them approximate particular types of functions into either a power series (in the case of Taylor's theorem) or a trigonometric series (in the case of Fourier theorem), with adequate conditions in both cases. Also in both cases approximations are made for the purpose of simplifying a function. The "transition" from Taylor series approximation to Fourier series approximation can be considered as a (stretched) abstraction, where specific properties have been abstracted and applied to a different context for similar purposes.
Recall that generalization is the process of forming general conclusions from particular instances while abstraction is the isolation of specific attributes of a concept so that they can be considered separately from the other attributes. Yet, abstraction is often coupled with generalization, but the two are by no means synonymous. Any arguments which apply to the abstracted properties apply to other instances where the abstracted properties hold, so (provided that there are other instances) the arguments are more general (Tall, 1988).

In lieu of a proof?

As instructors one should ask: is a mere display of the natural transition a substitute for a thorough proof or justification along the lines of a "scientific proof"? To what degree shall we be satisfied with "observing" a similarity in the results and just accept the extensions in all those listed illustrations as new natural facts? In the case of engineering students whose programs do not require knowledge of thorough justification in the form of formal proof, is observing the smoothness of the extensions between cases of functions of one variable and functions of several variables considered as a good enough justification of the new result as a substitute for what is referred to as "scientific proof". Is it enough to draw students' attention to the fact that Green's theorem is a generalization of the Fundamental Theorem of Calculus, and Stokes theorem is a generalization of Green's Theorem for different dimensions so to speak? No need to mention that some students are able to see the connection between those results with no help from the instructor. Once the generalized result is presented to them, they look back at what could have been its source and figure out in retrospect how it could have been anticipated. In all cases, isn't it an innate instinctive drive to want to connect and look at things as part of one big whole entity or behavior?

General questions:

1. To what extent does drawing the students' attention to the transition in a natural way a substitute for a formal thorough proof that highlights conditions and particularities of the two domains?
2. How much emphasis should be placed on the conditions and the particularities of both domains without menacing mathematical intuition and free "guessing" skills?
3. What are the cognitive skills involved in the generalizations and or abstraction? And are the students ready for that level of cognition at that stage?
4. Should transition be solicited or just lightly pointed out to (in retrospect)?

5. What instructional procedures should be followed that guide students to come to terms with the second type of transition on their own?

Conclusion:

One disadvantage of this behavior would be creating overconfident students. For instance, in probability a student might get tempted to expand the case of binary outcomes of a coin to the case of a die. As for most of us, we must have learned such transitions mostly in retrospect, since the traditional textbooks we used do not bother pointing to those connectives: once we saw the generalized result, we must have figured out its source of inspiration hoping that next time around, we can anticipate it ourselves. One positive outcome is bound to happen, and that is seeing mathematics as a collection of connected facts.

References

1. Beckmann, A., Michelsen, C., & Sriraman, B (2005) (Eds.). Expanding the domain - variables and functions in an interdisciplinary context between mathematics and physics. Proceedings of the 1st International Symposium of Mathematics and its Connections to the Arts and Sciences. The University ofEducation, Schwäbisch Gmünd, Germany, pp.201-214.
2. Freudenthal, H.(1991). Revisiting mathematics education: China lectures. Dordrecht (I believe this needs a country; at the very least, it needs a state if it is USA Kluwer Academic Publishers.
3. Gravemeijer, K. & Treffers, A. (2000). Hans Freudenthal: a mathematician on didactics and curriculum theory. Journal of Curriculum Studies 32(6), 777-796.
4. Jason Silverman (2006) A focus on variables as quantities of variable measure in covariational reasoning vol.2-174 PME-na 2006 Proceedings
5. Kerekes J. (2005), Using the Learners World to Construct and Think in a System of Mathematical Symbols College Teaching Methods & Styles Journal – Second Quarter 2005 Volume 1, Number 2
6. Lesh, Lester & Hjalmarson (2003). A Models and Modelling Perspective on Metacognitive Functioning in Everyday Situations Where Problem Solvers Develop Mathematical Constructs. In Lesh & Doerr (eds.) Beyond Constructivism. Models and Modeling Perspectives on Mathematical Problem Solving, Learning, and Teaching (pp. 383-404). Mahwah: Lawrence Erlbaum Associates

1. Niss, M. (1999). Aspects of the nature and state of research in mathematics education. Educational Studies in Mathematics 40 (pp. 1-24)
2. Streefland, L. (1991). Realistic mathematics education in primary school, Utrecht, The Netherlands Holland: Freudenthal Institute.
3. Tall D. (1988) The Nature of Advanced Mathematical Thinking a discussion paper for PME

The importance of using representations to help primary pupils give meaning to numerical concepts.

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Abstract

The workshop will be a practical one in which participants will have the opportunity to work on a suite of computer programmes which aims to help primary pupils and teacher trainees, in particular, to make sense of numerical concepts through an exploration of representations of these concepts. During the workshop we will also look at some data which illustrates the way in which both primary pupils and trainees have responded to the use of these ideas. There are 4 sets of programmes: early mathematics, addition and subtraction, multiplication and division, fractions.

In total the suite contains about 60 programmes. The writing below gives some background to the programmes and explains the philosophy underpinning their development.

Introduction

During their primary education, pupils are introduced to a number of 'big' ideas – for example addition and subtraction in early primary and multiplication and division later on. In teaching addition and subtraction, there has been a clear use of visual representations such as number squares and number lines, with number lines being viewed as the most appropriate representation for demonstrating the characteristics of these operations, for visualising the calculations and for developing flexible ways of executing the operations. It would appear that as pupils progress within the primary sector and explore other "big ideas", the use of representations is less prominent.

In developing this suite of programmes we have explored the use of visual representations in facilitating the understanding of other big ideas such as multiplication/division and fractions. In doing so, we examine the different aspects of these concepts that we can access through different representations.

The Importance of Representations

Shulman (1986) identified representations as being part of teachers' pedagogical knowledge. He defined these representations as "including analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others" (p.9). Specifically in mathematics, Ball *et al*. (2008) also highlighted representations as being part of the 'specialised content knowledge' of mathematics unique to teaching. This specialised knowledge included selecting representations for particular purposes, recognising what is involved in using a particular representation, and linking representations to underlying ideas and other representations. Teachers need to be able to draw on a variety of representations as there is "no single most powerful forms of representation" (Shulman, 1986, p. 9).

In particular, researchers have highlighted the role that representations play in the explanations of mathematical concepts by teachers (Leinhardt *et al*., 1991; Brophy, 1991; Fennema & Franke,1992).

> "Skilled teachers have a repertoire of such representations available for use when needed to elaborate their instruction in response to student comments or questions or to provide alternative explanations for students who were unable to follow the initial instruction" (Brophy, 1991, p. 352)

Leinhardt *et al*. (1991) also identified the skill and knowledge required by teachers in considering the suitability of particular representations, as “certain representations will take an instructor farther in his or her attempts to explain the to-be-learned material and still remain consistent and useful” (p.108). The effective use of representations therefore require that teachers have ‘deep understanding’ of the topics that they are teaching.

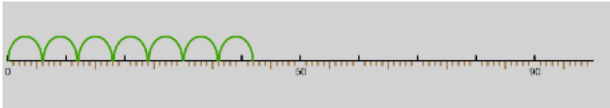
Representations also play an important role in the learning of mathematics by students: “An important educational goal is for students to learn to use multiple forms of representation in communicating with one another.” (Greeno & Hall, 1997, p. 363) More specifically, researchers have outlined the role that representations play in linking the abstract mathematics to the concrete experiences of learners (Bruner & Kenney, 1965; Post & Cramer, 1989; Fennema & Franke, 1992; Duval, 1999).

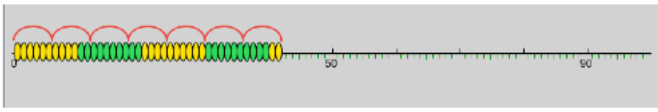
> “Mathematics is composed of a large set of highly related abstractions, and if teachers do not know how to translate those abstractions into a form that enables learners to relate the mathematics to what they already know, they will not learn with understanding.” (Fennema & Franke, 1992, p. 153)

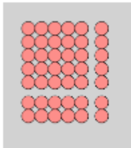
In addition, representations can support the working memory of learners (Paivio, 1969; Perkins & Unger, 1994), for example through ‘offloading’ elements of a given computation to externalized representations (Ainsworth, 2006). Related to the issue of explanation of mathematical concepts highlighted above, representations can be designed in order to constrain interpretation and to highlight particular properties of a mathematical concept (Kaput, 1991; Ainsworth, 1999).
More broadly, multiple representations play an important role in the development of learners’ mathematical understanding: “They can be considered as useful tools for constructing understanding and for communicating information and understanding.” (Greeno & Hall, 1997, p.362) In considering the role of representations within understanding, we make the distinction between *internal* and *external* manifestations of representations (Pape & Tchoshanov, 2001), or ‘mental structures’ and ‘notation systems’ respectively as referred to by Kaput (1991). Understanding of a mathematical concept is based on the internal representations of a concept, which are influenced by the external representations of the concept that are presented to learners (Hiebert & Wearne, 1992). Wood (1999) stated that conceptual understanding rests on a multiple system of ‘signs’ or representations. Lesh *et al*. (1983) used the definition that a student understands a mathematical concept if he or she could ‘translate’ or move between multiple representations. Hiebert & Carpenter (1992) defined mathematical understanding as being a network of internal representations, with more and stronger connections denoting greater understanding.

The medium for exploring representations

As a medium for exploring ideas on representation, we developed a suite of programmes which allowed representations to be explored in a dynamic and interactive way. Through the programmes the characteristics of these representations were explored. For example some of the representations for multiplication/division were:

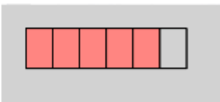


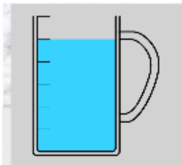


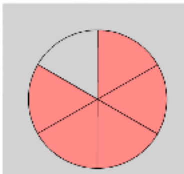


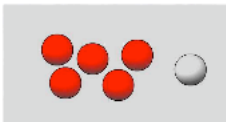
Using these representations we can ask what characteristics of multiplication are emphasised by a particular representation and can consider the possibility of there being a key representation. Further we can then explore how the representations could be used to make sense of the various procedures that need to be understood.

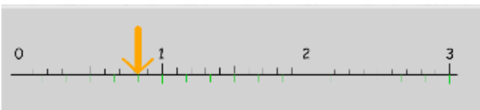
Similarly for exploring fractions the following representations were used:

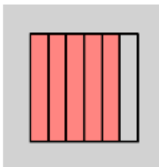












Through exploring these representations and the relationship between them the pupils/trainees are encouraged to build up a language which facilitated a discussion about the nature/characteristics of fractions.

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The programmes were created as a stimulus and scaffold for class discussion. The main guidelines for their design and development were:

- The diagrams and animation should illustrate concepts in a way which is impossible in any other medium;
- There should be a minimum of distraction;
- Pupils should be offered a choice of representation. By using several representations, they are encouraged to realise each way of looking at a problem has its strengths and limitations;
- When a mistake is made the computer should provide a clue (or clues) to the correct answer.

With animated visual representations it is first a question of seeing what is happening, then working out why this is happening, and finally developing robust procedures which will work without the diagrams. In this way the computer programs are envisaged as a bridge between physical manipulatives and abstract figures and symbols on paper. We conjecture that the use of animated visual representations with substantial discussion will allow more both pupils and trainees to see and understand their mathematics more deeply.

Anyone involved in educational research should make explicit their underlying assumptions and beliefs about the nature of knowledge and what conditions facilitate learning. There are fundamental philosophical questions here about the human mind and what exactly is involved in knowing. In a discussion of technology- enhanced learning, Derry (2009) highlights two crucial aspects of learning: its social nature and the central importance of knowledge. Offering a critique of some of the claims of technology- enhanced learning, she says "focus on the learners without recognition of knowledge domains offers no way forward" (p.153). She then quotes Balacheff to show that knowledge domains vary greatly:

> "The characteristics of the milieu for the learning of mathematics, of surgery or of foreign languages are fundamentally different. . . . One may say that the milieu of surgery is part of the 'material world' (here the human body), for foreign languages it includes human beings, for mathematics already a theoretical system." (p.154)

In conclusion Derry says that technology- enhanced learning should "turn attention away from technology to the knowledge domain, from here to questions of pedagogy and from there one step further back to epistemology" (p.154).

Conclusion

The process that pupils are encouraged to follow through using the programmes - of questioning or interrogating a representation- resonates with what Mason (2005) has called "structures of attention". He identifies 5 ways in which we can interrogate the representation: gazing (looking at the whole), discerning details, recognising relationships, perceiving properties, reasoning on the basis of the properties.
Within a representation this idea leads to such questions as:

- *What do you notice about the image/representation*
- *What are the characteristics of the image/representation?*
- *Can you explain how this image/representation shows us …..?*

When working across representations we have questions such as:

- *Why these representations show the same mathematical idea?*
- *What is the same about the different representations?*
- *What is different about the representations?*
- *What are the particular characteristics of the various representations?*
- *What aspects of the structure of fractions are emphasised by the representations?*
- *Can you explain how we move from one representation to another?*
- *What are the most useful characteristics of a particular representation?*

The workshop will allow participants to explore how the programmes can be used to help pupils/trainees to build up a language which facilitates discussion about the nature/characteristics of key numerical concepts within the primary Mathematics curriculum.

References

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education*, *33*, 131-152.

Ainsworth, S. (2006). DeFT: A conceptual framework for considering learning with multiple representations. *Learning and Instruction*, *16*(3), 183-198.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*(5), 389-407.

Brophy, J. (1991). Conclusion. In J. Brophy (Ed.), Advances in research on teaching (Vol. 2, pp. 349-364). Greenwich, CT: JAI Press.

Bruner, J. S., & Kenney, H. J. (1965). Representation and mathematics learning. *Monographs of the Society for Research in Child Development*, *30*(1), 50-59.

Derry J. (2009) Technology-enhanced learning: A question of Knowledge in Davis A and Cigman R (eds) *New Philosophies of Learning* (pp142-155). Chichester, Wiley-Blackwell

Duval, R. (1999). Representation, vision and visualization: Cognitive functions in mathematical thinking. In F. Hitt, & M. Santos (Eds.), *Proceedings of the Twenty-first Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 3-26). Columbus, Ohio: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In rouws, D. A (Ed.), Handbook of research on mathematics teaching and learning (pp. 147-164). New York: Macmillan Publishing Company.

Greeno, J. G., & Hall, R. P. (1997). Practicing representation: Learning with and about representational forms. *The Phi Delta Kappan*, 78(5), 361-367.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). New York: Macmillan.

Hiebert, J., & Wearne, D. (1992). Links between teaching and learning place value with understanding in first grade. *Journal for Research in Mathematics Education*, *23*(2), 98-122.

Kaput, J. J. (1991). Notations and representations as mediators of constructive processes. In E. von Glasersfeld (Ed.), *Radical constructivism in mathematics education* (pp. 53-74). Dordrecht: Kluwer.

Leinhardt, G., Putnam, R. T., Stein, M. K., & Baxter, J. (1991). Where subject knowledge matters. In J. Brophy (Ed.), *Advances in research on teaching: Vol. 2. Teachers' knowledge of subject matter as it relates to their teaching practice* (pp. 87-113). Greenwich, CT: JAI Press.

Lesh, R., Landau, M., & Hamilton, E. (1983). Conceptual models and applied mathematical problem-solving research. In R. Lesh & M. Landau (Eds.), *Acquisition of mathematics concepts and processes* (pp. 263-343). Orlando, Florida: Academic Press.

Mason, J. (2005). 'Micro-structure of attention in the teaching and learning of mathematics'. In *Mathematics Teaching 2005 Conference Report*. Edinburgh Centre for Mathematical Education, pp. 36-41

Paivio, A. (1969). Mental imagery in associative learning and memory. *Psychological Review*, *76*(3), 241-263.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice*, *40*(2), 118-127.

Perkins, D. N., & Unger, C. (1994). A new look in representations for mathematics and science learning. *Instructional Science*, *22*, 1-37.

Post, T. R., & Cramer, K. A. (1989). Knowledge, representation, and quantitative thinking. In M. C. Reynolds (Ed.), *Knowledge base for the beginning teacher* (pp. 221-232). New York: Pergamon.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.

Wood, D. (1999). Editorial: Representing, learning and understanding. *Computers & Education*, *33*, 83-90.

“Shuffle and Shake” and “Pay as you go” - The VG grade 8 experiment

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Abstract

The major aims of this paper is to present a new methodology of classroom practice, analyse the extent to which this new methodology was successful in terms of increasing the effectiveness of the teaching and learning of grade 8 Mathematics at our school; as well as analyzing the extent to which we have been successful in our aim of creating a paradigm shift in the minds of the learners to them seeing Mathematics as a dynamic exciting subject intrinsically associated with the 21st Century and 21st Century skills. The new methodology focuses primarily on the twinned concepts of changing the approach of the teachers and changing the approach of the learners. Educator changes include a lead teacher taking responsibility for each section; the introduction of more technology-based, games-based and kinesthetic-based approaches, and the changing of classes and teachers per section. Learner approaches and attitudes would (hopefully) be changed both by the constant classroom reshuffle, and changes in the way additional support is delivered. The success of implementing this system as well as the success of the system itself is discussed. Finally the transferability of this system is evaluated.

Introduction

The paradigm shift that we were hoping to achieve was essentially a psychological one: Mathematics is viewed by many in society as talent rather than a skill or set of skills. Many of our learners thus suffer from a double curse of a lack of basic Mathematical skills coupled with a societal excuse not to overcome this lack of skill. (The ‘No-one in my family is a “Maths person”’ syndrome.) In addition to, and ironically **despite** this perception, it is also common for both learners and families to blame the teacher if the child is struggling with Maths. Moreover, Mathematics is generally viewed in our society as something that is boring, difficult and only for the minority, a special elite of “Maths people”. More subtle societal prejudices are the concepts that “girls can’t do Maths” and that “black people can’t do Maths”. These are perhaps more insidious as it is politically incorrect to say such a thing in the 21st Century. The result is that the prejudice continues, largely unspoken; and because it is unspoken it cannot be rebutted; and because it cannot be rebutted, it continues.

As a result of combination of all of these factors, many learners struggle with Mathematics; struggling learners, need constant assistance, which causes a drag on the system, thus disadvantaging stronger learners.

Traditionally streaming has been used to address this problem. In South Africa, the Department of Education is against streaming, and for good reason: children who end up in “bottom” classes soon begin to identify with the stigma of being “bottom”. However, the advantages of being able to move at different paces with different classes mean that this approach is, in reality, often still used.

Aware though we are, as a department, of these factors, it is often hard to know how to redress them. We have for many years had, at the school, an **extensive** academic support system including extra lessons from staff, peer tutoring and computer-based support amounting to an average of 15hours of “extra Maths” on offer per week. However, many learners do not utilize the help on offer, and of those who do, many use the support as a continuous crutch, thus not taking responsibility for their own development, and underutilizing class time. Thus potential solutions become rather part of the problem.

In seeking ways to address these issues, in a way that would be beneficial to both staff and learners, bearing in mind the time constraints on staff, we came up with the "Shuffle and Shake"[i] and "Pay as you go" strategies.

In short we wanted to

- create a feeling of Maths being fun, relevant, and accessible to all
- tailor learning to be as individually paced as possible
- ensure that our lesson planning was as dynamic as possible, and in particular
    - ensure the inclusion of extension work to stimulate top learners
    - ensure the inclusion of technology in our teaching
    - ensure the inclusion of kinesthetic and model based learning[ii]
- make learners more accountable for their own successes and failures in Maths

The System (in theory)

"Shuffle and Shake" -instead dividing learners into classes assigned to a specific teacher, teachers take it in turns to introduce each new topic to the whole grade, then (after a short test following the introduction) learners are divided into working groups for that topic. So for each topic, learners will be with different classmates and have different teachers. Introductions are taped and made available on our network for revision. The "lead teacher" for the topic is also responsible for providing extension work and the end of topic test. Interns work as teacher assistants on certain days to assist individuals who are really struggling. At the end of the topic learners write a control test before returning to the lecture group for the next topic.

At the same time a "pay as you go" system has been introduced for extra lessons facilitated by teachers. Payment" is in the form of 10 mark worksheets generated using Microsoft Worksheet generator (or similar online versions) on the specific example that gave trouble. This approach is designed both to reinforce the correct methodology once remediation has happened, and also to encourage a more pro-active use of class time for solving problems rather than relying unnecessarily on extra lessons.

The advantages we anticipated were

- **Appropriate pacing without stigma**- dividing classes per topic rather than per year, provides all the advantages of traditional "streaming" in terms of being able to go at a different paces with different classes, without the disadvantages of the stigma of being in the "bottom class" as the classes are continuously in flux. In addition to this class sizes can be altered per topic to match the "natural break" in the test results. The extension work aspect, the use of computer lab (for Cami-Maths, online worksheets, World Maths day entries etc), as well as the use of teacher assistants would also play into our ability to pace work individually.
- **Label breaking** - by keeping the groups in flux children will be able to let go of labels like "Artsy not Mathsy" by realizing that Maths has many faces and that they can excel in some even if they struggle in others.

- **Learner accountability –** as the learners will largely have different teachers for each topic, they will not be able to attach either success or the lack of success to the teacher and will have to own their own triumphs and difficulties. Having the introductions available digitally means that learners can take the initiative to revise the section from the start in their own time. "Pay –as-you-go" results in less misuse of extra help, and good consolidation for those in genuine need.
- **Fun by association –** the Audio-Visual room (the venue for the introductory lectures) is associated with watching videos , we hoped that the positive association would rub off!
- **Teacher creativity -** as each teacher is responsible for only one in four topics, each has more time to be creative in preparation. Also, because of the experimental nature of the new system , teachers are given the freedom/permission (even the expectation) to experiment and be creative.
- **Teacher accountability** - introducing the topic means being watched by the rest of the department, you are accountable for delivering a dynamic lesson and extension work for your section, and your success or failure is very visible –thus ensuring the followed through on good intentions of making lessons dynamic; including more kinesthetic activities and more technology; and providing extension work.
- **Teacher in-service development -** teachers will learn from and critique each other in an informal and constructive manner, providing support and development

The disadvantages that we anticipated were

- **The personal touch -** not getting to know the learners as well as with a traditional class system.
- **Preparation time-** increased time in preparation in preparing dynamic introductory lessons for the whole grade
- **Class division pressure and admin –**the pressure of having to mark the whole grade overnight in order to be ready for new class divisions.
- **Multiple classes unsettling –**children might struggle to adapt to new teachers, and find the class swopping unsettling

The execution of the system in reality

At the time of going to print, certain aspects of the system have not been implemented because of practical/technical difficulties

- Videoing of lessons – video equipment failures, and videographer (i.e. me) failures have meant that this has not happened. Most of the introductory lessons have involved PowerPoint presentation, however, and these have been made available on the network, along with other resources and links to sites online.
- Because we have wanted to give ourselves flexibility in terms of when it is best to move on to the next section, rather stick to a rigid ("equations will take two weeks") time-table, our ability to utilize the interns has been restricted because of time-table clashes. Also some of the interns were daunted by the prospect of tutoring Maths .
- We have not included as many kinesthetic activites as we would have hoped (although this may be a more natural part of the geometry sections still to come).
- After the second session we realized that it would often be better to split the lead teachers'

lectures with a few days of small classes, to give learners a chance to master the basics before moving on, as there was a tendency for lead teachers to try to fit too much into the given time.

- We also needed to add a "correction and extension" lesson between the test and the returning to lecture group, in order for tests to be returned and corrections and remediation to happen before the next topic was started.
- The logistics of collating end of term marks, learners' portfolios and report comment slips was a stumbling block that we had only half anticipated.

Evaluation of the system after a term and a half

It is impossible to analyze the success of this system in terms of results. We have deliberately used some easy tests working on the principle that success breeds success, and we have reshuffled the order in which we teach the syllabus, so that tests from last year are no longer covering the same range of materials as we have covered. Also there is no way of knowing how strong this group would have been in the conventional system, as they are new to our school in grade 8 and we have multiple feeder schools so no "control" is available. Moreover the motivation behind the change of system was more about changing the way learners approach and think about Maths, than about improving marks. (Although we certainly hope that the long term effect of an improved attitude will be improved results!) Perhaps by the end of the year, we will be able to get a fair sense, by comparing exam results to those of the last few years. For now however, all we have to go on is impressions. To ascertain these, in addition to the organic daily and weekly discussions amongst staff , learners and staff were asked to respond to a questionnaire after the first term and a half. By this time 6 topics had been covered and classes had swopped around 5 times (learners stayed in the same classes for topics 4 and 5 as these were relatively short sections).

(1) Maps; direction and bearing; angles of inclination and declination; Cartesian planes and plotting

(2) Exponents

(3) Introduction to negative integers

(4) Factors LCM HCF

(5) Pattern

(6) Introduction to algebra

The learners' questionnaires were kept deliberately vague, and generalized, not referring directly to the new system, but trying go gauge their feelings in general while allowing enough space for the new system to be included in their responses. The teacher questionnaire was specific.

Learner questions:

In terms of my Maths marks so far this year I feel...
The Main difference between Maths this year and Maths last year is ...
What I really enjoy about Maths is ....
Sometimes I wish...

Teacher questions:

1. What has worked?

2. What hasn't worked?

3. a. Should we continue next term?

    a. If so with what modifications?

4. a. Should we do this system again with next years' grade 8s?

    b. If so with what modifications?

5. Other comments

Analysis

Learners were generally disappointed with their results. This is not surprising as the jump between primary and high school is always a shock. However, what was encouraging was that most learners were still enjoying Maths and /or were strongly motivated to improve their marks despite being disappointed by the mark itself.

18 out of a total of 63 questioned mentioned the 'class changing' system at all, 4 specified that they liked being taught be the different teachers; 7 mentioned without comment that the classes changed; 7 students wished that they could have the same teacher.

23 mentioned the level of complexity the detail and the speed of High School Maths. 8 said that Maths this year was easier.16 commented on Maths being more fun and enjoyable this year. (There was a overlap here of 9 with those commenting on complexity.) (There was no correlation between marks and any of the tendencies/preferences.)

The technological aspect was only mentioned by one learner who commented that she enjoyed Cami-Maths, though I have no doubt that this contributes generally to their sense of Maths being "fun"

The issue of favourite/preferred teacher was only raised by 3 learners. 1 learner commented that their current teacher didn't explain clearly enough (the only blaming the teacher type comment, although arguably the 7 wanting 1 teacher may be transferring blaming the teacher to blaming the system)

The response from the staff was strongly positive with the following pros and cons being identified.

Pros: general positive attitude about Maths, general desire to improve their marks; generally taking responsibility for their successes and failures; the diversity of teaching styles is positive; staff learn from each other, and feel supported and encouraged; staff pick favourite topics or areas of expertise which enhances teaching and satisfaction; development of staff skills facilitated organically; staff get to know the whole grade, and see the whole spectrum of ability levels within the grade

Cons: individual attention hard in lecture venue; time is sometimes lost setting up the data projector; centralized admin is needed and is hard to manage; knowledge of individual learners is reduced.

3/4 of us said we would like to use this system again next year with slight modifications, and the remaining person saying she wasn't sure. 4/4 of us opted to continue with the system this year, with modifications.

From my subjective perspective the system has been enormously beneficial despite its problems. The pros far outweigh the cons to my mind. The number of student who are clearly unsettled by the system is not as dramatic as we anticipated, and the generally positive effect is apparent. It is ideally suited to grade 8 which is a stage at which both the Maths, and the learners themselves change dramatically, so it is an ideal moment for a paradigm shift. Learners are also more open to a new way of doing things because they accept that High School is different from Primary School; and for many a paradigm shift has certainly been achieved.

In addition, I believe that the teaching staff have also undergone a paradigm shift. We have developed wonderful resources, enjoyed each other's lessons, and help each other to constantly improve our teaching styles and methodologies. We have seen both how easy and how powerful the inclusion of technology, games, and kinesthetic activities are, and we ourselves are less likely to label individuals learners as "strugglers".

Adaptability and applicability elsewhere

I have broken this experiment down into 5 components to analyze its adaptability, as this isn't an all or nothing system.**Class reshuffling:** adaptable provided the department is large enough and staff are willing, however I think that grade 8 is the ideal place for such a methodology. I am not convinced that it would be as successful in higher grades at school. At university level this could be effectively implemented in certain tutorial settings. Effective in terms of de-stigmatizing Maths ability and encouraging learners to take responsibility for their own learning. **Team teaching to introduction topics**: adaptable anywhere. Extremely valuable both for its developmental aspect and for encouraging top class creative work. Expectations need to be firmly laid out from the beginning.**Use of games, modeling and kinesthetic exercises**: adaptable anywhere. Definitely had a positive effect on learners perception of Maths.**Technological component**: adaptability depends on resources but powerpoint element requires only one computer and one data projector. Extremely valuable especially for visual learners and attention deficit learners. **"Pay as you go"** Easily adaptable at school level (though easier with the technology available) Adaptable to university. Extremely effective.

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[i] This system is an adaptation of a similar system was used a Westerford a number of years ago.
[ii] Based on Howard Gardner's "Multiple Intelligences" theory.

Left to their own devices: Student-led inquiry into mathematical ideas in kindergarten

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Absract
In this workshop, participants engaged with and reflected on authentic artifacts from student inquiry into measurement (4 years old) and data (5 years old). Participants analyzed and reflected on these authentic examples in order to discuss the children's learning processes that arose and what were the respective roles of the classroom environment and teachers in affording the students' self-directed inquiry. Participants were also invited to reflect on the radical implications of this for curriculum development and mathematics learning in school, even at the pre-primary level, as well as implications for teacher development.

Student-led Inquiry with Young Children

The mathematics education community is still engaged in a lively debate about the nature of mathematics, what it means to do mathematics, and how to best help students to become doers of mathematics in school. Many educators still believe that such a view applies only to older students who have already "mastered the basics" in order to move on to higher-level mathematical activity. The examples we explored in this workshop put the spotlight on how very young children might come to do mathematics and actively, purposefully and appropriately use it as a tool for learning on their own, without direct instruction from a teacher and well before having "mastered the basics."

Very recent studies in educational psychology by Bonawitz, et al. (in press) and Buchsbaum et al. (2011) have called into question the appropriateness of direct instruction for young children and also the role the teacher should have in the learning process. It turns out the very young children are capable of engaging more fully and fruitfully in authentic inquiry and exploration on their own than with explicit guidance from a teacher. Their work suggests that a more appropriate role for the teacher is to be a learner side by side with the children, a more implicit role as a learning partner in a process of inquiry. The Primary Years Program of the International Baccalaureate, if practiced as the documentation of it envisions, provides a potentially ideal programmatic school setting in which to explore these notions in practice because of the emphasis on student-led inquiry.

Indeed, the students in the present examples were able to pursue logical lines of inquiry on their own with little explicit teacher intervention. The children were working in the context of a pre-primary program housed within a larger international school in Lebanon that follows the International Baccalaureate Primary Years Program with some significant features of the Reggio Emilia educational approach. The program, if practiced as the documentation of it envisions, provides a potentially ideal programmatic school setting in which to explore these notions in practice because of the emphasis on student-led inquiry.

The four-year old students' inquiry into measurement began with noticing something placed in the environment by the teacher, namely a measuring tape posted vertically on the wall (the kind often used to keep track of changes in children's height). One student noticed it and began trying to make sense of it. He then called over other students until eventually several students were standing next to the measuring tape, comparing themselves to the lines drawn making the units, and comparing themselves to one another. The teacher stayed at a distance only taking photos and noting down what children were doing and saying. Later the same day there was an opportunity for students to recount what they had done and discovered. The teacher then elicited questions from the students about what they thought they should do next to find out more. Eventually this led to the students exploring the use of non-standard units for measuring length and making some connections about the concept of a measuring unit that we often think of as beyond their capability, such as discovering the idea of iteration of a unit and the relation between the unit size and the measure of the attribute. Student engagement in such an inquiry process constitutes significant learning and can be viewed as an essential foundation for students conceptual development in mathematics (Gravemeijer, 1999).

In the second set of examples, five-year old students engaged in a variety of authentic data collection and data analysis activities in order to pursue their own questions and to make sense of things in their environment. It is often hard for teachers to imagine how they can use data modeling (Lehrer & Schauble, 2002) with young children given all the traditional constraints and the tendency to view younger children as unable to deal with complex reasoning and problem solving. In our program, we have found that working with data is essential and comes very naturally for young children if they are truly engaged in trying to make sense of phenomena and understand how the world works, thus developing a better sense of the nature of mathematics and science and their role in our lives.

Implications for Curriculum and Teacher Development

Allowing children from such an early age to engage in more self-directed learning implies a radical shift in thinking about what mathematics learning should look like in school. The idea of engaging students in inquiry processes or in an inquiry cycle in school is aligned well with reform-oriented approaches to mathematics education as advocated for the past 20 years is not very radical by itself. However, I believe that taking it a step further and thinking about inquiry not as an instructional method, but as a *stance* toward mathematics curriculum (Short, 2009) represents a radical shift. Another aspect of the shift is from thinking of mathematics as a school subject toward thinking of it as a tool for learning about oneself, the human condition, and about the world in general. Such a shift has great implications for the design of school learning environments and materials, as well as for teacher education and ongoing development.

First, with respect to curriculum development, it is important to realize that inquiry is not just a way to make learning more fun; it is the embodiment of doing mathematics and science and the way children naturally learn and make sense of things. Any earnest attempt to engage children with authentic inquiry highlights the obvious tension between thinking of mathematics in school as a body of content knowledge that needs to be acquired by the child vs. mathematics as a tool for systematically understanding how the world works and imagining/inventing ways to improve upon it. This tension raises all the questions about the utility and role of textbooks as resources (and for whom should they be a resource) and what children "should" learn and how that should be determined and what really are "the basics" when it comes to mathematics in the 21st Century.

Second with respect to teacher development, the major challenge is helping teachers to get in touch with their own deep-seated beliefs about what mathematics is and why it is important to worry about whether children learn it and their beliefs about what children are capable of. Children do not know that they cannot direct their own learning and pursue their own ideas until they learn that through the schooling process—an assumption worth questioning. Teachers first need to believe it is possible for themselves and their students to generate legitimate questions that can lead to fruitful investigations and learning without always being told what the questions should be. Professional development programs need to address this explicitly and to engage teachers in authentic problem solving and inquiry processes as learners. Professional development experiences must be contextualized in teachers' practice as much as possible in order to impact on their practice (Smith, 2001) and it is further helpful if teachers actually come to view teaching as an important inquiry in and of itself. In other words, adopting inquiry as a stance toward curriculum implies a need to also adopt an inquiry stance toward one's teaching practice.

References

Bonawitz*, E.B., Shafto*, P., Gweon, H., Goodman, N., Spelke, E., & Schulz, L.E. (*in press*) The double-edged sword of pedagogy: Teaching limits children's spontaneous exploratoration and discovery. *Cognition*

Buchsbaum, D., Gopnik, A., Griffiths, T. L. & Shafto, P (2011). Children's imitation of causal action sequences is influenced by statistical and pedagogical evidence. *Cognition*.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning, 1(2)*, 155-177.

Lehrer, R. & Schauble, L. (2002). *Investigating Real Data in the Classroom: Expanding Children's Understanding of Math and Science*. New York: Teachers College Press

Smith, M.S. (2001). *Practice-based professional development for teachers of mathematics*. Reston, VA: NCTM

Adjusting the Mathematics Curriculum Into the 21st Century

Classroom Examples

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Abstract

In our efforts to improve the mathematics curriculum and adjust the mathematics education into the 21st century, we have developed a technology based course at a teacher education college. Technology enables us to integrate into the school curriculum topics which were until now only taught in higher math education courses. Raising students' awareness to the mutual relationship between math and technology will broaden the students' view of the nature of mathematics and its applications in the real world.
This paper focuses on one of the topics taught in the course- finding roots of various kinds of equations. We begin with computing square and cubic roots using the intuitive 'trial and error' method followed by Heron's method (100 a.d.). Then we generalize both to first and second order numerical methods which enable to solve even equations which have no analytic solving formulas.
We use the GeoGebra software to obtain the graphs of the functions or to check student's answers which were obtained using calculus. The students define the number of the solutions (if any). They get acquainted with the numerical methods, write their own algorithms, translate them into computer programs and get the solution by using Excel.
We hope that adding new and vital subjects, which are ordinarily absent from the regular school programs (in Israel), using technology and rich learning tasks, will make the shift in mathematics education.

Introduction

The last decades are characterized by rapid changes in mathematics teaching all over the world. Technology changes the way students think and learn (Dori, Barnea, Herschkovitz, Barak, Kaberman, and Sason, 2002) and demands adequate changes in the curriculum as well. "In mathematics instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics (NCTM, 1991).
In our efforts to improve the mathematics curriculum, students' math understanding and adjust the mathematics education into the 21st century, we have developed a technology based course**.** This course is taught to mathematics B.Ed and M.Ed students in a teacher training college.
Technology enables us to integrate into the school curriculum topics which were until now only taught in higher math education courses. Raising students' awareness to the mutual relationship between math and technology will broaden the students' view of the nature of mathematics and its applications in the real world. Lecturing at a teacher education college, we believe that after participating in the course, our students will incorporate its topics into schools.

This paper describes one of the topics taught in the course - finding roots of various kinds of equations. This is one of the issues mathematicians dealt with from the dawn of history, in order to solve various kinds of mathematical problems.
Students are not aware that (according to Abel, Galois and Lie) there does not exist and will never be found a closed formula for solving polynomial equations of an order greater than 4, and for other non algebraic equations such as exponential or trigonometric (Arbel, 2009). Working with advanced software (such as Matlab) one can get solutions to those problems. We Raise students' mathematical curiosity as to how the computer functions. In other words, they learn "the story behind the key" of calculators, graphic calculators and behind computer built in library functions.
During the course the students get acquainted with the mathematical ideas and numerical methods absent from the school curriculum (in Israel). At first the students use calculators, then they write an algorithm and translate it to a computer program using Excel. Realizing the "strength" of computers they construct permanent software that is both efficient and fully automatic.

Computing square and cubic roots

1. <u>The intuitive 'trial and error' method</u>

This method is based on finding two sequences of upper and lower bounds which get closer and closer to the root (until the desired accuracy is reached).
For example:
To compute $\sqrt{5}$ we start by intuitively finding two numbers a, b as lower and upper bounds for $\sqrt{5}$. In our example $2<\sqrt{5}<3$ (first approximation). Knowing that $\sqrt{5}$ is greater than 2 and smaller than 3, let us take 2.5 (the average) as our next intuitive approximation. Now we calculate the value of x^2.If x^2 is smaller than 5 the new approximation is the new lower bound a, otherwise it is the upper bound b. We continue until a is close enough to b. The algorithm is shown in figure 1.

```
COMPUTING THE SQUARE ROOT OF S Trial and Error
1. input s (positive number)
2. input  a, b  a< s^0.5 <b
3. while  a = b  (to a desired accuracy)  do
  3.1    x ◄— (a+b)/2
  3.2   if x^2 <s then a ◄— x
         else  b ◄— x
4. print x
5. end
```

Figure 1

The next stage is translating the algorithm to a computer program (using Excel) –see figure 2 below.

2. Heron's (100 a.d) iterative formula for computing the square root of s (a given positive number)

Heron's method is based on creating a sequence of rectangles, all with area S. In each new rectangle both sides are getting closer to each other. As a limit of the sequence we get a square. The sides of this square are the desired square root of S.

trial and error					Heron	
a	b	x	x2	S	a	b
2	3	2.5	6.25	5	1	5
2	2.5	2.25	5.0625		3	1.666666667
2	2.25	2.125	4.515625		2.333333333	2.142857143
2.125	2.25	2.1875	4.78515625		2.238095238	2.234042553
2.1875	2.25	2.21875	4.922851563		2.236068896	2.236067059
2.21875	2.25	2.234375	4.992431641		2.236067977	2.236067977
2.234375	2.25	2.2421875	5.027404785		2.236067977	2.236067977
2.234375	2.2421875	2.23828125	5.009902954			
2.234375	2.23828125	2.236328125	5.001163483			
2.234375	2.236328125	2.235351563	4.996796608			
2.235351563	2.236328125	2.235839844	4.998979807			
2.235839844	2.236328125	2.236083984	5.000071585			
2.235839844	2.236083984	2.235961914	4.999525681			
2.235961914	2.236083984	2.236022949	4.999798629			
2.236022949	2.236083984	2.236053467	4.999935106			
2.236053467	2.236083984	2.236068726	5.000003346			
2.236053467	2.236068726	2.236061096	4.999969226			
2.236061096	2.236068726	2.236064911	4.999986286			
2.236064911	2.236068726	2.236066818	4.999994816			
2.236066818	2.236068726	2.236067772	4.999999081			
2.236067772	2.236068726	2.236068249	5.000001213			
2.236067772	2.236068249	2.23606801	5.000000147			
2.236067772	2.23606801	2.236067891	4.999999614			
2.236067891	2.23606801	2.236067951	4.99999988			
2.236067951	2.23606801	2.236067981	5.000000014			
2.236067951	2.236067981	2.236067966	4.999999947			
2.236067966	2.236067981	2.236067973	4.99999998			
2.236067973	2.236067981	2.236067977	4.999999997			
2.236067977	2.236067981	2.236067979	5.000000005			
2.236067977	2.236067979	2.236067978	5.000000001			
2.236067977	2.236067978	2.236067977	4.999999999			
2.236067977	2.236067978	2.236067978	5			
2.236067977	2.236067978	2.236067977	4.999999999			
2.236067977	2.236067978	2.236067977	5			

COMPUTING THE SQUARE ROOT OF S –

HERON OF ALEXANDRIA

1. input q (desired no. of correct figures)

2. input s (positive no.)

3. a ← 1

4. b ← s

5. E ← abs(a-b)

6. <u>while</u> E>10^ -q <u>do</u>

6.1 a ← (a+b)/2

6.2 b ← s/a

7. <u>else</u> print a, b

8. end.

Figure 2

In our presentation we will show how the students expand Heron's algorithm to a fully automatic one that is stopped when a desired accuracy of q significant digits is obtained. We will also compute the cubic root similarly.
Another way of calculating the digits of the square root of a given number s, is by finding the roots of the equation x^2-s=0.

Solving equations

In order to solve f(x)=0, in other words to find the real roots of the equation, we look at the function y= f(x) and solve $\begin{cases} y= f(x) \\ y=0 \end{cases}$ (points of intersection with the x axis).

To investigate the given continuous function y=f(x) the students are asked to:

- Plot the graph of the given function (using calculus and/or GeoGebra software). The software is used either for checking students' answers which were obtained using calculus, or to obtain the graphs of the functions.
- Decide the number of zeros (if any).

We will now present two methods that enable to solve (numerically) all kinds of equations, even those which have no analytic solving formulas (exponential, trigonometric or polynomial of a degree greater than 4). These methods are a generalization of both methods discussed previously.

1. Bisection method

For each of the zeros of an increasing function

Choose a relevant interval [a,b] where f(a) <0 and f(b) >0 (switch f(a) with f(b) for a decreasing function). The required root lies between a and b (Cauchy's mean value theorem), precisely where the graph intersects the x axis.

Let x_m= (a+b)/2 be the midpoint of the interval. Compute y= f(x_m)
If y<o take x_m as the new a or else take x_m as the new b.

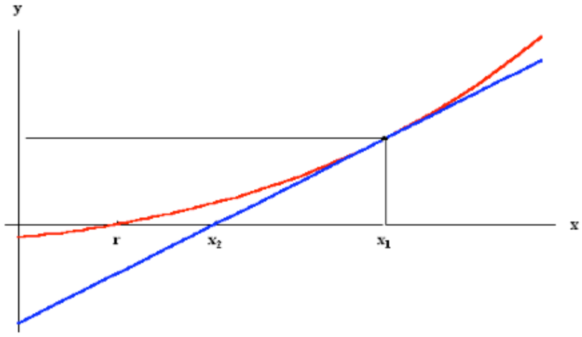
- Continue until the desired accuracy is reached. (Breuer and Zwas, 1993).

The algorithm and the program for calculating the square root:

1. Input s
2. Input a, b, f(a)<0, f(b)> 0
3. x $\longleftarrow$ (a+b)/2
4. f(x) $\leftarrow$ x^2 -s
5. While f(x)≠0 do
   5.1 if f(x)>0 x $\leftarrow$ b
   Else x $\leftarrow$ a
   5.2 x $\longleftarrow$ (a+b)/2
   5.3 f(x) $\longleftarrow$ x^2 -s
6. print x

Another example solving xe^{-x} -0.25=0, will be presented during the presentation.

3. Newton Raphson Method- using the tangent line for a differentiable function



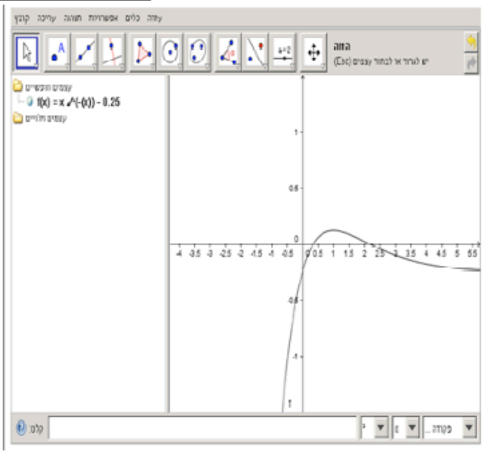
- Choose x_1 to be the first approximation for the root r. $y_1=f(x_1)$.
- Find $f'(x_1)$.
- At (x_1, y_1) calculate the tangent line.
- Find x_2, the point where the tangent line intersects the x axis. x_2 lies closer to r, therefore x_2 is chosen to be the next approximation of r.
- Continue similarly until f(x)=0 (the desired accuracy is obtained).
- For each iteration: Find $y=f'(x_n)$ and compute
  $\boxed{x_{n+1}=x_n-f(x_n)/f'(x_n) \qquad f'(x_n)\neq 0}$ **Newton Raphson** formula (1690).

$\boxed{\begin{array}{l} x_n+1=x_n-f(x_n)/f'(x_n) \\ f'(x_n)\neq 0 \end{array}}$

Newton Raphson Method **$f(x)=xe^{-x}-0.25$**

x	f(x)	f'(x)
0.5	0.05326533	0.3032653
0.324360635	-0.01549068	0.4884801
0.35607263	-0.00059899	0.4510207
0.357400699	-1.01E-06	0.4494932
0.357402956	-2.93E-12	0.4494906
0.357402956	0	0.4494906
0.357402956	0	0.4494906

increasing



x	f(x)	f'(x)
2.5	-0.0447875	-0.1231275
2.136251007	0.002284318	-0.1341876
2.153274331	2.41E-06	-0.133899
2.153292364	2.89E-12	-0.1338987
2.153292364	0	-0.1338987
2.153292364	0	-0.1338987

decreasing

the algorithm

```
1. a ← x1
2 b ← f(a)
3. c ← f'(a)
4.  while f(x)≠0   do
  4.1     print a
  4.2     a ← a-b/c
  4.3     b ← f(a)
  4.4     c ← f'(a)
5. end.
```

One can see:

- When dealing with the same equation $xe^{-x}-0.25=0$, the Newton Raphson Method (of the second order) gives the solution much quicker than the bisection method (of the first order).
  For $x^2-s=0$ Newton Raphson's method yields the same formula (and result) as Heron got without using calculus.

During the course, the students have learned many and varied numerical methods taken from different branches of mathematics. Emphasis is given to the mathematical knowledge and to accompanying justifications.
We believe that technological developments make it possible to incorporate selected chapters of this course in high school or even in the upper grades of the elementary school curriculum, by adapting the topics to students' knowledge.

We hope that these topics will be integrated into the curriculum and our students will be the agents who incorporate it into schools.

References

Arbel, B. (2009). *Mathematicians and great events in the history of mathematics*. Mofet institute, Israel. (in Hebrew).

Breuer, S., & Zwas, G.(1993). *Numerical Mathematics. A Laboratory Approach*, Cambridge University Press.

Dori,Y., J., Barnea, N., Herschkovitz, O., Barak, M., Kaberman, Z. and Sason, I. (2002). Teacher Education toward teaching in technology - enriched environment and developing higher - order thinking skills. *Proceedings of the 4th international conference on teacher education: Teacher education as a social mission*, vol.1, pp. 123. Achva college; Israel.

NCTM (1991). *Professional Standards for Teaching Mathematics*. Reston, VA.

Intervening for Success

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Abstract
Although New Zealand has been named in the top six countries in the world for achieving consistently highly in Mathematics, Reading and Writing according to PISA findings, 2009, there remains concern that a significant number of children are underachieving in Mathematics. Whilst it could be argued about the exact numbers it is patently clear that Maori and Pasfika children are over represented in the bottom 20%. This paper outlines a New Zealand pilot 'Accelerating Learning in Mathematics' (ALiM), trialled in 2010. The purpose was to support children by using a variety of approaches to accelerate children's mathematical learning over a short term period, thereby giving the children an equitable opportunity to achieve at their appropriate level.
Within the context of this paper the use of the word children is used to define primary school age children (5 – 13 years) as opposed to the older secondary students.

Introduction
It was not until the results from the Third International Mathematics and Science Studies were released in 1996 showing New Zealand children (in the studies) were found to be below the International average in mathematics that the government sprang into action by forming a Mathematics and Science Taskforce group, in 1997, to address the problem. Of special concern were the lower results of Maori and Pasifika children who are destined to become a significant component of the future workforce. Also, at that time, it was noted that teachers in New Zealand were experiencing difficulties with implementing problem solving reforms in mathematics teaching.
Although there were some strong mathematics educators in New Zealand they were few in number and disparate. Four of the Auckland College of Education mathematics lecturers and mathematics advisers from each of the regions in New Zealand under the guidance of the Ministry of Education formed the basis of the first co-ordinated numeracy community covering the whole of the country. The New South Wales Department of Education and Training initiative 'Count Me In Too' was seen as a worthwhile project to trial. An experienced team was brought out from Australia to demonstrate the work they had introduced to teachers of junior classes. That particular project provided a well researched base from which to develop the New Zealand Numeracy pilot in 2000, and later the growth of the Numeracy Development Project (NDP). Its aim from 2001 – 2009 was to raise children's achievements in mathematics by improving teachers' professional knowledge, skills and confidence. During those years every teacher was allocated 12 hours professional development time as part of NDP. Facilitators, principals, and teachers were interdependent in effectuating the successful implementation of the Numeracy Project. Schools were encouraged to either work in the project by syndicates or as whole school professional development. Having several teachers involved at once meant the commitment was easier to maintain, through the support they gave each other.
From 2000 to 2010 milestone reports and evaluations annually documented evidence from teachers, facilitators, researchers and policy analysts and those continued to inform any further development of the Numeracy Development Project and beyond 2009. Today the project has moved from phase 1 which was implementing the project into phase 2; sustainability. Whilst funding limits the number of school facilitators

work in, a substantial website means teachers have support at their fingertips (refer www.nzmaths.co.nz for further information).

Why Accelerating Learning in Mathematics (ALiM)?

Although there have been great successes in the Numeracy Project in New Zealand there remains an important issue. One of the aims was to close the gap, (identified through TIMSS and PISA reports) of the lowest performing students (NZ) as compared to the top performers (NZ) and this has not been sufficiently attended to. Maori and Pasifika students, after ten years in the numeracy development project, still remain a significant number of the 'below expectations' group. In an attempt to focus on the issue the government has released funding solely to address such a problem. Along with the funding National Standards were introduced in 2010 to highlight at each school year (Year 1-8) the expected levels of achievement for Mathematics, Reading and Writing. National standards were put in place to help counteract the slowly diminishing tail with urgency. The numeracy development project was about raising children's achievement through improved teacher capability whereas ALiM is about attending to the child.

ALiM

The purpose of the pilot was to see if the children's learning could be accelerated with targeted teaching and learning for a period of ten weeks. The project involved thirty nine schools with funding from the Ministry of Education. Numeracy facilitators were given twenty hours to offer support and guidance per teacher. In the Otago/Southland region four schools were involved in the pilot and were selected by facilitators on the basis that the classroom teachers were good classroom teachers but still had small groups of children who were not achieving to expectations.

The children were selected for this project by principals and classroom teachers in consultation with numeracy facilitators. Factors that were taken into consideration were; regular school attendance, status in relation to the national standards, behavioural issues and a desire to learn. The teachers were selected for their effective classroom teaching practice, and had demonstrated a willingness to be part of the project.

At a national meeting, mathematics education researchers were called to present research ideas about accelerating learners who were not achieving to expectations. Several aspects arose from the discussion. Alton-Lee mentioned the emotional environment, "children will not learn if they do not feel safe and that we need to strengthen valued social outcomes and that an ethic of care needs to be created" (A. Alton-Lee, personal communication, April 26, 2010). Bullying is a huge issue in primary and secondary schools and scared children often think of the next break outside the classroom when they are at their most vulnerable rather than on the tasks at hand. Anthony shared her thoughts that making connections is central to the learning process and learning mathematics is not a linear process as children do not move along the same path (G. Anthony, personal communication, April 26, 2010). Attention was drawn to other researchers outside the group such as Mulligan's work on patterning which can lead to a significant improvement in mathematical outcomes, the importance of building harmonious relationships between school, families and communities which can have reciprocal benefits for all concerned (Ministry of Education, 2008, p.3) and reference to Wright's address at a numeracy conference in Auckland where he stated any teaching/tasks must develop their number knowledge to support non-count-by-ones strategies (R. Wright, personal communication, February 20, 2008).

With those considerations in mind facilitators wrote 16 small resources that could be used as a basis for planning with teachers. An initial two day ministry funded seminar was held where teachers and facilitators, from all over New Zealand, worked together to look at effective pedagogy and select an appropriate intervention which was then tailored to meet the particular needs of the targeted children. Throughout the months teachers and facilitators were mainly guided by ten principles:

- An ethic of care
- Arranging for Learning
- Building on students' thinking
- Worthwhile mathematical tasks
- Making connections
- Assessment for learning
- Mathematical communication
- Mathematical language
- Tools and representations
- Teacher knowledge (Anthony & Walmshaw, 2009)

The principles focus on effective teaching and although intended to be 'nested within a larger network', in general teachers found some practices were more applicable to their children's needs.

The most common mathematical concepts covered were place value in the senior classes and addition/subtraction in junior classes. Most teachers chose to run an extra four or five sessions for the targeted children, over and above the normal classroom mathematics programme. The sessions ranged from an extra 20 – 30 minutes depending on teachers' release time. Teachers were assigned a facilitator who would mentor and support them, and on their return to school began to implement their planned intervention.

Success of ALiM

The New Zealand Council for Educational Research (NZCER) was contracted to supervise the data gathering, analysis and evaluation of the exploratory study. The children were assessed using NumPA (a diagnostic interview), a Progressive Achievement Test (norm-referenced assessment tool) and an attitudinal survey. These assessments were administered prior to the intervention and at the end of the intervention (usually about ten weeks later). The PAT assessment was debated as to the appropriateness of having the children, who usually were at the lowest percentile; sit a test in exam like conditions. PAT involves a significant amount of reading therefore teachers were encouraged to read the questions for the children.

Findings showed that achievement levels increased (NZCER, 2010). Most of the children moved at least one numeracy stage, and surprisingly in the PAT Mathematics test, scores increased by an average of 80% of a year's growth over the ten weeks involvement in the pilot. When comparing ethnic groups results Pasifika children also averaged 80% with Maori children increasing by 40%. An achievement gain for Maori children but the questions remaining unanswered were; why was it that they didn't make the same gains and what were some of the variables that impacted on their lesser achievement?

Children's attitudes to mathematics also showed a positive shift, their confidence and self-belief showed gains.

Reasons identified by teachers and facilitators for these successes were; regular well-structured sessions which made effective use of concrete materials, repetition of material learned at previous sessions, connections between knowledge and strategy, knowledge gaps identified and addressed, small groups where the children were actively involved and could take risks and be free from other children's/ teachers preconceived notions of their ability in mathematics, more complex mathematical language being explored and used by the children to explain their thinking, teachers reflecting on their practice and having an active involvement from their facilitator, communication and support from whanau (family).

ALiM as Seen Through One School's Success

Throughout New Zealand thirty nine schools implemented the project in 2010 but for the purpose of this paper the picture can be seen through the story of one of the four schools in the southern reaches of the country and how they implemented ALiM (further stories can be found on the website www.nzmaths.co.nz).

One School's Story

The small school is situated about one and a half hour's drive north of Dunedin with a population of approximately13,000. It is a decile 4 school (decile 10 relates to a high socio-economic area), with 156 pupils, 25% of the students identifying as Maori and 20% identifying as Pasifika. It is a contributing school which has children from five years old to eleven years old. The school has a high emphasis on family values and has very strong connections with the community. With the downturn in the economic climate in NZ there has been a greater transience of pupils in the last three years.

Participants and Data Gathering

Two groups of students were chosen. Six children who were five years old in one group (Group A) and five children, seven and eight year olds, were in the other group (Group B). The small sized groups were deliberately chosen as the teacher felt they were more likely to take risks and were able to make better interactions with the teacher and other children.

Group A was taken by a teacher aide and happened for 38 of the sessions over the same period. This teacher aide (TA) has a TA certificate and was guided by the teacher. Those sessions were held from 2.30pm to 3.00pm, therefore, the children received extra mathematics lessons as they had their own class session daily from 12.40pm to 1.40pm.

Group B was taken by the teacher responsible for this project and happened on 40 x 30 minutes sessions. This was the highest possible number of sessions that could be taken over the eight week period. The sessions were held from 9.00am to 9.30am with those children also receiving extra mathematics lessons as they had their own class session daily from 12.40pm to 1.40pm.

Initial and final data was collected, as set out by NZCER and teachers administered the Numeracy Diagnostic Interview which they were very familiar with because of their involvement in the Numeracy Development Project. PAT assessment was not administered in this school due to the age of the young children. Records taken were put onto the school's files for future reference and target setting.

Facilitation of the Exploratory Study

The teacher, an experienced teacher of nearly 40 years chosen by the Principal, was coupled with a numeracy facilitator who also had many years in the classroom environment. The teacher was able to offer many ideas and worked well with the facilitator to fine tune the plans for both groups. Assistance to the teacher was also given through visits, email and phone.

As the support resource for 'children after one month at school' was well structured the experienced teacher felt that her teacher aide would be able to follow the format with encouragement from the teacher. The focus for that work was a) to increase students' every day and mathematical language through exploratory activities and b) to develop an understanding of numbers to 5. The lesson structure was based around six short 5 minute periods and always included: counting, matching, physical, sorting and copying activities as well as action rhymes (see nzmaths.co.nz\ALiM for further information).

Four resources for Group B children were used: after 6 months, after 9 months, after 18 months 'moving children to simple additive strategies' which the group had just been introduced to when they finished in the study.

The teacher and the teacher aide started each lesson with tasks the children had already experienced success with and made opportunities to connect their knowledge with other 'harder' aspects. An example was to use their knowledge of counting backwards 9-0 to connect with bigger numbers. If the children started at 27 the teacher would point to the 7 then the 6 with 26 and 5 with 25 and so on. A variety of activities were required to be adapted and made for the children's use. This proved to be a time consuming effort but worthwhile as the end products were able to be used time and time again

In the 7th week both the teacher and the teacher aide were videoed working with the children and gave interviews. The DVD has proved an inspiration to many other teachers who have used and adapted many of the ideas the teacher, teacher aide and facilitator worked on.

Results

The results (see Table 1) show where the children were at the start of the programme on the New Zealand Framework, and where they were after 8 weeks. All stages are counting stages from Stage 0 where children cannot one to one count, to Stage 4 where they can 'hold a set in their head' and count on. For example, to solve 8 + 5 they would say 8 and count on 9, 10, 11, 12, 13 to solve the problem. In New Zealand children are expected to reach Stage 2 or 3 by the end of Year 1 and Stage 4 by the end of Year 2.

Table 1

Number of students	**Year**	**Initial stage Add/Sub**	**Final stage Add/Sub**	**Time in programme**	**Predominant Focus**
Group A 6	**0/1**	3 students - stage 0 3 students - stage 1	6 students - stage 2	5x30mins weekly 7.5 weeks	Language development & Number Knowledge to 5
Group B 5	**2/3**	1 student - stage1 4 students - stage 2	5 students - stage 4	5x30mins weekly 8 weeks	Number Knowledge & Strategy

The group A children in 7.5 weeks were able to reach a stage on the New Zealand Framework which is an expectation for a child who has been at school for a year. The group B children who were well below expectation for their year group managed to achieve at a comparative stage with their peers. In 8 weeks they made the equivalent progress of 12 months at school

Reasons for success

The teacher identified several factors for success:

- Extremely supportive principal and facilitator
- Parental involvement with close communication between the teacher and the parents
- The interest, involvement and support of other teachers in the school. They observed some lessons and were able to apply some of the concepts to their own classroom practice
- The environment where the sessions took place was set up specifically for mathematics and showed mathematics was valued
- The group size was ideal, children were able to take risks without other children making value judgements. Problem solving and working together as a group increased the children's confidence in their own ability. The children's image of themselves as being able to solve problems became much more positive

- Connections between known material and new material were made explicit. The children became much more confident to share their thinking using more complex mathematical language.
- Having a variety of materials was essential so that children had a great deal of repetition of the same concept but in a variety of ways so concepts were able to be generalized

Conclusion

The teacher gave several reasons why ALiM was successful in the small school but failed to mention the pivotal factor and that was, of course, herself. She is a highly effective, experienced classroom teacher and was prepared to take on a challenge to improve outcomes for children in mathematics.

The exploratory study was so successful in her school that the principal has decided that all five year olds will be given the opportunity of 'after one month at school' sessions and the ALiM resources will be made available for other teachers on their staff. The TA has decided to enter teacher training through an online course with practicuum in the school where she is familiar. The teacher has since incorporated many of the ideas into her teaching approaches. The school's story is only one small part of the wider exploratory study but it is a story that is repeated in many other schools.

Following the successful report from NZCER the Ministry of Education through the government has secured funding for a pilot to be held in 2011 involving 160 schools working in ALiM. A new component is the appointment of specialist mathematics teachers in each region who are appointed 0.5 time in their schools and will work with the 'well below expectations' children who have had few or no opportunities for mathematics interventions. New approaches need to be found for those children as opposed to serving the same 'diet' year after year with little success.

As ALiM showed that children's learning could be accelerated after interventions it is with interest that mathematics educators await the next evaluations. From the work in 2010 one factor stands out: When principals, teachers, parents, and outside facilitators work together, alongside the children, achievement improves.

Acknowledgements

Further information on ALiM exploratory study, 2010 and ALiM pilot, 2011 can be found on www.nzmaths.co.nz The views expressed in this paper do not necessarily reflect that of University of Otago College of Education or the Ministry of Education. Thanks are conveyed to the school involved, the staff and the children whose story has been told in this paper.

References

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. Educational Practices Series-19. Belgium:IAE & Switzerland:IBE.

Ministry of Education (2008). *Home-school partnership: Numeracy*. Learning Media: Wellington.

Neill, A., Fisher, J., & Dingle, R. (2010). *Exploring Mathematics Interventions: Exploratory evaluation of the Accelerating Learning in Mathematics pilot study*. NZCER:Wellington.

What can be Learned from Comparing Performance of Mathematical Knowledge for Teaching Items found in Norway and in the U.S.?

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Abstract
This paper reports from a Norwegian research project, where a U.S. developed model for teachers' mathematical knowledge for teaching (MKT) was studied. Part of this project included the adaption of MKT measures developed in the U.S. to gauge teachers' MKT. We present results from a pilot study where 149 Norwegian teachers were tested, and where 10 teachers were interviewed in 5 focus group interviews. We discuss how these measures can be used as a tool in relation to professional development of teachers in Norway.

Introduction

Teachers play an important role in determining the quality of graduates, and there is widespread agreement that teachers' understanding of content matter is important for their teaching (Askew, 2008). Still, exactly what knowledge teachers need to have in order to teach is continually discussed (e.g. Hill, Schilling, & Ball, 2004; Rowland & Ruthven, 2011). Researchers at the University of Michigan in the U.S. have contributed to this discussion by developing a framework referred to as "mathematical knowledge for teaching" (MKT. Ball, Thames, & Phelps, 2008). From studies of mathematics classrooms they have identified specific tasks that are involved in mathematics teaching and the mathematical demands behind those tasks (ibid.). Based on this, they have developed measures of teachers' MKT. Their studies have shown that a high MKT score among teachers can be positively associated with increased learning by their students and with higher quality of instruction (Hill et al., 2008; Hill, Rowan, & Ball, 2005).

Knowledge about the topics that teachers struggle with is useful when preparing professional development programs (Hill, 2010). Some research has already been done within this area in the U.S., but there is a need for more research concerning in-service teachers' MKT in other countries. Investigations of how the MKT measures can be used in professional development of teachers in other countries will be an important contribution to this field of research. This paper aims to contribute to an investigation of how the MKT measures can be used in professional development of teachers in Norway. When attempting at using these measures in connection with professional development of teachers, it is interesting to learn more about the connection between teachers' MKT score and their teaching experience. It is also interesting to learn more about the connection between the amount of professional development that teachers have had and their MKT ability. Our research question for this paper is:

> What is the connection between teachers' MKT, their experience and professional development?

This question is virtually impossible to approach on a general level, but we investigate these connections in a sample of Norwegian teachers and discuss possible implications of these findings.

Theoretical Background
After having developed the MKT framework, Ball and her colleagues (2008) have developed items that can be used to measure teachers' MKT. These measures include forms of teaching-specific knowledge (Hill, 2010). One such aspect of MKT is related to purely mathematical knowledge which is specific to the work of teaching or "specialized content knowledge" (Ball, Thames, & Phelps, 2008). Hill (2010) recommends that specialized and pedagogical content knowledge (Shulman, 1986) have particular focus in professional development.

The MKT framework, is a further development of Shulman's (1986) model of teacher knowledge. The MKT model (see Figure 1), which is still in development, consists of a number of knowledge domains describing two of Shulman's initial categories in more detail.

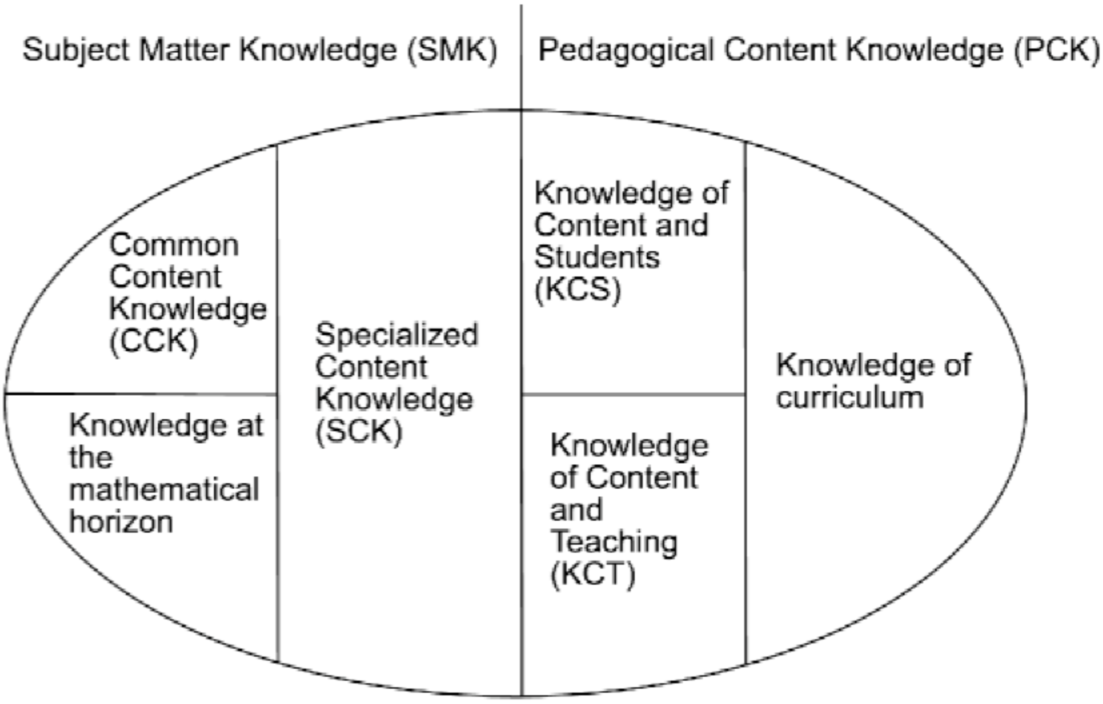

Figure 1: Domains of MKT (see Ball et al., 2008, p. 403 for a definition and discussion of the domains)

The assessment of practicing teachers' knowledge is not a widely accepted practice (e.g. Hill, Sleep, Lewis, & Ball, 2007), at least not in Norway (Lysne, 2006). The goal of Hill and her colleagues (2007) is to move the debate concerning assessment of teachers ". . . from one of argument and opinion to one of professional responsibility and evidence" (ibid., p. 112). To make advances in developing instruments to study teachers' knowledge, a set of agreed-upon, reliable and valid methods for assessing teachers' MKT is required (Hill, et al., 2007). This is in line with Shulman's (1986) initial aim, which was to develop tests where those educated to teach would get high scores.

Hill, Sleep, Lewis and Ball (2007) argue that assessing teachers' knowledge:

> ". . . can be done in ways that honor and define the work of teaching, ratify teachers' expertise, and help to ensure that every child has a qualified teacher. Doing so requires carefully constructed instruments that take seriously the work of teaching and that can be used at scale" (ibid., p. 150).

Hill and colleagues (ibid.) see further development of the MKT measures as one attempt to attain this goal. These measures can, to a certain extent, help in-service educators to identify teachers' lack of knowledge (e.g. Hill, 2010) and thus identify opportunities for teachers to learn (Hiebert & Grouws, 2007). Research so far has indicated that the MKT instrument can be relevant for use in professional development in the U.S., but little has been done to investigate its use in other countries. Norwegian teacher education has until recently certified teachers to teach any subject in grades 1-10, and this differs from the situation in many countries. It is therefore interesting to investigate if the U.S. developed measures can give the same useful information when used in a Norwegian context and if the measures can be a relevant tool for in service educators planning professional development.

Methods

After having decided to use the MKT measures in Norway, our first step was to translate and adapt measures for use in a Norwegian context. The 2004 elementary form A (MSP_A04) from the LMT project[1] was translated and adapted (Mosvold, Fauskanger, Jakobsen, & Melhus, 2009). The process of translating and adapting items was conducted based on recommendations from Delaney and colleagues (2008). When the entire set of items was translated, a pilot study was conducted. The pilot study included a quantitative as well as a qualitative part. Mathematics teachers at our partner schools were invited to participate (grade 1 to 10), and 142 teachers from 17 schools participated in the initial phase. In a second phase two new partner schools were added, and the number of participating teachers was extended to 149. In the quantitative part of the study, all participating teachers completed the test individually. All tests were conducted at the teachers' respective schools, and the testing situation was organized in order to be as similar as possible. Among the participating schools, teachers at five schools were selected for participation in semi-structured focus group interviews (FGIs). These interviews were held directly after the teachers had completed the test, and ten teachers participated in the interviews altogether.

The final form used consisted of two parts. Part 1 included the translated and adapted MKT items[2], a total of 61 items (30 item stems). Of the 61 items, 26 items were from the content domain number concept and operations (NCOP), 19 from geometry (GEOM), and 16 from the domain patterns functions and algebra (PFA). In Figure 2, one of the released items is shown in order to illustrate the nature of the items.[3] This item asks teachers to respond to a mathematical task situated in a teaching context. In part 2 of the form, teachers were asked about factual information concerning their gender, their teaching experience, their mathematical background, and their participation in professional development courses.

The MKT items are meant to relate to the underlying MKT construct and can be viewed as one possible operationalization of the construct. An item response theory (IRT) model can serve as a link to the observed latent world (Edwards, 2009). A basic idea in IRT is that an observed item response is a function of person properties and item properties (ibid.). To estimate teachers' MKT score and item characteristics, we have, in the same manner as initially done in the U.S., used a two parameter IRT model.

[1] See http://sitemaker.umich.edu/lmt

[2] For sake of simplicity, we refer to items from the LMT project as "MKT items" in this paper.

[3] The items used in the test are not released and not available for publication.

2. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

Figure 2: Example from the set of released items (Ball & Hill, 2008).

We have used the program BILOG-MG (Zimowski, Muraki, Islevy, & Bock, 2003) for the estimation of teachers' MKT score and testing of IRT models. For the calculation of correlations, we have used PASW Statistic 18 (formerly known as SPSS statistics).

Results

In our analyses of the data, we looked for correlations between the teachers' MKT score and answers in Part 2 of the form. First, we did not find any significant correlation between teachers' MKT and their experience. When taking a closer look at the number of years they had worked as teachers, however, we found out that our data sample consisted of a rather experienced group of teachers with 80 percent of the teachers having more than six years of work experience. Only 1.4 percent from this convenience sample of teachers was in their first year of teaching.

Second, we studied correlations between teachers' MKT and the grades in which they where teaching. Here we found that there was a significant correlation between the level in which the teachers had teaching experience and their MKT score. Teachers with experience from grades 5-7 or 8-10 (or both) had significant higher MKT score than those with experience only from grades 1-4 (p-value < 0.0005), but the correlation factor was low (Pearson correlation 0.462). If we looked at 1-7 teachers as one group and compared to teachers with experience in grades 8-10, the latest group had significant higher MKT (1.005) and with higher correlation factor (Pearson correlation 0.522, p-value < 0.005).

Third, we studied the correlation between teachers' MKT score and the number of days they had participated in professional development in the years they had worked as teachers. This variable only informed about the total number of days with professional development, and did not say anything about when this professional development took place or what kind of professional development this was. First we considered teachers that had participated in professional development as one big group and compared their MKT score with teachers that had never participated in any professional development program. We did find that this group had a significantly higher MKT score (0.349 higher) compared to teachers without any professional development, but the correlation was weak (Pearson correlation 0.194, p-value < 0.05). Second we grouped the respondents into 6 subgroups: Those who had a) 1-5 days of professional development; b) 6-10 days; c) 11-15 days, d) 16-20 days; e) 21-25 days and e) more than 25 days of professional development. For all of these subgroups we found that the correlation between teachers MKT score and professional development was close to what was found for the big group. However, the subgroups containing teachers with more professional development (e.g. group d), e) and f)) had on average a higher MKT score (0.541 for group d), e) and f)).

Concluding Discussion

In previous analyses, we found that the Norwegian adapted item characteristics are strongly correlated to what is reported in the U.S. (Jakobsen, Fauskanger, Mosvold, & Bjuland, 2011). For item difficulty, the correlation was strong (0.804, p-value < 0.0005). We have also found strong correlation between teachers' MKT in the three content areas (ibid.). Building upon these results we have now analyzed the correlation between the teachers' experience and their MKT score.

We studied a convenience sample of relatively experienced teachers. Despite the limitations of this present study, it has given some indications of issues that should be further investigated. A larger and representative sample of teachers should then be studied.

The results from our study indicate that teachers with experience in teaching higher grade levels have stronger MKT. It should be emphasized, however, that the results from our study cannot be used to argue that experience from higher or varied grade levels produces higher MKT score, only that there is a correlation.

In our study we did not gather information about what kind of professional development courses the teachers had taken or when. The weak correlation between professional development and MKT can thus be interpreted as an area that needs to be investigated further. From our data, we cannot say if there was a change in teachers' MKT after taking part in professional development. Future studies should be conducted in order to learn more about what kind of professional development courses produce stronger MKT, and more generally to investigate the connection between professional development and the development of MKT.

References

Askew, M. (2008). Mathematical discipline knowledge requirements for prospective primary teachers, and the structure and teaching approaches of programs designed to develop that knowledge. In P. Sullivan & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (Vol. 1, pp. 13-35). Rotterdam, The Netherlands: Sense Publishers.

Ball, D. L., & Hill, H. C. (2008). Mathematical Knowledge for Teaching (MKT) measures. Mathematics released items 2008. Retrieved from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Delaney, S., Ball, D., Hill, H., Schilling, S., & Zopf, D. (2008). "Mathematical knowledge for teaching": adapting U.S. measures for use in Ireland. *Journal of Mathematics Teacher Education, 11*(3), 171-197.

Edwards, M. C. (2009). An introduction to Item Response Theory using the need for cognition scale. *Social and Personality Psychology Compass, 3*(4), 507-529.

Hiebert, J., & Grouws, D. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 371-404). NCTM: Information Age Publishing.

Hill, H. C. (2010). The nature and predictors of elementary teachers' mathematical knowledge for teaching. *Journal for Research in Mathematics Education, 41*(5), 513-545.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., et al. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26*(4), 430-511.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing measures of teachers' mathematical knowledge for teaching. *The Elementary School Journal, 105*(1), 11-30.

Hill, H. C., Sleep, L., Lewis, J. M., & Ball, D. L. (2007). Assessing teachers' mathematical knowledge. What knowledge matters and what evidence counts? In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 111-156). Charlotte, NC: Information Age Publishing.

Jakobsen, A., Fauskanger, J., Mosvold, R., & Bjuland, R. (2011). *Comparison of item performance in a Norwegian study using U.S. developed mathematical knowledge for teaching measures*. Paper presented at the Seventh Congress of the European Society for Research in Mathematics Education. Retrieved from http://www.cerme7.univ.rzeszow.pl/WG/11/CERME7_WG11_Jakobsen.pdf

Lysne, A. (2006). Assessment theory and practice of students' outcomes in Nordic countries. *Scandinavian Journal of Educational Research, 50*(3), 327-359.

Rowland, T., & Ruthven, K. (Eds.). (2011). *Mathematical knowledge in teaching*. London: Springer.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Zimowski, M. F., Muraki, E., Islevy, R. J., & Bock, R. D. (2003). *BILOG-MG 3 for Windows: Multiple-group IRT analysis and test maintenance for binary items [Computer software]*. Lincolnwood, IL: Scientific Software International, Inc.

A Comprehensive Model for Examining Pre-Service Teachers' Knowledge of Technology Tools for Mathematical Learning: *The T-MATH Framework*

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Abstract
This paper proposes a comprehensive model for understanding the multiple dimensions of knowledge employed by pre-service elementary teachers' when they choose technology for teaching mathematics (Johnston & Moyer-Packenham, in press). The *T-MATH Framework (Teachers' Mathematics and Technology Holistic Framework)* integrates several frameworks, including TPACK (Mishra & Koehler, 2006), MKT (Ball, Thames, & Phelps, 2008), and technology evaluation criteria (Battey, Kafai, & Franke, 2005). This model, which can be used to examine the manner in which pre-service elementary teachers rank and evaluate technology tools for mathematical learning, suggests that there are multiple dimensions to understanding teachers' knowledge of technology for teaching mathematics. The paper reports recommendations for mathematics teacher educators and researchers.

Introduction

This paper posits an integrated model of teachers' *technology knowledge for teaching mathematics*. The model is based on the integration of the relevant literature on technology for teaching mathematics including: TPACK (Technological Pedagogical Content Knowledge) as it applies to the teaching and learning of mathematics (Mishra & Koehler, 2006; Niess, Suharwoto, Lee, & Sadri, 2006); the TPACK framework proposed by Mishra and Koehler (2007); the Domains of Mathematical Knowledge for Teaching (MKT) framework proposed by Ball, Thames, and Phelps (2008); and, the mathematics and technology evaluation criteria proposed by Battey, Kafai, and Franke (2005). By integrating these frameworks and evaluation criteria, this model can be used to investigate pre-service teachers' knowledge as they develop in their evaluation and use of technological tools for mathematics teaching.

Frameworks, Models and Constructs Used to Build the Teachers' Mathematics and Technology Holistic Framework (T-MATH Framework)

The literature includes several theoretical and graphical frameworks which are used to explain the relationships among technology knowledge, pedagogical knowledge, and mathematical content knowledge within the context of mathematics education. Elements in all of these frameworks are valuable in understanding the multi-dimensional nature of teachers' technology knowledge for teaching mathematics. For that reason, we have chosen to integrate these various models in a comprehensive model that is specific to the use of technology for mathematics teaching, which we call the *Teachers' Mathematics and Technology Holistic (T-MATH) Framework*.

The first framework used to develop the T-MATH Framework proposed in this paper is *technology, content, and pedagogical content knowledge,* or simply TPACK. TPACK for mathematics is defined as "the intersection of the knowledge of mathematics with the knowledge of technology and with the knowledge of teaching and learning" (Niess et al., 2006, p. 3750). Mishra and Koehler (2007) represent this intersection of knowledge as a Venn diagram, where each of three circles contains pedagogical knowledge, technological knowledge, and content knowledge, with each circle intersecting the others. At first glance, one might assume that these three components are distinct entities, but Mishra and Koehler remind researchers that "TP[A]CK is different from knowledge of all three concepts individually" (2007, p. 8). The representation of concepts using the technology, based on pedagogical strategies for teaching the content, and an understanding of how to use the technology to develop those concepts in children demonstrates the complexity of the integrated nature of TPACK.

Early research on TPACK referred to "content" in general; Niess (2008) suggested identifying how TPACK can be expressed within mathematics education. Specifically, she identified various components of TPACK within mathematics education, namely the importance of teachers' knowledge of students, curriculum, and instructional strategies for teaching and learning mathematics with technology. Niess notes that TPACK requires teachers to consider "what the teacher knows and believes about the nature of mathematics, what is important for students to learn, and how technology supports learning mathematics" (p. 2-3). To support this view, Niess (2005) designed a course where pre-service teachers identified technology tools for mathematical learning as well corresponding mathematics and technology standards which could be supported by the technology tools. The results of this study suggest that the pre-service teachers meaningfully engaged in reflection while considering appropriate technology tools for mathematical learning. Although the TPACK model is an integrated model for pedagogical knowledge, technological knowledge, and content knowledge, the T-MATH Framework proposed in this paper goes beyond the TPACK model to make the mathematics in the TPACK model more explicit by aligning types of mathematical knowledge and fidelity with the elements in the TPACK model.

A second framework used to develop the T-MATH Framework proposed in this paper is the Domains of Mathematical Knowledge for Teaching (MKT) graphical framework described by Ball et al. (2008). In describing this framework they note that, based on their empirical results, "content knowledge for teaching is multidimensional" (p. 403). Within the context of their work, three of their six domains are important for the T-MATH Framework that we propose. These three domains are Common Content Knowledge (CCK; "the mathematical knowledge and skill used in settings other than teaching," p. 399), Specialized Content Knowledge (SCK; "the mathematical knowledge and skill unique to teaching," p. 400), and Knowledge of Content and Teaching (KCT; "combines knowing about teaching and knowing about mathematics," p. 401). These three domains of MKT are important for the T-MATH Framework because they provide much greater explication of the type of mathematical knowledge than that which is described in the TPACK framework (where mathematical knowledge is not described at this level of specificity). What the MKT domains bring to the framework is that there are different types of mathematical knowledge in interaction with the different elements of technological, pedagogical, and content knowledge proposed by Niess.

The third model used to develop the T-MATH Framework proposed in this paper is centered around the criteria proposed by Battey et al. (2005). In their study of pre-service elementary teachers, they identified four main criteria used by the teachers for evaluating mathematics software for use with students: software features, mathematics features, learning features, and motivation features. Further studies which used these four criteria among pre-service elementary teachers noted similar results. These studies found that pre-service teachers emphasized Software Features most often over all other criteria. This finding is important because it suggests that pre-service teachers should consider technology use in mathematics teaching situations as a *mathematical instrument*, not simply as a stand-alone tool. It further demonstrates the challenge that this type of integrative thinking poses for pre-service elementary teachers. In the T-MATH Framework, these criteria highlight a teachers' focus, and that focus reveals the complexity of a teachers' thinking on the use of technology in mathematics teaching. For example, focusing on a motivation feature indicates less complexity because the teacher is considering pedagogy only, while focusing on a mathematics feature indicates more complexity because the teacher is simultaneously considering pedagogy, technology and the mathematical content.

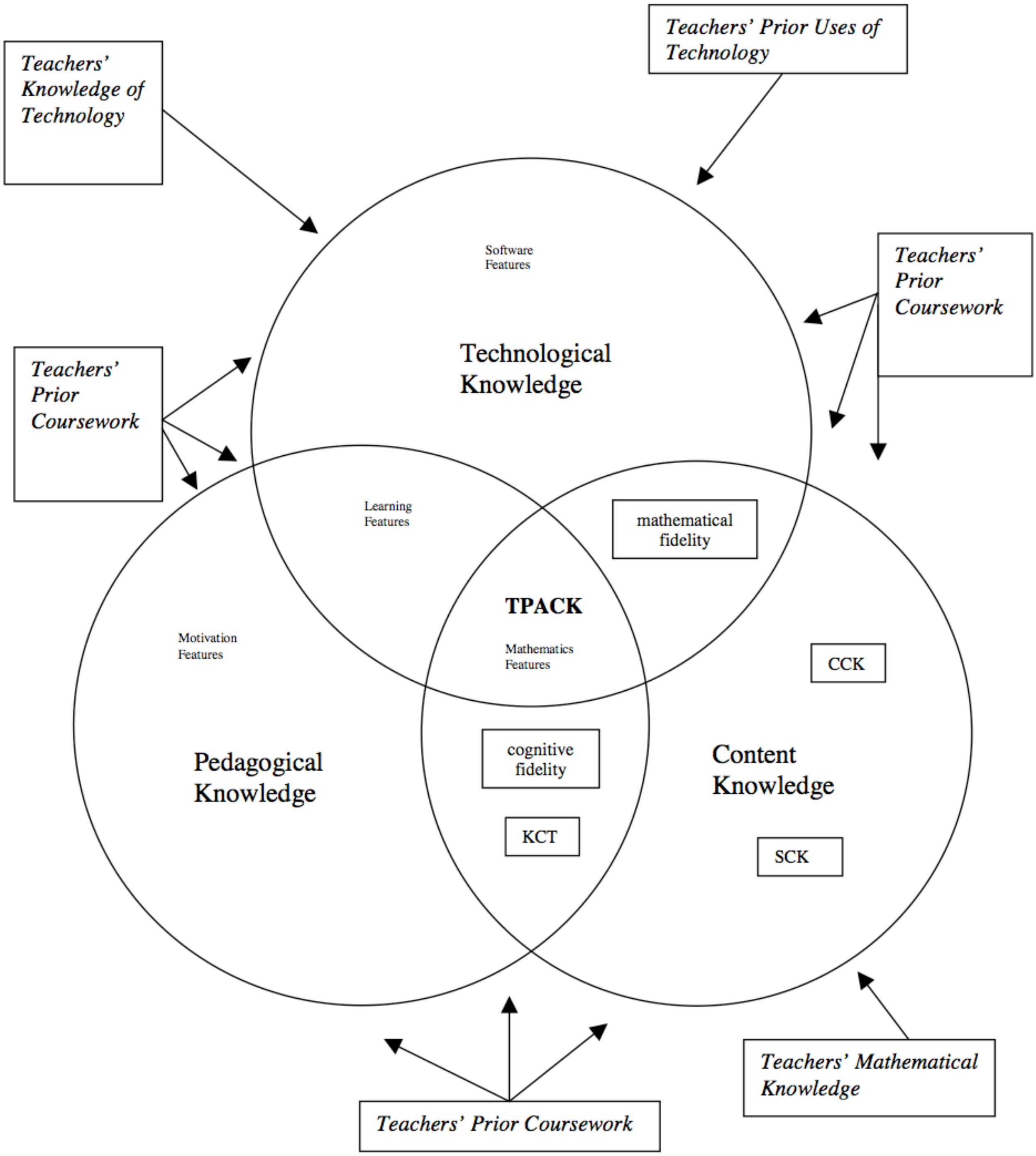
An additional construct used to develop the T-MATH Framework proposed in this paper is *fidelity*. If a tool has high *mathematical fidelity*, "the characteristics of a technology-generated external representation must be faithful to the underlying mathematical properties of that object" (Zbiek, Heid, Blume, & Dick, 2007, p. 1174). Thus, the technology should model procedures and structures of the mathematical system, and be mathematically accurate. A tool is considered to have high *cognitive fidelity* "if the external representations afforded by a cognitive tool are meant to provide a glimpse into the mental representations of the learner, then the cognitive fidelity of the tool reflects the faithfulness of the match between the two" (Zbiek et al., 2007, p. 1176). Thus, a tool which matches the thought processes and procedures of the user has high cognitive fidelity.

The Teachers' Mathematics & Technology Holistic Framework (T-MATH Framework)

The frameworks and constructs discussed in the previous section demonstrate the *integrated* nature of knowledge and the *complexity* of knowledge specifically as it is needed by teachers who want to use technology to teach mathematics effectively. Because each framework and construct informs different aspects of teachers learning to use technology for teaching mathematics, we propose a comprehensive model that takes into account elements of each framework in an integrated way which we call the *Technology Knowledge for Teaching Mathematics (T-MATH) Framework*.

This model in Figure 1 is a specific extension of TPACK, forming a model of teachers' knowledge of mathematical TPACK. The proposed model begins with the Mishra and Koehler TPACK framework (2007), which includes Technological Knowledge, Pedagogical Knowledge, and Content Knowledge in three circles in a Venn diagram. Next we map onto this framework Ball et al.'s (2008) three domains of knowledge including: Common Content Knowledge (Content Knowledge circle), Specialized Content Knowledge (Content Knowledge circle), and Knowledge of Content and Teaching (intersection of the Pedagogical and Content Knowledge circles). Finally we consider the constructs of mathematical fidelity (intersection of the Technological and Content Knowledge circles) and cognitive fidelity (intersection of the Pedagogical and Content Knowledge circles).

Figure 1. Teachers' Mathematics and Technology Holistic (T-MATH) Framework



Interpreting a Teacher's Location in the T-MATH Framework

In the T-MATH Framework, we propose that when prior researchers (Battey et al., 2005; Johnston, 2008; 2009) reported that pre-service elementary teachers focused on various features of teaching mathematics with technology (e.g., software, mathematics, learning, motivation), their focus on these features reflects important information about the dimensions of their technology knowledge for teaching mathematics. For example, when pre-service teachers focus their selection of technology tools for mathematics teaching on Software Features (which shows their Technological Knowledge) or Motivation Features (which shows their Pedagogical Knowledge), this is a "one dimensional" focus. That is, they are interested solely in an aspect of the technology tool that is not explicitly linked to students' learning or the learning of mathematics. For example, identifying features such as "has clear directions" (Software Feature) or "is fun for students to use" (Motivation Feature) could apply to many different technologies or learning situations and does not consider how the features are related to teaching and learning mathematics concepts. On Figure 1, we have positioned Software Features and Motivation features in the Technological Knowledge and Pedagogical Knowledge circles, respectively. Pre-service teachers who focus on these features are exhibiting a singular focus and no intersection with other knowledge areas in the model, thus reflecting a less integrated knowledge.

Within the T-MATH Framework, we propose that when pre-service teachers focus their selection of technology tools for mathematics teaching on Learning Features (which shows their knowledge of how the technology is related to student learning), this is a "two dimensional" or integrated focus. That is, they are connecting features of the technology to students' learning with the technology. For example, identifying a feature such as "applicable to what we are learning in the classroom" connects the technology with the pedagogy of the classroom, considering the implications of both the technology and the pedagogy. On Figure 1, we have positioned Learning Features in the intersection of the two circles for Technological Knowledge and Pedagogical Knowledge. We propose that identifying a Learning Feature requires teachers to make a connection between technology and pedagogy, and thus reflects a more integrated type of knowledge with respect to learning and the use of technology. For example, when pre-service teachers consider the use of embedded buttons on an applet which allow the selection of "easy" or "difficult" mathematics problem items, this allows for learning differentiation afforded by the technology.

Finally, in the T-MATH Framework, we propose that when pre-service teachers focus their selection of technology tools for mathematics teaching on Mathematics Features, this is highly complex, representing a "multi-dimensional" focus and requiring highly specialized and integrated knowledge. On Figure 1, we have positioned Mathematics Features at the intersection of the three circles. We propose that identifying a Mathematics Feature requires teachers to make multiple connections among technology, pedagogy, mathematics (including CCK, KCT, and SCK), and mathematical and cognitive fidelity. Let us further examine the complexity of this placement. The circle of Content Knowledge itself, specific to our model, includes Common Content Knowledge and Specialized Content Knowledge (i.e., mathematical knowledge and skill; Ball et al., 2008). At the intersection of Technological and Content Knowledge is mathematical fidelity. The intersection of the Content Knowledge circle and the Pedagogical Knowledge circle integrates knowledge, and is reflective of Knowledge of Content and Teaching (i.e., combines knowing about teaching and knowing about mathematics; Ball et al., 2008) and cognitive fidelity. The intersection of the Content, Pedagogy, and Technology circles are at the highest levels of complexity because they integrate each of the types of knowledge discussed here, thereby forming the total package.

The positioning of Mathematics Features in the three-circle intersection indicates the complexity of this focus for teachers. We propose that when pre-service teachers focus on Mathematics Features, they must have a deep understanding of technology, pedagogy, *and* mathematics. Specifically, Mathematics Features can focus on three primary areas, namely: *Provides multiple representations of mathematical concepts; Links conceptual understanding with procedural knowledge*; and *Connects multiple mathematical concepts*. In addition, their mathematical knowledge can be the type of knowledge that is used in settings other than teaching (CCK), the mathematical knowledge unique to teaching (SCK), or the knowledge that combines teaching and knowing about mathematics (KCT) (Ball et al., 2008). The identification of a teacher located in the proposed framework and focused on the mathematics features would indicate much more complexity in that teacher's technology knowledge for teaching mathematics.

Conclusion

In this paper we have proposed the *Technology Knowledge for Teaching Mathematics (T-MATH) Framework*. This framework is a specific extension of TPACK that integrates MKT, fidelity (i.e., cognitive and mathematical), and criteria for evaluating technology (i.e., motivation, learning, software, and mathematics features). This framework demonstrates the complex, multi-dimensional nature of a teacher using technology to teach mathematics. It can be used to examine teachers' mathematics lessons or to observe mathematics instruction that integrates technology to determine the complexity of a teacher's knowledge in planning and teaching mathematics with technology. It can also be a framework for teaching pre-service teachers to design technology experiences for their K-12 students that integrate technology in mathematics teaching. Our hope is that this framework illuminates that complexity, and by making these complex elements explicit, helps to focus on the critical elements necessary for consideration and integration when teaching mathematics with technology.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.

Battey, D., Kafai, Y., & Franke, M. (2005). Evaluation of mathematical inquiry in commercial rational number software. In C. Vrasidas & G. Glass (Eds.), *Preparing teachers to teach with technology* (pp. 241-256). Greenwich, CT: Information Age Publishing.

Johnston, C. J. (2009). Pre-service elementary teachers planning for mathematics instruction: The role and evaluation of technology tools and their influence on lesson design. Unpublished doctoral dissertation, George Mason University.

Johnston, C. J. (2008). Pre-service teachers' criteria for evaluating technology tools for mathematics learning. Unpublished doctoral pilot study, George Mason University.

Johnston, C. J. and Moyer-Packenham, P. S. (In press). A model for examining the criteria used by pre-service elementary teachers in their evaluation of technology for mathematics teaching. In R. Ronau, C. Rakes, and M. Niess (Eds.), *Educational technology, teacher knowledge, and classroom impact: A research handbook on frameworks and approaches*. Hershey, PA: IGI Global.

Mishra, P., & Koehler, M. (2007). Technological pedagogical content knowledge (TPCK): Confronting the wicked problems of teaching with technology. In C. Crawford et al. (Eds.), *Proceedings of Society for Information Technology and Teacher Education International Conference 2007* (pp. 2214-2226). Chesapeake, VA: Association for the Advancement of Computing in Education.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*, 1017-1054.

Niess, M. L. (2008, June). Developing Teachers' Technological Pedagogical Content Knowledge (TPCK) with Spreadsheets. Paper presented at the annual meeting of the International Society for Technology in Education, National Educational Computing Conference, San Antonio, TX.

Niess, M. L., Suharwoto, G., Lee, K., & Sadri, P. (2006, April). Guiding inservice mathematics teachers in developing TPCK. Paper presented at the annual meeting of the American Education Research Association, San Francisco, CA.

Zbiek, R., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: A perspective of constructs. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1169-1206). Reston, VA: National Council of Teachers of Mathematics.

Using Large-Scale Datasets to Teach Abstract Statistical Concepts: Sampling Distribution

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Abstract

With the advancement in computer technology, more and more statistics instructors now rely on simulations, web-based statistical applets and other artificially-generated datasets to teach abstract statistical concepts. While this approach may be useful, this paper shows how instructors can use large-scale datasets to make these concepts more real for students thereby facilitating their understanding of the concepts. The paper uses the Education Longitudinal Study of 2002 (ELS: 2002), a large-scale real dataset to demonstrate the concept of sampling distribution and standard errors. The paper focuses on the distribution of sample means and sample standard deviations.

Introduction

Statistics is used in almost our everyday lives and it has applications in a wide variety of settings, for example in weather reports, in business, in crime reports, in finance, and in education. This wide application of statistics makes it essential for students' to have a good understanding of statistical concepts and become statistically literate. However, students often do not grasp certain statistical concepts due to the abstract nature of the subject matter. According to the Guidelines for Assessment and Instruction in Statistics Education (GAISE), an introductory statistics course should: (1) promote statistical literacy and statistical thinking; (2) use real data; (3) promote conceptual understanding; (4) foster active learning; (5) use technology for developing conceptual understanding and analyzing data; (6) use assessments to improve and evaluate student learning. This paper will focus on the second guideline, the use of real data in teaching statistics.

One of the ways to enhance the understanding of statistical concepts, one needs to run real experiments that generate reliable data (Akram, Siddiqui, & Yasmeen, 2004). In most cases, the main limitation for running real experiments is the lack of reliable real data. Even if in cases where real data may be available, the challenge then becomes how to effectively integrate those data in a statistics curriculum. It is difficult to run real experiments during the teaching period in the university. Because of the challenges in using real datasets in teaching statistics, one option that is commonly used by statistics instructors is the use of artificial data in conducting statistical experiments. Statisticians developed simple and very economical experiments, which can be performed in the class by the students (Akram, Siddiqui, & Yasmeen, 2004). A number of studies have been conducted over the years that demonstrate improved students' learning through the use of statistical experiments and simulations. For example, Larwin & Larwin (2010) conducted an experimental study in which they examined the effect of teaching undergraduate business statistics students using random distribution and bootstrapping simulations. Their results indicate that students in the experimental group-where random distribution and bootstrapping simulations were used to reinforce learning demonstrated significantly greater gains in learning as indicated by both gain scores on the Assessment of Statistical Inference and Reasoning Ability and final course grade point averages, relative to students in the control group.

In another study, Raffle & Brooks (2005) reported results of the effectiveness of a Monte Carlo simulation method in teaching a graduate-level statistics course. In their paper, they reported how computer software (MC4G, Brooks 2004) can be used to teach the concepts of violations of assumptions, inflated Type I error rates, and robustness in an introductory graduate course statistics course.

While simulations and simulated data are valuable tools in teaching statistical concepts, the Guidelines for Assessment and Instruction in Statistics Education (GAISE) calls for the use of real datasets to teach these concepts. In line with this guideline, there are several studies that report results for using real-life situations or examples in teaching statistical concepts. In one such study, Leech (2008) used the game of poker and small group activities to teach basic statistical concepts. He noted that this approach helped to reinforce the understanding of basic statistical concepts and helped students who had high anxiety and it made learning these concepts more interesting and fun. In another study, Connor (2003) illustrated statistical concepts using students' bodies and the physical space in the classroom. The concepts covered included: central tendency, variability, correlation, and regression are among those illustrated. In this study, these exercises encouraged both active learning and the visual-spatial representation of data and quantitative relations. Connor (2003) reported that students evaluated the exercises as both interesting and useful. Makar & Rubin (2009) presented a framework for students to think about informal statistical inferential reasoning covering three key principles of informal inference--generalizations "beyond the data," probabilistic language, and data as evidence. In their study, they used primary school classroom episodes and excerpts of interviews with the teachers to illustrate the framework and reiterate the importance of embedding statistical learning within the context of statistical inquiry.

In their paper, Schumm, Webb, Castelo, Akagi, Jensen, Ditto, Spencer Carver, & Brown (2002) use of historical events as *examples* for *teaching* college level *statistics* courses. They focused on *examples* of the space shuttle Challenger, Pearl Harbor (Hawaii), and the RMS Titanic. Finds *real life examples* can bridge a link to short term experiential learning and provide a means for long term understanding of *statistics*. Stork (2003) describes a pedagogy project that keeps students interest. He used student survey to gather data on students' university experience and demographics. The class used the Statistical Package for the Social Sciences (SPSS) data set for demonstration of a range of statistical techniques. The student survey instrument and data set provided examples and problems for each of the major topics of the course. He concluded that student comments and performance demonstrated the project's positive results.

In most cases when talking about sampling distribution, most students think this only applies to the distribution of sample means. The main reason for this is that most textbooks explain this concept using the distribution of sample means. Yet, in actual fact sampling distribution applies to any sample statistic that we calculate to estimate population parameters. It is important for students to understand that when talking about sampling distribution, this applies to standard deviation, variance, correlation coefficient, and many other statistics that we can calculate. What is also important for students to understand is that the sampling distribution is a model of a distribution of scores, just like the population distribution, except that the scores are not raw scores but statistics. The resulting distribution of statistics is called the sampling distribution of that statistic. For example, if standard deviation is the statistic of interest, the distribution is known as the sampling distribution of standard deviation. In this paper, sampling distribution is illustrated for sample means and standard deviations. While this may not be new, what's unique is that a large-scale real dataset is used do illustrate these concepts.

Method

Data used in this paper came from the Education Longitudinal Study of 2002 (ELS: 2002). This survey was designed to monitor the transition of a national sample of young people as they progress from tenth grade through high school and on to postsecondary education and/or the world of work (National Center for Education Statistics 2008). During the 2002 base year, students were measured on achievement, and information was also obtained about their attitudes and experiences. These same students were surveyed and tested again, two years later in 2004 to measure their achievement gains in mathematics, as well as changes in their status, such as transfer to another high school, early completion of high school, or leaving high school before graduation, and also in 2006. This cohort will be interviewed again in 2012 to measure their transition into the job market (National Center for Education Statistics 2008). For the purpose of this paper, only data collected during the base year (2002) was be used.

Procedure

One of the several variables contained in ELS: 2002 is a measure of mathematics achievement of the cohort of 2002 10th graders using a standardized mathematics score (MathScore). This variable was selected for use in this paper. The entire ELS: 2002 dataset contains 10, 094 cases, and this represents the population (N=10,094) for the purpose of this paper. Initially, the population distribution of MathScore was obtained, and this is displayed using a histogram (μ= 51.40, σ = 10.094) in Figure 1. Next, using an SPSS syntax, 100 random samples each of size 100 were drawn from the population and an SPSS dataset with n=100 was created with mean and standard deviation for each sample produced, resulting in 100 means and 100 standard deviations. This was repeated for 500 samples, and 1000 samples, and in each case, the size of each sample drawn was 100 generating means and standard deviations.

Results

The population distribution is shown in Figure 1 while the sampling distribution of the means and standard deviations for the samples on n=100, n=500, and n=1000 are presented in Figures 2 to 4. Population parameters, expected values and standard errors are presented in Tables 1 below. The mean of the sample means in these distributions is the expected value of the sampling distribution of the mean, and the mean of the sampling distribution of the standard deviation is the expected value of the sampling distribution of the standard deviation. Both of these parameters are estimators of the corresponding parameters. When the expected value of a statistic equals a population parameter, the statistic is called an unbiased estimator of that parameter. Students should note that the standard deviation of the sampling distribution of the mean is called the *standard error of the mean*. Similarly, the standard deviation of the sampling distribution of the standard deviation is called the *standard error of the standard deviation*. Students will have the opportunity to learn that sample statistics are most likely to be different from the population parameters due to random sampling errors. However, as sample size increases the sample statistics approach the population parameters, thereby reducing the discrepancy between the statistic and the parameter. This discrepancy is known as the sampling error, and students will be able to see that large samples yield small sampling error. In addition, the histograms show that sampling distribution of the mean will approach a normal distribution as sample size increases.

Figure 1. Population Distribution

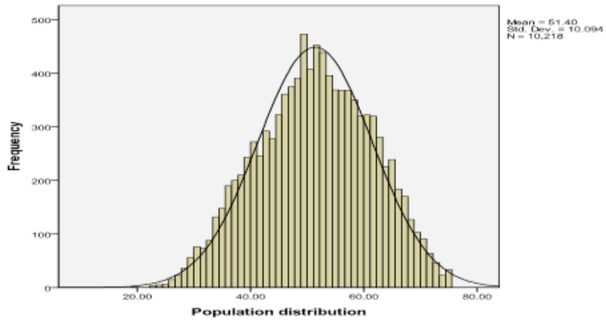


Figure 2a. Distribution of Sample Means (n=100)

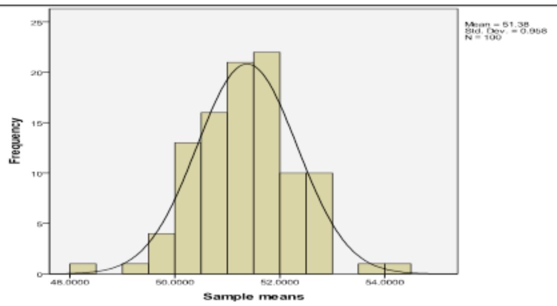


Figure 2b. Distribution of Sample Standard Deviations (n=100)

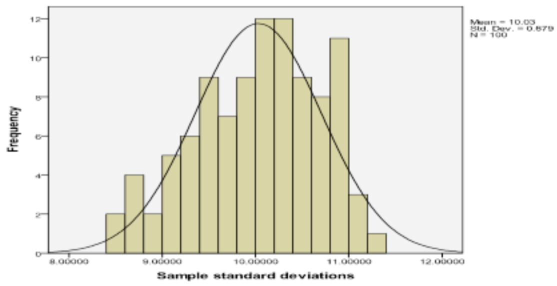


Figure 3a. Distribution of Sample Means (n=500)

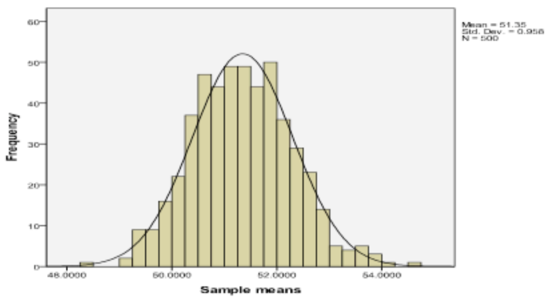


Figure 3b. Distribution of Sample Standard Deviations (n=500)

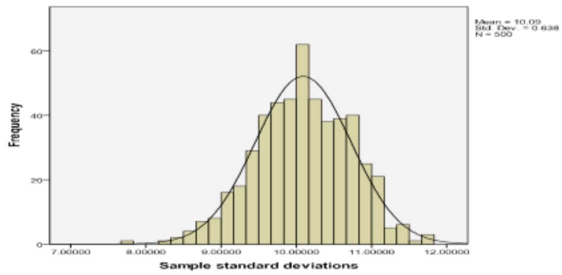
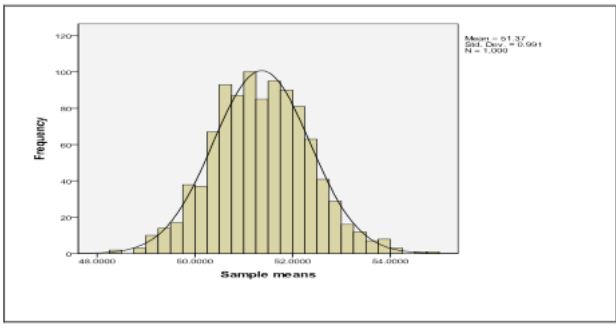
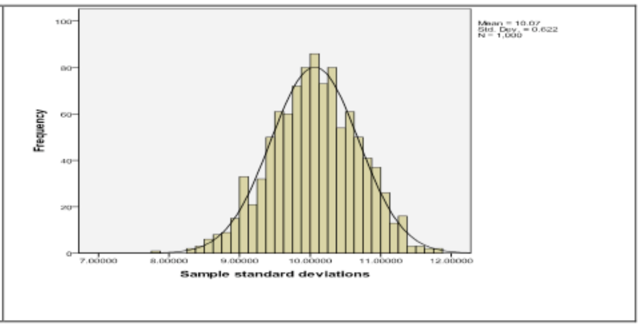


Figure 4a. Distribution of Sample Means (n=1000)

Figure 4b. Distribution of Sample Standard Deviations (n=1000)





Conclusion

In this paper, I have demonstrated the use of a real dataset to teach sampling distribution. While artificial data is usually used to demonstrate these concepts, this paper showed that it's possible to use real data to achieve the same objective with the added advantages that real data bring to students' learning experiences. The dataset ELS: 2002 contains thousands of variables, and students will be able to select variable(s) of their choice, that make sense to them. The fact that ELS: 2002 and other large-scale datasets are publicly available should be an incentive for statistics instructors to use them as teaching tools.

References

Akram, M., Siddiqui, A. J., and Yasmeen, F. (2004). Learning statistical concepts. *International Journal of Mathematical Education in Science and Technology, 35*(1), 65-72.

Brooks, G. P. (2004). MC4G: Monte Carlo Analyses for up to 4 Groups [Computer software and manuals].

Connor, J. M. (2003). Making statistics come alive: Using space and students' bodies to illustrate statistical concepts. *Teaching of Psychology, 30*(2), 141-143.

Larwin, K. H. and Larwin, D. A. (2010). Evaluating the use of random distribution theory to introduce statistical inference concepts to business students. *Journal of Education forBusiness, 86*(1), 1-9.

Leech, N. L. (2008). Statistics poker: Reinforcing basic statistical concepts. *Teaching Statistics:An International Journal for Teachers, 30*(1), 26-28.

Makar, K. and Rubin, A. (2009). A framework for thinking about informal statistical inference. *Statistics Education Research Journal, 8*(1), 82-105.

National Center for Education Statistics [cited November 11, 2010]. Education Longitudinal Study of 2002[Online]. Available at http://nces.ed.gov/surveys/els2002/

Raffle, H. and Brooks, G. P. (2005). Using Monte Carlo software to teach abstract statistical concepts: A case study. *Teaching of Psychology, 32*(3), 193-195.

Schumm, W. R., Webb,F. J., Castelo,C. S., Akagi, C. G, Jensen, E. J., Ditto, R. M.,Spencer, C.E., and Brown, B. F. (2002). Enhancing learning in statistics through the use of concrete historical examples: The Space Shuttle Challenger, Pearl Harbor, and the RMS Titanic. *Teaching Sociology, 30*(3), 361-375.

Stork, D. (2003). Teaching statistics with student survey data: A pedagogical innovation in support of student learning. *Journal of Education for Business, 78*(6), 335-

Transforming Instruction and Assessment Using Student-created Video

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Abstract of Workshop: The major aim of this workshop is to provide an opportunity for participants to assess the potential for the use of student-created videos in their instruction and assessment activities. This will be accomplished through direct experience with sample student videos. Information about the development of the student-video project will also be shared. **It is recommended that attendees bring a computer with video capability to the session, if at all possible.**

Introduction

The transformation of students, with narrow views of what mathematics is and how to learn it, into thoughtful mathematics learners is a major focus of mathematics for prospective elementary teachers courses. Mathematics teacher educators have used a variety of teaching strategies and provided many traditional and alternative learning and assessment opportunities for students with the goal of enhancing the learning of meaningful mathematics by prospective teachers of young children. Video technology has been used in a supportive role, with videos created by knowledgeable-others shown for this purpose.

Building on the work presented in my Dresden paper (Keen 2009), this session serves as an opportunity for others to observe how I use *student*-created video for both instructional and assessment purposes. The dream of creating knowledgeable teachers becomes a reality as the students begin to think like teachers, developing teaching video vignettes that provide opportunities for peers to learn about the mathematics that is the subject of each video. The videos serve as evidence of the student-creator's learning and understanding in very rich, personal and candid ways.

The video project is designed as a way to invite students into the study of mathematics while experimenting with the role of teacher. So, rather than using videos commercially produced, students experience first-hand the preparation and understanding required to present their thinking about a mathematical concept. The concepts to be "taught" are part of the regular course content and can then serve as tools for later study and revisiting of content by all class members.

Workshop Activities

In this workshop, participants view student-created videos. By viewing examples of videos showing a variety of evidence of content knowledge for teaching of mathematics, attendees will determine first-hand whether this type of activity appears to have value as a vehicle for student learning and assessment. Collegial discussion of the knowledge, skills, and mathematical dispositions in evidence on videos will serve as the basis for a professional analysis of the potential for the tool as a transformational vehicle for the student *and* the mathematics teacher educator.

Using the scoring rubric used for the student videos, participants will both scrutinize the video evidence for student understanding of the mathematics presented as well as critique the rubric designed to assess the students' videos. Of interest is whether this form of student learning and assessment appears to serve as a useful and transformational tool.

Extensions

Participants will be asked to use their on-board video capability to create a short video vignette about a topic that students often find confusing or in some way problematic. They will gain a clearer sense of the potential this assignment has for enhancing both student understanding of mathematics as well as assessment alternatives for the instructor.

Reference

Keen, V. (2009). "Using Digital Video to Strengthen Student Learning of Mathematics." Included in *Proceedings of the Tenth International Conference Mathematics Education in the 21st Century*, Dresden, Germany. Available online at http://math.unipa.it/~grim/21_project/21Project_dresden_sept_2009.htm

A case study of a teacher professional development programme for rural teachers

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Introduction

Since 1994, the South African school education system has experienced many curriculum waves. Several universities have configured their curriculum to suit the needs of the country as it moves through these curriculum waves. Many universities developed Advanced Certificate in Education (ACE) programmes in various specializations to help teachers prepare for the demands of the new curriculum. Teachers join the ACE programmes expecting that their acceptance into the programme will start the process of them turning the dreams into reality. They have dreams of improving their qualifications, or re-skilling themselves so that they are more relevant to the education system. A further incentive for the teachers is that the programmes have been funded by the Education department, thus removing from them the financial burden of covering the costs of the programme.

However for many teachers these dreams are shattered early in their journey and they drop out without completing the programme. Most full time undergraduate students in South Africa who drop out of their studies, do so because of an inability to pay for their fees and living costsThe high drop-out rate of students enrolled in various programmes offered by higher education (HE) institutions in South Africa has been a concern for many years. For instance, South Africa's graduation rate of 15% is one of the lowest in the world, according the National Plan for Higher Education compiled by the Department of Education (DOE) in 2001. In 2005 the DOE reported that of the 120 000 students who enrolled in HE in 2000, 36 000 (30%) dropped out in their first year of study. A further 24 000 (20%) dropped out during their second and third years. Of the remaining 60 000, 22% graduated within the specified three years duration for a generic Bachelor's degree (Letseka & Maile, 2005).

In this case of funded ACE programmes, the teachers' studies are funded, so they do not have the financial burden of paying for their fees. However the dropout rate for the ACE programmes has not been investigated since it was introduced in 2000. We felt that it was important for us to investigate why teachers drop out of such funded programmes. We engaged in a preliminary drop out analysis of the ACE programmes, and it emerged that the ACE (Mathematics, Science and Technology) programmes offered by the University of Zululand (UNIZULU), a historically black university[1] had a very low drop put rate. This ACE (MST) programme, is offered to teachers from rural areas north of the province of KZN, who teach in the General Education and Training (GET) phase.

The purpose of this small scale study was to explore the effectiveness of the programme, with particular reference to the low drop out rate. The research question guiding this study is : 1. What are some factors that contribute to the low drop out rate?

History of ACE programmes in South Africa

Prior to the introduction of ACE, a similar programme known as Further Diploma in Education (FDE) existed. The FDE was the state's first major intervention to re-skill or upgrade teachers from their initial qualification in teaching. Teachers with a three-year teaching diploma were eligible to enroll. The FDE was introduced in the late 1980's and early 1990s by the old distance education college of education such as the South African College of Education as a way of upgrading teachers to a qualified teacher status. With the introduction of the NSE (Norms and Standards for Educators; DoE, 2000) framework, the FDE was renamed the ACE. The ACE's purposes were similar to those of the FDE and it soon became the multi-purpose qualification for teacher upgrading, re-skilling and access to higher level programmes. During the review process instituted by the Council for Higher Education (CHE) in 2006, it became clear that there were 69 different kinds of ACE's in the country and that over 290 specializations were being offered (CHE, 2010).

Profile of the ACE (MST) at the University of Zululand

The three ACE programmes offered at the University of Zululand (UNIZULU) are a dual combination: Maths/Science; Science/Technology & Maths/Technology. These programmes were designed to deepen the knowledge of Senior Phase educators in the three areas; Mathematics, Science and Technology. The contact sessions were delivered on weekends or during block sessions during school holidays to cater for the needs of the teachers who came from rural areas situated far away from the campus.

The study

The design of this study was a naturalistic, qualitative, interpretive case study. A naturalistic inquiry was used as it has an emphasis on interpretive dimensions where the goal of the researcher is to understand reality (Cohen, Manion & Morrison, 2007). In this case we wanted to understand the experiences of the ACE teachers to find out why they persevered in their studies. A qualitative case study approach provided the opportunity to concentrate on a specific instance or situation (Cohen *et al*., 2007), namely the particular ACE programme. Data for this study was generated from the student's examinations results (from the university archives), an interview with the programme coordinator, questionnaires to 8 past students and a focus group interview with the same students. These 8 students were selected because they have continued with their studies after completing the ACE programme.

Results

In this section we first report on the throughput rate for these ACE students, before discussing some factors accounting for the high perseverance rate. Thereafter we share some suggestions made by the teachers on how the programme could be improved.

Throughput rate.

The original group consisted of 234 students which was reduced to 217 after the students who were registered for another qualification were excluded. Of these 181 (83%) students graduated within the minimum time of 2 years while 50 (23%) are still in the system. There were six students who dropped our three of whom passed away. This is a very high throughput rate as compared to another ACE programme which had a 50% graduation rate in minimum time rising to 60% over a 3 year period (Bansilal, 2010). When compared to the reported 22% throughput rate for the undergraduate students reported by Moketsi and Maile (2005), the 83% graduation rate in minimum time is impressive.

Factors which contributed to the high throughput rate

Residential Nature of Contact Sessions

The coordinators of the programme decided to run the lectures in block sessions during holidays or weekends. Students (who are practicing teachers) were transported from their local areas to a hotel close to the university. They were accommodated during the sessions and were also transported to the university for the lectures. The reasoning behind this arrangement was that since the teachers on the programme were from rural areas, public transport was notoriously unreliable and the teachers would have spent much of their time just travelling to the lectures. Although regular, day–long or shorter sessions would have been preferable to avoid mental fatigue, the residential block sessions were selected as the mode of delivery to alleviate the transport and isolation problems of the students.

This decision seems to have paid off in many positive ways, some of which were identified by the coordinators as well as the teachers. We outline two of the benefits below.

Contact sessions were intensive learning opportunities for the teachers:

Because of the residential nature of the sessions, lectures and tutorials were spread out throughout the day from 8 in the morning to 7 in the night. All eight respondents indicated that the intensity of the contact sessions helped them to succeed in their studies. The planning of the sessions was such that the teachers made optimal use of their available time. In fact the teachers indicated that as adults, they had family duties and these often interfered with their studies. By taking them away from their everyday situations, studying was made easier because they did not have to worry about their family responsibilities while they attended the sessions. The teachers spoke about the value of working in groups. After dinner, they would meet in groups and go over the work that they learnt. Ambiguities and misunderstanding of content was often cleared up by colleagues during these informal evening discussions.

The student representatives were able to meet often to discuss issues of concern across the different classes as well as to disseminate information to people in the classes.

The teachers were comfortable during their contact sessions

This theme may seem to be out of place in a study of an academic programme. However the coordinator and the teachers both stressed that the comfortable accommodation served as inspiration for the teachers to work harder. Because they did not have to stress about arranging or paying for their own accommodation, all their efforts could be concentrated on their studies. The coordinator commented that the teachers were very appreciative that the university provided the accommodation for them.

Student support

The analysis of the data also revealed many different support mechanisms which improved the students' experiences. We discuss five of these:

Academic Support

The coordinator stressed that at least 50% of the people who taught on the programme, were full time university staff because of the stipulations of the Council for Higher Education (CHE). This meant that the teachers were taught by experienced and well qualified staff, who were available for consultations during the block sessions. The particpants were positive about the academic support offered by the staff and named particular lecturers who made a big impact on them.

The teachers also felt good that the university considered them as important by ensuring that the teaching was done mainly by permanent academic staff.

Attitudes of staff

The drop out study by Letseka and Maile (2005) revealed that some of the reasons for students dropping out was linked the institutional culture of the HEI's which did made them feel ill at ease. They spoke about students' problems being dismissed as 'someone else's problem". In this case the teachers spoke about the warmth and respect they experienced during their studies which made them feel at home and comfortable. One teacher was impressed at the personal attention they got when the "coordinator sat with all [the teachers] who were going to be involved [in the programme].

One student said that in his first year, they found that certain staff had an "attitude" towards them and they found it very hard to work under those circumstances. However after approaching the lecturer as a group, he was willing to hear their concerns and thereafter they had no problems. This solution helped them to become more confident as well.

The teachers considered this approach to the problem as a learning curve. One student stated that " so if ever there are any problems, we can come to the lecturer and then we can try and discuss the problem". These comments suggest that the students felt welcome and respected at the institution.

Tutorial Support

One innovation of this programme was the use of full- time young university students as student tutors in the afternoons. As mentioned earlier, the programme ran until 7p.m. This was because the formal lectures lasted until 4pm, and thereafter the student tutors were available to work with the teachers in an informal basis, by helping them to work through particular concepts, practice exercises and tasks. The teachers were very pleased with this support. One teacher commented that the students "explain everything". She felt that the students explanation were so good that they should "come to our area to teach". This additional facet of support was seen as a reason for the high pass rate in the modules because teachers were given the opportunity of working through the materials while having students available for their assistance.

Financial support

The teachers were appreciative of the government's commitment to improving their skills by finding their studies. The teachers are mature students who have their own families and funding for their studies is not prioritized in the hierarchy of their family needs, so this was a welcome opportunity of many. In the words of one teacher: " When entering this course, I am fully capacitated, we are bread winners, we are having children who need to go to university and it is not easy for me to pay for my child at university and also to pay myself and upgrade myself" This comment captures the students' gratitude about being offered a funded opportunity to upgrade herself. The other benefits of free transport, accommodation and food made the teachers even more determined to make the most of the opportunity.

Classroom support visits

The programme had a classroom support component, where lecturers visited the teachers at their schools and watched them teach and thereafter held individual discussions about their practices. All the teachers found this aspect very helpful and useful. Although one teacher said that she was nervous initially when she knew the lecturer was going to watch her teach, but the lecturer was friendly and helpful and "there just to help you improve". They found the discussion after the lessons useful. Some mentioned that the discussions helped them to concretise the outcomes that they were expected to plan for, and to explore alternate strategies.

Relevance of curriculum and materials

The teachers found the curriculum relevant and all expressed the opinion that the programme had changed the way they teach. They learnt alternative teaching strategies. Two teachers mentioned that the section they struggled with was drawings in technology and this was addressed by the programme. Another spoke glowingly about how the lecturer for science "gave us tips on the designing material in how she was doing it". The teachers also mentioned that what they learnt was relevant to their teaching and they found the materials and some activities useful in their classrooms.

Flexibility of programme

The university policy for such funded joint programmes was that money generated from such programmes would be used for the programmes only. Thus the university deducted a minimal percentage for administration fees. This left the planning and administration of the programme to the departments. The coordinator emphasized that this approach reduced the bureacratic load as well. The department was able to plan the delivery of the programme around the needs of their students. They had the freedom to allocate the funds in an optimal manner and chose to organise the transport and accommodation for the students, a decision that led to much of the success. In addition sufficient staff were employed to mange particular aspects of the programme such as transport and accommodation, administration coordinator an dan academic coordinator. This meant that the academics were freed up to concentrate on the academic delivery of the programme.

An additional advantage of the institutional arrangement was that the coordinators worked directly and closely with the national department of education who sponsored the programme. The department representative met often with lecturers and teachers. Complaints from teachers were taken up quickly and resolved together. The department representative was also pleased with the attention given by the programme coordinators to the teachers' needs.

Suggestions for improvement

There were some suggestions for improvement. The administration was noted as an area of concern because a few teachers spoke about late release of assessaiment marks. One teacher mentioned that the mark he received was initially a pass and then the next day it was reported as a fail. The teachers were also frustrated about the pacing of the examinations especially when they had more than one examination scheduled on one day.

Concluding remarks

In this paper we looked at the throughput rate of an inservice qualification programme (designed for rural Senior Phase Mathematics, Science and Technology teachers) administered by the University of Zululand. We then analysed students' responses to a questionnaire and a focus group interview to identify possible factors which contributed to the high throughput rate. This data was supplemented by an interview with the programme coordinator. One of the aims of the study is to contribute to knowledge about successful practices in designing inservice programmes for teachers. The study identified three main factors which accounted for the teachers' satisfaction with the programme —the residential contact sessions, the student support and the freedom given to the programme coordinators to meet the teachers ' needs.

Although accommodation for in-service teachers is an expensive programme investment, the teachers' responses indicate that this facet of the programme helped them to get more work done without being distracted by their family duties. The evening discussions with their peers also contributed positively to their learning experiences. The teachers and coordinator also commented that the financial support via the bursaries, the welcoming attitudes of the institution, the tutorial support as well as the classroom visits were all beneficial to them.

However the success of the whole programme cannot just be judged on the basis of the successful delivery and high throughput rate. A crucial indicator of the success is the degree to which the teachers are able to improve the teaching and learning of mathematics in their classrooms. It is important to investigate whether the programme translates to sustained improvement in classroom practice. This is one of the focuses of the current research project which aims to investigate the impact of various ACE programmes on the teaching and learning in KZN.

References:

Bansilal, S (2010). A throughput analysis of the ACE (Mathematical Literacy) programme. Unpublished report presented to the Deputy Dean, Faculty of Education, UKZN

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education*. New York: Routledge.

Council for Higher Education (2010). Report of the National Review of Academic and Professional Programmes in Education . *HE Monitor, No 11*

Letseka, M & Maile, S (2005).High University drop –out rates: a threat to SA's future. HSRC Policy Brief March 2005.

Mathematics through Language

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Abstract
This workshop explored the importance of mathematics in written and oral language development. Mathematics has its own unique words and expressions, but is learned through a student's natural language. Writing in mathematics can help students grasp concepts better, explain and assess their thinking, and bring more enjoyment to the subject. When a student writes about math, he/she gains a deeper understanding of what he/she knows and is able to do. In the workshop we explored the following questions: How does writing in mathematics develop language? When should students write in mathematics? What does good mathematical writing look like? How can mathematical writing be used to assess students? Participants analyzed an article, investigated lesson ideas, reflected on student work, and generated ideas for their own practice.

Introduction: How is language developed through the use of writing in mathematics?
Traditionally mathematics has been taught through repetition and memorization. Classroom practices must be transformed by teaching concepts and strategies through meaningful learning experiences. The use of language and specifically written language in mathematics is a valuable paradigm switch in teaching and learning. Instead of assessing the right answer to a problem, teachers should assess the ideas and the strategies behind finding the solution. To solve a problem in mathematics students need to use their language skills to comprehend the problem, reason the best strategy for solving it and to communicate how they found the solution to others. Integrating writing into this process allows the student to be reflective of their understanding and abilities.

Studies show that effective teaching in mathematics includes a focus on the development of conceptual knowledge and language that requires the teacher to use clear and understandable dialogue with the students. This supports them in learning new ways of expressing their thinking and models the appropriate uses of the mathematics register; the use of the special vocabulary that is particularly to math as well as natural language. Even natural language takes on special meaning within the mathematics register. Everyday words may be transformed or broadened to include new meaning peculiar to the concepts of mathematics. At times, students may struggle with understanding concepts described within the mathematical register. Integration of writing allows for more exposure, repetition and reflection on the use of the appropriate register while students are developing their natural language.

Benefits and challenges of using writing in mathematics

Most educators were taught mathematics through traditional means, yet recognize the need to be more progressive in their own teaching practices. Conceptual based teaching and the integration of disciplines benefits the learner. This requires action research, innovation, sharing of ideas, the battling of misconceptions and collaboration. Transforming teaching practices in the classroom requires taking risks and spending more time in preparation and planning. In addition, when students write in mathematics, more time is needed to read and reflect on their work.

There are many ways that writing and mathematics can be integrated. Students can write short reflections or explanations of their problem solving, which can make over a more traditional approach to teaching math. They can be incorporated into an interdisciplinary unit project. Mathematics can be used as a theme for creative writing. Some of the more practical math writings can be journals or logs, writing about the strategies used during the problem solving process, writing about understanding of concepts, and writing about feelings or attitudes towards math. Each form of writing has its purpose and value. Writing can be used to bring enjoyment to the subject or to reveal the knowledge and misconceptions a student may have.

Understanding the Approach
Workshop attendees read article selections and shared their reflections with the group. Half of the attendees read an article by Gilberto J. Cuevas. Cuevas asserts that the student's ability in language not only determines performance in mathematics but also in the acquiring of conceptual learning and therefore is an integral part of the teaching process. Although there is not a clear understanding of the relationship between language factors and mathematical achievement, research has shown that there is a correlation between mathematical achievement and reading ability. This speaks of the need for language development. Teachers must support students in their development of written and oral language and be mindful of the students in the class that need additional support in this area. Mathematical language has its own functional register. It is important to help students develop the language so that they can adequately describe the problem solving process and explain their thinking. They must use a combination of mathematical terms and expressions along with their natural language to communicate effectively. Their ability to understand and share their thinking helps them in processing their reasoning. Naturally, language has a role in assessment.

The other half read an article by Dr. Marcia Frank. Frank concluded that integration of writing with mathematics is integral for teaching concepts. Although there are limitations to how writing can be used, it is one of the best ways to assess whether students understand the concepts and the process of the mathematics learned in class. It is a means to conference with each individual student when time is limited. It is through their writing that students can demonstrate what they know and what they can do. By asking students to respond to simple writing prompts, much of their understanding can be revealed.

Following the analysis of the articles, we focused on how different forms of writing can contribute to helping students process their own reasoning, demonstrate their learning and develop their language skills. The use of journals can help students reflect on their learning and monitor their growth. In class writing prompts can assist in working through a process, communicating to the teacher individual needs and demonstrating understanding of a newly learned skill. Portfolios are also great for monitoring growth and to give the students an opportunity to highlight work that they are proud of. Graphic organizers can be useful in developing vocabulary and working through a problem solving process. In everyday class work, homework, test and quizzes, writing can be incorporated to develop language and make the task more enjoyable for the students.
Attendees also looked at some sample student work from an international school in Beirut, Lebanon. The artifacts represented various types of mathematical writing. Participants were given a rubric to assess the work. This activity was followed by a discussion of how to assess mathematical writing and the useful data that can be gained from it. Finally, attendees were asked to reflect on their teaching practices or the practices of their teaching community and set goals for the future.

Conclusion
Students are expected to use language appropriately when communicating mathematical ideas, reasoning and findings – both orally and in writing. Through mathematics students have access to a unique and powerful universal language while developing their primary academic language. Students should be able to communicate a coherent mathematical line of reasoning using different forms of representation when investigating complex problems. Writing in mathematics is integral to working through a process, communicating to the teacher individual needs and demonstrating understanding of newly learned skill.

References

Andrew Sterrett (Ed.), *Using Writing to Teach Mathematics*, Mathematical Association of America, 1992.
Dr. Marcia Frank, Writing in Math – Should it Have a Home in Today's Curriculums, University of Maryland, 2001.
Gilberto J. Cuevas, Mathematics Learning in English as a Second Language**,** *Journal for Research in Mathematics Education*, Vol. 15, No. 2, Minorities and Mathematics. pp. 134-144, Mar., 1984.
Bernadette Russek, *Writing to Learn Mathematics*, Writing Across the Curriculum, Vol 9, August 1998.

An action research study of the growth and development of teacher proficiency in mathematics in the intermediate phase – an enactivist perspective. Work-in-progress

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Abstract
This paper reports on a current research project that focuses on an action research study of a pre-service mathematics education module to grow and develop proficient teaching in mathematics in the intermediate/middle school phase. The aim of this research study is to ascertain if a fourth year mathematics education module whose teaching pedagogy is informed by the underlying themes of enactivism will develop and grow pre-service teachers' proficiency in teaching mathematics in the intermediate phase. The focus of this presentation will be to report on the outcome and my analysis of the first research cycle and to highlight the way forward in the second cycle of this action research project.

Introduction
The intention of this research project is to determine if a mathematics education module informed by an enactivist orientation and teaching pedagogy will enable pre-service teachers participating in the module to unpack the reality of their teaching practice in terms of proficient teaching through active participation in lectures and practical tutorial sessions. My theoretical framework is informed firstly by enactivism and secondly by the use of the underlying themes of enactivism, as introduced by Di Paolo, Rohde and De Jaegher (2007), as a vehicle to develop teaching proficiency.
The investigation is an action research study that encompasses a fourth year mathematics module for BEd. Degree pre-service teachers training to teach in the foundation phase. The module has been designed to equip the foundation phase pre-service teachers with the necessary skills and content to teach mathematics in the intermediate phase. The aim of this mathematics module is to develop and grow pre-service teachers who will one day be role models for their learners and instil in their learners a love and interest in mathematics. Thus the module has been designed to augment the pre-service teachers' basic skills in mathematics and provide them with opportunities to become competent in basic mathematics and pedagogic skills for the Intermediate Phase. The intention of this module is to develop both their confidence and proficiency in mathematics and their proficiency in teaching mathematics in the Intermediate Phase.

Enactivism
Enactivism is a philosophy that views the body, mind and world as inseparable and was originally developed by Merleau – Ponty. It was then further developed by Maturana and Varela; Thompson and Rosch (Davis, 1996), as an alternative to the bipolar and divisive nature traditionally associated with western thinking. As a philosophy enactivism recognises the structure of an individual to be a combination of both their biological composition and their personal history of interacting with the world. Enactivism also "focuses on the importance of embodiment and action to cognition" (Li, Clark & Winchester, 2010:403). Consequently the structure of the individual is considered to be adaptable as it changes to make sense of new experiences and challenges.
Since enactivism considers cognition to be a complex co-evolving process of systems interacting and affecting each other, cognition is deemed to be a producer of meaning and not a processor of information. Therefore an individual's interactions with the world and their past experiences will shape and influence the meaning that they make of their world (Lozano, 2005). Thus any perturbation will present an opportunity for an individual to act according to his/her structure and it is the structural make up of the individual that determines what and the extent of the change that occurs. Enactivism encourages learning and the construction of knowledge by means of a collaborative process, thus any given learning situation must encompass the lecturer, the student, the content and the context in order for some form of interaction to take place.
From an enactivist perspective the role of the lecturer is that of a "perturbator" in order to encourage and provoke learners into "thinking differently" about mathematical concepts that may not form part of their personal

construct. With this in mind, I have chosen to use five underlying themes of enactivism (Di Paolo, Rohde and De Jaegher, 2007; McGann, 2008) namely autonomy, sense-making, emergence, embodiment and experience to underpin my pedagogic practice. A number of tasks and practice will be developed or drawn from the literature where they have been applied in other mathematics contexts and adapted with the intention of developing teaching for mathematical proficiency skills.

Underlying themes

I have used the five underlying themes to assist pre-service teachers to determine what embodied views of cognition reveal about their personal proficiency in mathematics, their mathematical identity and their self development. Furthermore the themes will help to ascertain how the pre-service teachers' embodied perceptions of their mathematical proficiency support their own teaching for mathematical proficiency during the practical tutorial sessions.

Di Paolo, Rohde and De Jaegher (2007) indicate that living organisms are autonomous due to their ability to create their own identity that characterises them as a unique entity. In this research study the pre-service teachers' mathematical identity is representative of autonomy since enactivism argues that one's identity is enacted and therefore determined by the interplay between biology and human culture and the individual's manner of dealing with life's experiences.

With regard to the second theme, namely sense-making, according to Di Paolo, Rohde and De Jaegher (2007) this refers to the way an individual actively participates in generating meaning of their world with changes occurring as a result of dialogue between the individual and the environment. Davis (1996) introduces the notion of listening as an embodied action that enables one to understand human communication and collective action. Thus sense making will manifest in my pedagogy through encouraging and creating opportunities for the pre-service teachers to play an active and participatory role in emerging conversations and to engage in different types of listening.

The third theme, emergence, is explained by Davis (2004) as the notion that understanding and interpretations are generated through shared activities over a period of time, as opposed to predetermined learning objectives. In addition, Davis (2004) raises the point that learning is not the site of the individual, be it learner centred or teacher centred, but rather the collective generation of knowledge and understanding. Therefore this research study will use various reflective techniques to encourage the students to reflect on what they have learnt during a particular lecture or tutorial and to acknowledge the role that the community of practice, autonomy and sense making have played in their understanding of and culmination of the learning process.

According to Li, Clark and Winchester (2010) the embodied experience that an individual undergoes is due to the bringing together of the mind and body by means of reflection. Di Paolo, Rohde and De Jaegher (2007) augment this notion indicating that cognition is an "embodied action" (p.11) in that the mind and the body form part on a living system composed of various "autonomous layers of self-coordination and self-organization" (p.12) which allow it to interact with the world in "sense-making" activities, with the understanding that the body is not controlled by the brain. Davis (2004) suggests that embodiment refers to the idea that individuals are all part of a larger collective system and cites Kauffman's description of the relationship between the collective and the individual as one in which "the collective is enfolded in and unfolds from the individual" (p. 213). With this in mind an aspect of the pedagogical practice is to encourage the formation of a mathematical community of practice amongst the pre-service teachers to research its influence on the lesson preparation and sense making processes.

Finally the pre-service teachers will be given the opportunity to experience developing lessons for the intermediate phase and teaching for proficiency. Furthermore, it is anticipated that these experiences will change through embodied action and the emergent process. In the initial stages of the module,

activities are incorporated to encourage pre-service teachers to discuss their experiences and critical incidents of mathematics teaching. Further the students will need to determine the influence that these experiences have had in determining what they understand by proficiency and their role as a teacher in trying to teach for mathematical proficiency.
In order to analyse the growth in proficiency, Kilpatrick, Swafford & Findell's (2001) framework for mathematical proficiency is the core analytical framework for the research project, since the authors are cognisant of the importance of noticing and analysing learners' interpretations and of the role that different contexts, especially environmental and situational elements, play in the teaching and learning process. Kilpatrick et al (2001) identify mathematical proficiency as encompassing five interwoven and interdependent strands, namely, conceptual understanding; procedural fluency; strategic competence; adaptive reasoning and a productive disposition.

Action Research Cycle

In this study the pre-service teachers' involvement in the decision process entails providing feedback on the tutorial sessions in the form of a class group interview at the conclusion of each tutorial session and through their reflective journals. Enactivism views cognition as a complex co-evolving process of systems interacting and affecting each other, not as processors of information but as producers of meaning. Lozano (2005) explains this as the manner in which an individual's interactions with the world and their past experiences shape and influence the meaning that they make of their world. The pre-service teachers will reflect on their experiences at each tutorial session and identify what aspects or critical incidents they believe have influenced their proficiency. As the researcher, I will analyse this data using Kilpatrick et al's (2001) five strands of mathematical proficiency to determine the growth and development of proficiency. This information in turn will be used to inform the development of the next lecture so that different activities and tasks can be developed to encourage and grow proficiency.

The module comprises a double lecture weekly in which enactivist pedagogy is role-modelled and mathematical theory and content discussed. In addition to the lectures, weekly practical tutorials are held during which the pre-service teachers are given the opportunity to develop their mathematics teaching skills. Since there are fifty pre-service teachers, the class has been divided into two groups with each group attending fortnightly tutorials. Each of the tutorial groups is further subdivided into five groups with 5 pre-service teachers in each, three teaching groups, one learner group and one observation group (see figure 1). Therefore at each tutorial session the three teaching groups are given the opportunity to teach a mathematical concept, the learner group takes on the role of the learners and the observation group monitors and analyses the teaching process identifying critical incidents and completing an observation sheet underpinned by the 5 strands of proficiency. Over the duration of the year the groups in each tutorial session will rotate so that each pre-service teacher will have the opportunity to be either in a teaching, learner or observation group. The pre-service teacher will remain in the same group of 5, both for tutorial sessions and lectures, thus forming their own mathematical community to assist with the sense making process.

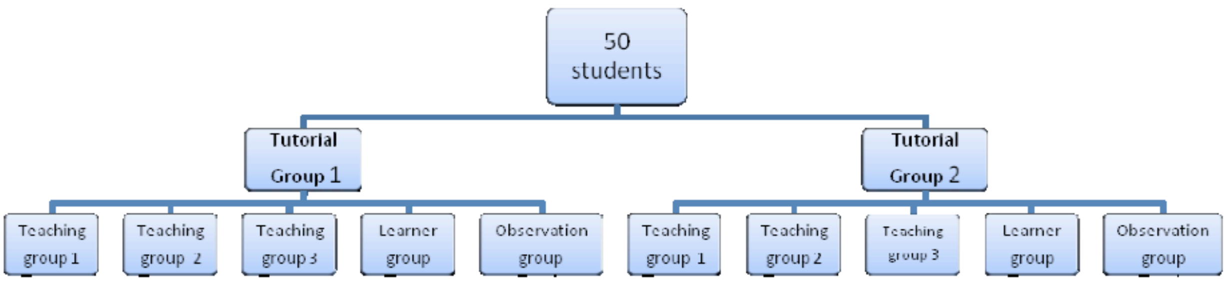

Figure 1

The three teaching groups will be given prior warning of the content that they will be required to teach with the expectation that they will then develop a 15 minute micro lesson that encompasses some of the strands of mathematical proficiency. All the members of the teaching groups contribute to the preparation of the lesson and resources, although only one member from each teaching group is given the opportunity to teach the lesson at the tutorial session. During the course of the year a number of pre-service teachers will be given an opportunity to teach a lesson. At the conclusion of the first micro lesson, the pre-service teachers will be given a few minutes entering into conversation as to the outcome of the lesson with regard to teaching for proficiency. The teaching group and the learner group will discuss what strands of proficiency they believed had been addressed, while the observation group will identify critical incidents that they deem to have impacted on and affected teaching for proficiency. The role of the observation group is to notice and identify which strands of proficiency have been addressed, what enactivist pedagogic strategies and themes have had a positive effect and where and what difficulties have arisen in the lesson delivery and activities. This same process will then be repeated for the second and third teaching group. Following the final lesson, the tutorial session concludes with a class group interview with all the pre-service teachers, relating to the lessons observed and the perceived growth and development of teaching for mathematical proficiency and the contributing factors.

Conclusion

In presenting this paper it is my intention to discuss and reflect on my experiences pertaining to the first cycle of the research. Since enactivism is a theory that has not been researched widely in South Africa, I hope to engage the audience in a conversation as to the merits of my research to date and my objectives for the second cycle.

References

Davis, B. (2004). Inventions of Teaching. A Genealogy. New York: Lawrence Erlbaum Associates.

Davis, B. (1996). Teaching mathematics. Toward a sound alternative. London: Garland Publishing, Inc.

Di Paolo, E, M Rohde and H De Jaegher (2007) Horizons for the enactive mind: Values, social interaction and play. In Enaction: Towards a New Paradigm for Cognitive Science. Stewart, J, O Gapenne and E Di Paolo (eds). Cambridge, MA: MIT Press

Kilpatrick, J., Swafford, J. & Findell, B. (2001). Adding it up: helping children learn mathematics. Washington, DC: National Academy Press

Li, Q., Clark, B., & Winchester, I. (2010). Instructional design and technology with enactivism: A shift of paradigm? British Journal of Educational Technology. Retrieved June 12, 2010, from http://people.ucalgary.ca/~qinli/publications2.html

Lozano, M. D. (2005). Mathematics learning: Ideas from neuroscience and the enactivist approach to cognition. For the learning of mathematics, 25(3), 24 -27.

McGann, M. (2008). What is “Enactive” Cognition? A Cognition Briefing. Retrieved 9 January, 2011, from euCognition The European Network for the Advancement of Artificial Cognitive Systems: http://www.eucognition.org/euCognition_2006-2008/cognition_briefing_enactive_cognition.htm

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MATHEMATICAL COMPETENCE ASSESMENT OF A LARGE GROUPS OF STUDENTS IN A DISTANCE EDUCATION SYSTEM

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Abstract

We present a model for the assessment of mathematical competence of a large group of students in a distance education system. The model can be applied automatically and allows not only a summative evaluation type, but it has also educational character. We present some results of applying the model to a real situation for the Course *Applied Mathematics to Social Sciences* taught at UNED of Spain as a part of the Foundation Course for access to University for people 25 years old and over.

Mathematical Competence

The use of the term competence has set out as the cornerstone for the design of teaching and learning process in diverse areas and educational levels. Currently, in Spanish universities, all degrees must be designed in accordance with the teaching and learning model aimed to developing learning skills. It is not possible here to make a profound discussion on the various meanings of the term. For purposes of this paper we understand that "*competence is values, attitudes and motivations, as well as knowledge, skills and abilities, all as part of a person who is insert in a given context, considering also that he/she learns continuously and progressively throughout life*" (Sevillano, 2009).

According to the above idea, to define the mathematical competence it is necessary to identify the components that comprise it. That is, mathematical competence is developed when successfully integrated mathematical knowledge and skills, so they emerged in different areas of personal performance directed by certain values and attitudes. Ramos, 2009, includes an extensive discussion on how to achieve the mathematical competence. The determination of the components leads to the following specifications, Ramos *et al.*, 2010:

- *Mathematical knowledge* formed around certain key ideas that have historically set the scope of mathematics and the core of mathematical thinking. These ideas include: i) *Mathematical language*, ii) *Quantity*, iii) *Space and shape*, iv) *Change and relationships*, v) *Uncertainty*. Each of these sections includes a set of concepts that formalize the different experiences of men in real situations that present aspects of mathematical nature. It is clear that among those sections there are many common points and any attempt to strictly separate them leads only to artificial situations. However, the above classification is useful not only for its traditional character, but also for its important methodological implications. The description and contents of the five sections can be found in the text by Hernández, Ramos, Velez and Yáñez, 2008.

- *Mathematical capabilities* that are required at different stages of the mathematical process, putting into action the mathematical knowledge in a given context. These capabilities, according to PISA, 2003, 2006, 2009, can be stated as: i) *Thinking and reasoning*, ii) *Argumentation,* iii) *Communication* iv) *Modelling*, v) *Problem posing and solving*, vi) *Representation*. vii) *Using symbolic, formal and technical language and operations,* viii) *Use of aids and tools*. The cognitive activities that include these capabilities are three levels of development or skill clusters: *reproduction* cluster, the *connections* cluster and the *reflection* cluster, each of which represents, respectively, a higher level of development of mathematical competence. It is also possible to consider the following contexts*: personal, educational/occupational, public and scientific*. The meaning and scope of the above expressions is conveniently detailed in PISA 2003, 2006, 2009, (*vid*. Ramos 2009, Ramos *et al*. 2010).
- *Attitudes* that show: i) *Security and confidence with information and situations that contain mathematical elements*, ii) *Respect and enthusiasm for certainty through the correct reasoning*.

Evaluation of mathematical competence

One of the key issues of a teaching and learning model designed to develop skills is how to truly evaluate the achievement of the desired competences. Traditional models aimed at acquiring knowledge include an evaluation system designed to observe mainly whether or not a student has certain knowledge. In a model of competences we must go further and try to assess whether or not students have acquired the expected competences. In the assessment of competences we should keep in mind that these, in addition to knowledge, include skills, abilities, attitudes and values that the student has to develop. We then face the problem of assessing the level a student show in certain qualities, that can only be seen with a degree of subjectivity and are more difficult to detect by the usual assessment methods.

Generally, the competence models use assessment systems such as oral presentations, analysis of data or texts, skills practice while the individual is under observation, professional folder (portfolio), reports field work, written thesis, or similar. However, these activities encounter practical problems that can make them inapplicable in certain situations. In particular, we note that they are usually only practicable in groups of few students, they represent a major increase in workload for both students and faculty and are difficult to apply in some contexts such as distance education model. If we do not want to refuse to use a competence model, the alternative is to investigate the possibility of adapting the traditional assessment schemes so that they can be used as an acceptable method for assessing competence.

Ramos *et al*., 2010, present the theoretical ideas for the development of a competence evaluation system inspired by traditional systems, assessing not only knowledge but also skills and attitudes. The idea is to design assessment activities provided with a number of appropriate characteristics so that it is possible to assess the degree of acquisition of the various components and subcomponents of the competences.

The practical application of this theoretical model of evaluation to the case of the mathematical competence, in a large group of students and in a distance education system, leads to designing evaluation activities with certain attributes that allow the combination of simplicity of the traditional evaluation with the objectives of the competency assessment. We use automatic evaluation tests in order to obtain an objective qualification within a reasonable period of time. Moreover, the design of these tests include a quantification of all aspects that make up the components and subcomponents of the competences, that is, the acquisition of knowledge, skills and attitudes within a given context. Finally, it is necessary to consider an indicator for the accuracy or adequacy of the response to a particular reference. The above considerations lead us to define a test or evaluation activity as an object identified by the attributes that are listed in Table 1. Specific examples of such tests can be seen in Ramos *et. al.*, 2010

TEXT	ASSESSMENT			CORRECTION INDICATOR
	Knowledge	Capabilities	Attitudes	
Statement (TST).	Mathematical Language (VKL)	Thinking and reasoning (VCT)	Security and Confidence (VAS)	Success. (S)
Correct alternative (TCA)	Quantity (VKQ).	Argumentation (VCA)	Enthusiasm for certainty and correct reasoning (VAE)	Failure (F)
Distractor 1 (TD1)	Space and Shape (VKS)	Communication (VCC)		No answer (W)
Distractor 2 (TD2)	Change and Relationships (VKC)	Modelling (VCM)		
	Uncertainty (VKU)	Problem posing and solving (VCP)		
		Representation (VCR)		
		Using symbolic, formal and technical language and operations (VCST)		
		Use of aids and tools (VCU)		

Table 1

Thus, mathematical competence can be assessed by applying an evaluation form consisting of a number N of activities with the characteristics in Table 1. To do this, we must have a function of the attributes that, based on indicators of correctness of each of the N tests, provide an overall score, either quantitative or qualitative, in each of the components and subcomponents of the competences.

The quality of an evaluation form can be assessed by considering the overall characteristics of the attributes of the activities that it comprises, like the mean, minimum, maximum, range, etc. of the valuation of knowledge and skills. Thus, it is possible to compare different forms and even prepare forms that meet certain requirements desired by the teacher.

Case study

We present in this section the results of applying in practice the model described above. The framework is the subject *Mathematics Applied to Social Sciences* in the exam of June 2010. The examination form consists of ten evaluation questions. We use a numerical scale from 0 to 4 to assess the intensity with which a question measures the level of acquisition of each of the subcomponents of the competences. The attributes of each question in the form are summarized in Table 2. The indicator correction is the same for all question: S = 1, F =- 0.25 and W = 0.

Question	Knowledge					Capabilities								Attitudes	
	VKL	VKQ	VKS	VKC	VKU	VCT	VCA	VCC	VCM	VCP	VCR	VCS	VCU	VAS	VAE
1	1	2	1	4	0	2	1	1	1	1	2	3	1	1	0
2	1	4	0	0	0	1	0	2	1	2	2	3	3	1	0
3	4	2	0	0	0	4	1	2	3	2	2	2	2	1	3
4	0	0	0	0	4	4	2	2	4	2	1	1	0	4	1
5	2	2	4	1	0	2	2	2	3	2	3	2	2	1	1
6	2	1	1	4	0	2	2	2	3	3	2	3	2	1	1
7	1	0	4	0	0	3	3	3	3	2	3	2	0	3	0
8	4	0	0	0	0	3	4	3	1	1	1	2	0	1	4
9	0	3	0	0	4	3	2	3	4	3	4	2	4	4	1
10	0	4	0	0	0	1	1	1	2	2	2	2	4	1	1

Table 2

The student responses are collected by automatic reading sheets and form a vector $R=(R_1,R_2,\ldots,R_{10})$, where, R_i, *i=1,...,10*, takes one of the values *TCA(i), TD1(i), TD2(i)* or it is a blank or null answer. If we denote

$$E_i=\begin{cases} 1 & \text{if } R_i=TCA(i) \\ -0.25 & \text{if } R_i=TD1(i) \text{ or } R_i=TD2(i) \\ 0 & \text{otherwise} \end{cases}$$

then the traditional numerical grade, on a scale of 0 to 10, is calculated by the formula $E=\text{Máx}\left\{0,\sum_{i=1}^{10}E_i\right\}$.

For an independent assessment of each of the components of the competences, the above expression can be extended in several ways. For instance, it is possible to assign a different weight to each response, according to different attributes of the question. However, the current assessment model considered only assesses the different subcomponents of competence, using the data included in Table 2. Thus, the score obtained in knowledge, K_j, *j=1,...5*, capability C_k, *k=1,...,8*, and attitude A_l, *l=1,2* can be expressed as

$$K_j=\text{Max}\left\{0,\frac{10}{k_{.j}}\sum_{i=1}^{10}k_{ij}E_i\right\}\quad C_k=\text{Max}\left\{0,\frac{10}{c_{.k}}\sum_{i=1}^{10}c_{ik}E_i\right\}\quad A_l=\text{Max}\left\{0,\frac{10}{a_{.l}}\sum_{i=1}^{10}a_{il}E_i\right\}$$

where k_{ij} is the valuation of question *i* in knowledge *j*, c_{ik} is the valuation of question *i* in capability *k* and a_{il} is the valuation of question *i* in attitude *l*, given by Table 2; moreover, $k_{.j}=\sum_{i=1}^{10}k_{ij}$, $c_{.k}=\sum_{i=1}^{10}c_{ij}$ and $a_{.l}=\sum_{i=1}^{10}a_{il}$ are, respectively, the total of the values of the knowledge (*j=1,...,5*), capabilities (*k=1,...,8*), and attitudes (*l=1,2*) in the form. Finally, the average values can be calculated:

$$K = \sum_{j=1}^{5} \frac{K_j}{5}; \qquad C = \sum_{k=1}^{8} \frac{C_j}{8}; \qquad A = \sum_{l=1}^{2} \frac{A_l}{2}; \qquad EC = \frac{K + C + A}{3}$$

providing a measure of the level of each component of competence, (*Knowledge K*, *Capabilities C*, *Attitudes A*) and overall assessment of the mathematical competence *EC*.

Table 3 shows some records in the database of results. Each row corresponds to a particular student, while the columns include identification (ID), the valuation of each of the five subcomponents of knowledge, the eight subcomponents of capabilities and the two subcomponents of attitudes, the average valuation on knowledge *(K)*, capability (*C*) and attitude *(A)*, as well as traditional grade *(E)*.

ID	VKL	VKQ	VKS	VKC	VKU	VCT	VCA	VCC	VCM	VCP	VCR	VCS	VCU	VAS	VAE	K	C	A	EC	E
1	8,53	10,0	10,0	5,45	10,0	8,71	8,30	8,91	9,11	9,43	8,21	8,44	8,96	9,34	9,22	8,80	8,76	9,28	8,95	8,75
2	7,06	3,75	2,05	3,18	3,75	5,69	5,45	4,57	3,75	4,32	3,75	5,31	4,27	4,08	6,09	3,96	4,64	5,09	4,56	5,00
3	6,67	4,44	4,00	1,11	,00	4,40	4,44	3,81	3,60	3,50	3,64	3,64	4,44	2,22	7,50	3,24	3,93	4,86	4,01	4,00
4	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,00	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0
5	3,83	6,39	,00	4,44	5,00	3,30	4,44	4,64	3,80	5,00	4,43	5,00	6,94	3,89	5,21	3,93	4,70	4,55	4,39	4,50
6	4,17	3,75	3,75	3,06	3,75	5,00	5,83	5,24	4,50	5,00	3,75	4,89	3,75	5,14	4,79	3,69	4,74	4,97	4,47	5,00

Table 3: Some records in the database of results

Table 3 illustrates the possibilities of the evaluation model. Instead of a single data about each student, the standard grade *E*, we have a complete information on each competence level. For example, in the student with ID #5 has a grade of 4.50, typically assessed as insufficient. However, we can see that has an acceptable development of knowledge *Quantity* (VKQ=6.39) and *Uncertainty* (VKU=5.00). Similarly, he/she has a satisfactory level on *Problem posing and solving* (VCP = 5.00) and *Use of aids and tools* (VCU=6.94). On the other hand, the student with ID #6 has a classical grade satisfactory (E=5.00); however we see a lack in many components of the mathematical competence, since most of her scores are less than five. In both cases, we may recommend conducting some activities which had the objective of strengthening the shortfalls.

Conclusions

The assessment model we present has several advantages. We can highlight that it can be easily applied in educational contexts where it is difficult to make a more personalized assessment of the student, as may occur when the group is too big, or in a distance learning system. It is also possible to obtain not only a summative assessment, but also a formative evaluation designed to assess the individual student's level in each of the competences, setting minimum standards in each one of them to meet the objectives of the curriculum, identify gaps, to be able to recommend learning activities needed to achieve the desired level in each component of competence: knowledge, ability or others that we may consider.

References

Hernández, V.; E. Ramos; R. Vélez and I. Yáñez (2008): *Introducción a las Matemáticas*, Ediasa, Madrid.

PISA (2003): *Learning Mathematics for Life: A Perspective from PISA*, OECD. http://www.oecd.org/dataoecd/53/32/44203966.pdf

PISA (2006): *Assessing Scientific, Reading and Mathematical Literacy A Framework for PISA 2006,* OECD, http://www.oecd.org/dataoecd/63/35/37464175.pdf

PISA (2009): *PISA 2009 Assessment Framework. Key competencies in reading, mathematics and science,* OECD, http://www.oecd.org/dataoecd/11/40/44455820.pdf

Ramos, E. (2009): La competencia matemática, In Medina (Ed,): *Formación y desarrollo de las competencias básicas* , Universitas, Madrid.

Ramos, E; R. Vélez; V. Hernández; J. Navarro; E. Carmena and J.A. Carrillo (2009): *Sistemas inteligentes para el diseño de procedimientos equilibrados para la evaluación de competencias*, In M. Santamaría y A. Sánchez-Elvira (coord.): *La UNED ante el EESS. Redes de investigación en innovación docente 2006/2007*, Colección Estudios de la UNED, UNED, Madrid, pp.597-610.

Ramos, E; R. Vélez; V. Hernández; J. Navarro; E. Carmena and J.A. Carrillo (2010): *Competencias en Matemáticas Aplicadas a las Ciencias Sociales y su Evaluación Inteligente*, In M. Santamaría y A. Sánchez-Elvira (coord.): *La UNED ante el EESS. Redes de investigación en innovación docente 2007/2008*, Colección Estudios de la UNED, UNED, to be published.

Sevillano, M.L. (Dir.) (2009): *Competencias para el uso de herramientas virtuales en la vida, trabajo y formación permanentes*, Pearson, Prentice Hall, Madrid.

THE INFLUENCE OF GEOGRAPHICAL, SOCIAL AND CULTURAL FACTORS IN THE MATHEMATICAL COMPETENCE LEVEL

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Abstract

In this paper we present some results on the influence of different geographical, cultural and social conditions in the observed level of mathematical competence in a group of students. The experimental situation refers to the Course *Mathematics Applied to Social Sciences* taught at UNED of Spain as a part of the Foundation Course for access to the University of people 25 years old and over. As a tool, we used an evaluation model designed by the authors for the assessment of competences.

Introduction

The implementation of teaching models aimed at developing competences has been generalized to all levels of education in diverse fields. One of the keys to the effectiveness of these models is to find a solution to the problem of evaluating competence, i.e., it is necessary to have useful tools to adequately assess whether students have acquired the competences designed by the curriculum.

Typical evaluation systems of competence models, such as oral presentations, skills practice while the individual is under observation, or others, are difficult to apply in many real situations because the circumstances of the teaching-learning process make it complicated to follow the student on an individual basis. A particular case is a distance education system with large groups of students distributed around the world. In this context, if we do not want to give up the use of a competences model, it is necessary to resort to adapting traditional evaluation systems by adding certain features to measure the level that the student has in each of the competences set in the curriculum.

In previous work, Ramos *et al*. 2009, 2010, Leví and Ramos 2011, we presented various ideas for developing a competence assessment model capable of being applied in situations such as the noted above. We present an application of this model to a real situation, consisting of evaluating the mathematical competence shown by students in a course that is part of *Direct Access Course for over 25* years taught at the National University of Distance Education (UNED) from Spain. The main results focus on a quantitative study of mathematical competence of students grouped by various characteristics of geographical, social and cultural factors. In general, we obtained satisfactory levels of competence, although there are minimal differences between groups.

Objectives and Hypothesis

Mathematical competence is developed when mathematical knowledge and skills are integrated successfully so that they emerge in different areas of personal performance directed by certain values and attitudes, Leví and Ramos 2011. The components and subcomponents of mathematical competence that are considered are the following:

- *Knowledge*: i) Mathematical language, ii) Quantity, iii) Space and shape, iv) Change and relationships, v) Uncertainty.
- *Capabilities*: i) Thinking and reasoning, ii) Argumentation, iii) Communication iv) Modelling, v) Problem posing and solving, vi) Representation. vii) Using symbolic, formal and technical language and operations, viii) Use of aids and tools.
- *Attitudes*: i) Security and confidence with information and situations that contain mathematical elements, ii) Respect and enthusiasm for certainty through the correct reasoning.

The main objective of this paper is to analyze the level of mathematical competence, expressed in terms of its components and subcomponents, which reaches a group of students from the *Direct Access Course for over 25* years taught at the UNED of Spain. This course serves as preparation for the entrance exam to the university for students who have not finished the usual way of the "*Selectividad*" to complete secondary education. The analyzed matter is *Mathematics Applied to Social Sciences;* this matter is mandatory for students following options for Social Sciences and Health Sciences, while it is optional for students of Arts and Humanities. Students in the field of Science, Architecture and Engineering were not included in the study.

The course is taught according to the methodology of distance education distinctive of UNED. Basically, it uses a set of studying materials developed by faculty and designed for individual study, while monitoring of learning takes place through a series of technological means of all kinds, which include the so-called virtual course through internet. In addition, most students have in person tutorial support in study centers spread throughout the country and even internationally, although in some cases there are several factors that hinder or even preclude the effective use of such support. In any case, students can always address the teaching staff directly to solve any difficulties they may encounter in their study.

Although most students are Spanish and live in the country, there is a significant participation of foreign students spread mainly around various countries in Europe and America. Special mention may be made for a group of students from Equatorial Guinea (Africa). UNED has maintained since its inception a historic commitment to the training of Guinean higher education, with significant contribution of Access Course. Another peculiar group is the group of students boarders in a prison. UNED has a program of collaboration with the *Directorate General of Prisons of Spain* to facilitate access to higher education of students in prison. The Access Course is also a way used by many inmates to start university studies.

In our study we intend to analyzed mathematical competence of the different groups of students mentioned above. The objective of the analysis is to compare the influence of different social, cultural and geographical factors in the acquisition of mathematical competence.

Since Mathematics is a basic science and uses universal methods for the rational man, we want to corroborate the hypotheses that there should be no difference in math competence levels among different groups due to the influence of social, cultural or geographical factors.

Methods

The target population was the group of students who take the examination in June 2010. Considering the social, cultural and geographical factors outlined above, the students were classified into four groups: *Foreing, Guinea, Prison, Spain*.

The assessment of mathematical competence was carried out by considering a form of exam consisting of ten questions of objective-type with one foot and three alternatives of which only one is correct. The correction answer indicator for the question $i = 1,...,10$, is $E_i = 1$ if the right choice is selected, $E_i = -0.25$ when a wrong alternative is selected and $E_i = 0$ otherwise. Traditional grade is $E = \text{Máx}\left\{0, \sum_{i=1}^{10} E_i\right\}$. Moreover, each question of the form has an assessment value in each of the subcomponents of the competence, given on a scale of 0 to 4 and prepared by the faculty. From these, it is possible to calculate the score that the student achieves in each subcomponent of the mathematical competence by using the expressions:

$$K_j = \text{Max}\left\{0, \frac{10}{k_{.j}} \sum_{i=1}^{10} k_{ij} E_i\right\} \quad C_k = \text{Max}\left\{0, \frac{10}{c_{.k}} \sum_{i=1}^{10} c_{ik} E_i\right\} \quad A_l = \text{Max}\left\{0, \frac{10}{a_{.l}} \sum_{i=1}^{10} a_{il} E_i\right\}$$

where k_{ij} is the valuation of question i in knowledge j, c_{ik} is the valuation of question i in capability k and a_{il} is the valuation of question i in attitude l; moreover, $k_{.j} = \sum_{i=1}^{10} k_{ij}$, $c_{.k} = \sum_{i=1}^{10} c_{ij}$ and $a_{.l} = \sum_{i=1}^{10} a_{il}$ are, respectively, the total of the values of the knowledge $j=1,...,5$, capabilities $k=1,...,8$, and attitudes $l=1,2$ in the form. Finally, the average values can be calculated:

$$K = \sum_{j=1}^{5} \frac{K_j}{5}; \qquad C = \sum_{k=1}^{8} \frac{C_j}{8}; \qquad A = \sum_{l=1}^{2} \frac{A_l}{2}; \qquad EC = \frac{K + C + A}{3}$$

providing a measure of the level of each component of competence, (*Knowledge K, Capabilities C, Attitudes A*) and overall assessment of the mathematical competence *EC*.

Because multiple meetings for the examination with different locations and times are required, it is necessary to use various forms of examination, identified by a code called *Type*. To eliminate possible biases that may result from this fact, the analysis will be presented later in place of direct scores we used the standardized scores, i.e., those obtained by subtracting the mean and dividing by the standard deviation of the results of each type of form. Thus, all assessments of the components and subcomponents of the competence have a zero average and standard deviation equal to one.