse than those reported by Bird (2003, p. 62), in Rwandan and Burundian refugees classes at

Goma and Ngara refugee camps in Tanzania where children were reported to have learnt in "cramped, underequipped, poorly lighted classes with fixed benches". The subdivided Form 2 classroom was cramped with learners, dirty, dusty, too small and poorly aerated. The Grade 4 to 7 primary school classes used the 'Chapel' and the 'Main Sanctuary' halls at the CMC church. The Main Sanctuary accommodated three primary classes whilst the Chapel Hall had two classes. These halls were poorly ventilated and unpartitioned thus teaching in one class would interfere with other classes. The Grade 1 class at the St Albert Street School had 43 learners yet the UNHCR (1995) recommends a class of not more than 35 pupils in crisis situations. Therefore the greatest challenge to teaching and learning as well as effective maths classroom practices at the CMC Refugee School is inadequate space and poor physical learning conditions.

Like any other refugee programmes, the St Albert School faced high staff turnover as teachers complained of poor incentives, the teachers received a R3 000 monthly stipend. Poor staff retention can disrupt learning as was the case in one Bhutanese refugee class where it was reported that there had been five different teachers for one class in a single year (Brown, 2001). At the Refugee school the most affected by an absence of qualified teachers were Maths and Science classes as Maths and Science teachers are most sought after in South Africa. Most Maths and Science teachers lasted for only one term before seeking for higher paying jobs in government schools. Teaching staff attrition severely affected the teaching of maths and science at the refugee school. The progressive and cumulative nature of maths teaching make high teacher turnover in this subject particularly problematic.

In addition the choice of language of instruction is a problematic issue in the refugee curriculum. Sinclair (2001) argues that the issue of language instruction is a human rights issue and advocates for the use of the mother tongue medium of instruction amongst refugee children. On this issue the Refugee school's official and agreed medium of instruction and communication is English. Teachers were not allowed to teach in vernacular languages, in the three maths classes observed in Grade 7, in Form 2 and in Form 3, the different Maths subject teachers always used English. However it emerged during my fieldwork observation that the Grade 1 teacher at times used Shona in a class of 43 learners where 15 children spoke Ndebele and 3 students were Congolese with the remainder being Shona speaking learners. The issue of the language of teaching and learning was problematic at the refugee centre and is a persistent problem even in South African multi-lingual Maths classes (Setati, 2005).

Teaching and Learning approaches and Maths classroom practices at the CMC Refugee School.

Refugee education is also embedded in contested pedagogical issues that have characterised the education terrain in the 20th and 21st centuries. The question has been; should refugee teaching and learning methods be child-centred or teacher centred? (Williams 2001). Kagawa (2005) argues for learner-centred progressive pedagogies, which are closely linked to the democratisation of society and also allows for the learner's critical thinking and exploration. However, democratic teaching methods are difficult to carry out in overcrowded classrooms, are seen as time consuming in a congested curriculum and do contradict with the drive for examination passing (Williams 2001). While many curricula are moving towards participatory methods of teaching and learning, limited space and resources, the nature of the curriculum and assessment methods also determine the pedagogical approach to be employed. Whilst the school principal in the School Council meeting dated 16 November 2008 had recommended for the enhancement of teacher-pupils interaction in classrooms, in practice the opposite seemed to prevail. At the Refugee School, the maths classrooms were characterised by authoritative teacher centred teaching practices that inhibited learner participation thus depriving the refugee learners of maths discussion and engagement.

In the observed Grade 1 class Numeracy lesson the teacher encouraged rote learning and employed closed questions. It was characteristic of the, Grade 7 and Form 2 maths teachers to pose "funnelling questions" thereby inhibiting extended learners' participation in the lessons. In the observed Form 3 maths class lesson, on Matrices, teaching was teacher dominated with students' participation being limited to closed questions which were chorused and chanted by the learners (Fieldwork observation notes, 1 October 2009). Such maths teaching and learning approaches are problematic and a challenge to democratic learning that aims to empower learners and ensure that students are given opportunities to explain and make sense of mathematical ideas and procedures. According to Kilpatrick, Swafford and Findell (2001) traditional methods of instruction in maths classes generally lead students to develop procedural fluency at the expense of conceptual understanding, strategic competence, adaptive reasoning and productive disposition. The observed Form 3 "Matrices" maths lesson mostly involved simple addition rather than relating to why, when and what to do next, thus limiting student access and engagement even with procedural knowledge of matrices. The usage of traditional methods of teaching and learning maths at the refugee school deprived learners of a holistic understanding of mathematics.

Direct Instructional/ Active Teaching/Mastery Learning Model

Whilst recommending more participatory approaches to maths teaching and learning at the refugee school, it is important to note that the Cambridge curriculum is a "collection code" that primarily focuses on disseminating the structure of the discipline to the students and has strong pacing and sequencing (Bernstein, 2000). Democratic learning approaches could be difficult to carry out at the Refugee school which is characterised by overcrowding and lack of space. The best teaching and learning approach for use in refugee contexts and most suited for the Refugee community school and for effective maths classroom practices is what the UNHCR (2003, p. 43) calls the "Direct Instructional Model" (DIM).

The DIM had been piloted and introduced in Afghan Refugee Camp schools in the Peshawar region in Pakistani (UNHCR, 2003). This model is teacher directed but in a positive manner that ensures student engagement and is mostly used to teach in difficult circumstances with limited space and resources (UNHCR, 2003). The teacher under this approach is expected to carefully structure every skill and concept, yet ensuring student engagement through the use of task-orientated approaches (UNHCR, 2003). Such an approach to teaching and learning suits the adopted Cambridge Refugee school curriculum which is knowledge focused and would also ensure that learners are engaged in extended subject practices. The Direct Instructional model augurs well with highly regarded maths practices which value learners' mathematical contributions in classes and at the same time "keeping an eye" on maths conceptual knowledge. The refugee school needs to consider such a teaching and learning possibility both for their school classrooms and maths lessons practices.

Possibilities in Refugee education: Implications for maths teaching and learning.

This part of the paper provides possible pathways for the challenges facing the Refugee schools. The problems facing the refugee school also spilled into maths lessons thereby affecting the effective teaching and learning of maths. To overcome the problem of textbooks at the CMC Refugee School, I borrowed textbooks from the university libraries which the School administrators photocopied for teaching purposes. A well-resourced independent school that used the Cambridge curriculum, St Johns College had expressed interest in helping the refugee school with textbooks. Maths textbooks enable learners to work independently and at different paces. The supply of teaching and learning materials and stationery had improved substantially at the Refugee School, thanks to the numerous donors which were beginning to support the refugees' education initiatives.

The greatest challenge to teaching and learning at the CMC Refugee School is inadequate space and poor physical learning conditions. The "double-shift system", could be one solution to this problem as was the case in Rwandan refugee schools in Tanzania where such a practice was introduced to overcome overcrowding (Bird, 2003, p. 65). However even if this system is introduced there would still be need for more teachers.

One possibility of overcoming the problem of multi-lingual classes has been suggested by Gutierrez, Baquedano-Lopez & Tejeda, (1999). Gutierrez et al, (1999) argue for the utilisation of 'hybrid languages' under which educators use different languages from the learners' community when teaching basic concepts, this increases the possibility of classroom dialogue. In the same vein Setati (2005) calls for "code-switching" in South African multi-lingual maths classes. This ability to draw on learners' languages as a resource for learning and teaching mathematics has positive implication for learners' access to Mathematical knowledge (Setati, 2005). Applying this theory into practice in Refugee maths classes implies that Shona, Ndebele, Zulu and Xhosa could be used to teach maths concepts to learners. I think such a measure would be very helpful to support learner engagement in mathematical discussions at the Refugee School.

To overcome the challenge of teaching staff turnover the Refugee school administrators had resorted to recruiting untrained teachers who had passed 'Advanced level' from the refugee community. Four such volunteer teachers are teaching at the St Albert Street School. The move of recruiting temporary teachers from the emergency population, according to the UNHCR (1995), ensures sustainability of the education system. The recruitment of untrained Maths teachers from the refugee could possibly lead to the normalisation and stabilisation in maths classes. There is however tension between avoiding teacher attrition by employing temporary untrained teachers at the expense of qualified experienced teachers.

A move towards progressive teaching approaches (similar to the Direct Instructional model) was being spearheaded by a group of lecturers from the Wits School of Education which was holding fortnightly workshops on teaching and learning (Slo, personal communication, 8 October 2009). Such staff development initiatives are highly appreciated and valuable in schools and are more than helpful in fragile education contexts. On this note the UNHCR (1995) recommends that there be in-service training for teachers in emergency situations. We hope that the refugee staff development initiatives offers one viable avenue that would change classroom practices and also impact positively on maths teaching and learning.

Conclusion

The emergency education field is new, full of possibilities and one area of study that is under researched in South Africa. Refugee education borrows theories from other fields. Just as refugee education borrows from other fields, classroom and maths teaching and learning practices can also be viewed within the frameworks of emergency education approaches such as the Direct Instructional Model. Such approaches offer possible ways and new horizons for emergency education. Of key importance is the need for methodologies to be employed which enable active democratic participation of learners since it is often the undemocratic circumstances in refugee home countries which have led to the influx of refugees. We hope that the possible alternatives discussed in this paper flow into as well as influence the refugee classrooms and maths teaching and learning practices resulting in quality effective education that leads to mathematically proficient and competent learners.

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Mathematics Connections to Current Events

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Abstract

The "Mathematics Connections to Current Events" provides pedagogy for introducing current events into Mathematics courses thus providing humanistic mathematics examples and discussions in an instructional environment. The outcome is instructional approaches that result in student's gaining not only an understanding of mathematical concepts but how to apply the concepts to current events thus engaging students in and beyond the classroom. This allows students to apply their knowledge to new mathematical situations providing a sense of empowerment and active participation in social environments with mathematics as their basis of putting current events into proper perspective.

Introduction

Mathematics is occurring all around us. It is veiled under the covering of everyday life and current events. Reaching students with mathematics occurs by lifting that veil to uncover mathematics as a humanistic endeavor. Mathematics curriculum must be broadened in its connections to relevant global societal events. This is accomplished using global events that occur in selected subject areas of society. Mathematics is presented as a relevant and useful tool that is used to identify and assist in resolving global problems, concerns, and crises.

Mathematics in proper perspective is mathematics in personification. It is put into perspective as a tool for viewing our daily environment and making decisions that personal lives and society in general. A paradigm shift occurs as mathematics is portrayed in a humanistic view, rather than as depersonalized, asocial, and without much human context or relevance.[1] Mathematics is learned and performed within the context of human purpose and meaningful human enterprise.

Richard Felder, North Carolina State University and Rebecca Brent, EDI - Education Designs Inc. developed a list of pedagogical mistakes that are made in the presentation of Mathematics. Mistake #5 on the list is the failure to establish relevance.[2] Establishing relevance takes into consideration both the student and the mathematics curriculum content. The curriculum is built around the student not the student molded and shaped around the curriculum.

Students learn best when they clearly perceive the relevance of course content to their lives. Connecting theory and practice must be achieved by bringing classroom instruction to bear on real world events. To foster motivation you must begin the course by describing how the content

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[1] Brown, S. I., "Towards Humanistic Mathematics Education", Mathematics Ulterior Motives, http://mumnet.easyquestion.net/sibrown/sib003.htm

[2] Felder, R.M., and R. Brent, "The 10 Worst Teaching Mistakes. Mistakes 5–10," *Chem. Engr. Education*, 42(4) 201 (2008) <*http://www. ncsu.edu/felder-public/Columns/BadIdeas1.pdf*>

relates to important societal events to include whatever you know of the students' experience, interests, and career goals.[3]

To find current events related to mathematics there are several techniques that can be used. These techniques locate the events but do not provide the mathematical context for them in a curricular format. It is up to the professor, teacher, instructor to place the current event into a humanistic context in which mathematics can be drawn out. The steps of doing this include:

- Determine categories of Mathematics of Interest
- Provide search categories for Media outlets
- Develop Unique search criteria

The humanistic categories upon which the current events can be placed are taken from mathematics topics to include:

- Mathematics of Social Choice
- Mathematics of Finance and Economics
- Mathematics of Shape and Form
- Mathematics of Change and Growth

This is not an all inclusive list of categories. Others may be added based on curriculum and context of instruction.

Descriptions of each of these topics are as follows:

Mathematics of Social Choice – mathematics represented in government, voting, sharing, and apportionment

Mathematics of Finance and Economics – mathematics represented in money and commerce

Mathematics of Symmetry – mathematics represented in nature, art, human body, and architecture to include fractals

Mathematics of Change – mathematics of growth and measurement

These categories link current events with the recognitions of mathematics in daily life. Mathematics is involved in governance, finance, nature, art, music, growth, data collection and interpretations. Each of these daily recognitions interacts with our lives and enriches them with awareness of humanity.

To find current events the common and advanced Internet search engines can be used. The search engines containing "Alert" systems which provide notification when a topical subject of your interest is posted to a public database of information and accumulated knowledge are used. The alerts are triggered by "Key Words" based on meta-tags. So, the choice of key words is critical in notification of the alerted current events, as well as, syntactical limiters. The

[3] Dewar J., Loyola Marymount University, CA. –AWM Newsletter Nov- Dec 2009