Mathematical Practices and the Role of Interactive Dynamic Technology

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Abstract:
The Common Core State Standards in Mathematics adopted by most of the states in the United States offer a set of mathematical practice standards as part of the expectations for all students. The practice standards suggest mathematical "habits of mind" teachers should cultivate in their students about ways of thinking and doing mathematics. With the teacher as a facilitator and using the right questions, dynamic interactive technology can be an effective tool in providing opportunities for students to engage in tasks that make these practices central in reasoning and doing the mathematics.

Introduction

Until 2010, each state in the United States had its own version of what mathematics was important to teach in K-12 classrooms and its own ways of measuring whether students were meeting their standards. The consequences of this were vast: enormous text books designed to meet every standard from any state but providing teachers little direction for making the text applicable in their state; assessments varying from multiple choice questions focused on procedures to using college entrance examinations as high school exit examinations. This variability in standards and assessments resulted in very different benchmarks for quality performance - one state's "A" was another state's "D". Comparing student performance on state assessments to the results of the National Assessment of Educational Progress, given to randomly selected students across the United States, makes this clear. Overall, individual states' level of satisfactory performance for students has been consistently lower than that of NAEP but varies greatly from state to state; for example in 2009, the difference in the percent of eighth grade (age 13-14) students who achieved a basic level of proficiency on the two tests was 11% in Massachusetts (43% to 32%) and 45% in New York (71% to 26%).

Deeming these results unacceptable and responding to the poor showing of United States students on TIMMS and the Programme for International Student Assessment (PISA), the Council of Chief State School Officers, the directors of education in each of the states, commissioned a set of national standards, the Common Core State Standards in Mathematics (CCSS), written by a small team of mathematicians and educators and released in 2010. As of April 2011, the standards were accepted by 42 of the 50 states. In addition to a set of content standards, one central and potentially significant component of the CCSS is a set of standards that focus on the mathematical "habits of mind" teachers should cultivate in their students about ways of thinking and doing mathematics, beginning in primary grades and continuing throughout secondary school. If implemented, these standards have the potential to significantly change how teachers in the United States approach teaching mathematics and what students take from their mathematics education. The eight practices are:

MP1: Make sense of problems and persevere in solving them
MP2: Reason abstractly and quantitatively
MP3: Construct viable arguments and critique the reasoning of others
MP4: Model with mathematics

MP5: Use appropriate tools strategically
MP6: Attend to precision
MP7: Look for and make use of structure
MP8: Look for and express regularity in repeated reasoning

Elaborations of the practice standards can be found at www.corestandards.org/assets/CCSSI_Math%20Standards.pdf. While the practices are part of the standards in the United States, in essence they seem universal. Cuoco and colleagues (1996) claim that the central aim of mathematics instruction should be to give students mental habits that allow them to develop a repertoire of general ways of thinking, strategies and tools they will need to use and understand in order to do mathematics and approaches that can be applied in many different situations. The mathematical practices are an attempt to make this visible to teachers – to call attention to ways of reasoning and sense making in mathematics.

The focus of this paper is to suggest how the practices can be used to leverage deeper understanding of core mathematical content by elaborating on two examples, one from calculus and one from mathematics. The discussion below describes some of the research about learning and how the use of dynamic interactive technology can support the implementation of the examples in classrooms.

Research and Dynamic Interactive Technology

Research suggests learning takes place when students engage in concrete experiences, observe reflectively, develop abstract conceptualizations based upon the reflection, and actively experiment or test the abstraction (Zull, 2002). The kind of learning that supports the transfer of concepts and ways of thinking occurs when students are actively involved in choosing and evaluating strategies, considering assumptions, and receiving feedback. They should encounter contrasting cases, noticing new features and identifying important ones (NRC, 1999). Dynamic interactive technology provides an environment in which these kinds of learning opportunities can take place. They allow students to deliberately take a mathematically meaningful action on a mathematical object, observe the consequences of that action, and reflect on the mathematical implications of those consequences, an "Action/Consequence" principle (Dick & Burrill, 2008).

In such environments, the task and role of the teacher are central as interactive technology alone is not sufficient for students to learn. Research about effective use of interactive applets in learning statistical concepts suggests teachers should engage students in activities that help them confront their misconceptions and provide them with feedback (delMas, Garfield, & Chance, 1999). Students' work needs to be both structured and unstructured to maximize learning opportunities; even a well-designed simulation is unlikely to be an effective teaching tool unless students' interaction with it is carefully structured (Lane & Peres 2006). In addition, students need to discuss observations after an activity to focus on important observations, become aware of missed observations, and reflect on how important observations are connected (Chance et al, 2007).

The following suggests how the mathematical practices might be realized in a calculus task. (Note that the figures were produced on a TI Nspire; could also be reproduced in any applet.)

Example 1: The Urn

Students are given an interactive file displaying an urn that can be filled with water using a clicker. A side panel displays a graph of the height vs. the volume as the urn is filled (Figure 1). To help students make sense of the problem and become familiar with the context (MP1: make sense of problems), they are asked questions such as, "Will the graph ever be a straight line? Why or why not?"; "If the urn were in the shape of a cone, what do you think the graph would look like? Why?"; "How would the graph of the height vs. the volume for a cone differ from the graph for an inverted cone?" Where is

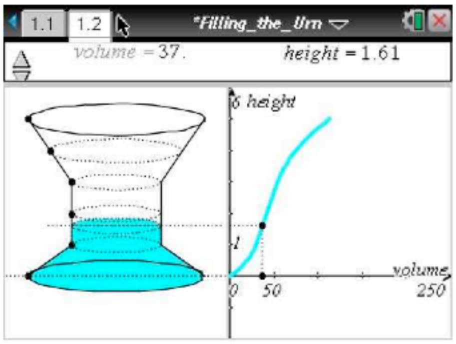

Figure 1 Filling the urn

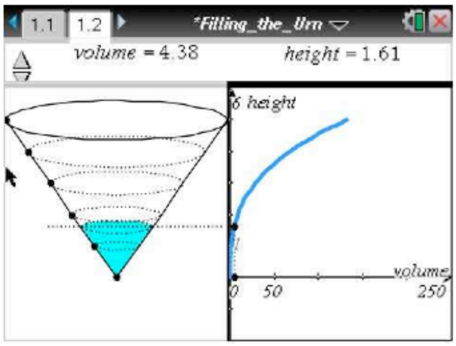

Figure 2 Filling a cone

the graph of height vs. volume the steepest? Explain your thinking." Once students have considered how the shape of the urn might affect the volume and made some conjectures (MP2: Reasoning), they "fill" the urn with water and check their reasoning. Students can be given time to experiment with different shapes and by describing them to their peers, can strengthen their understanding and ability to talk about the critical points of a function and the characteristics of its first and second derivatives.

Evidence from an analysis of student assessments in calculus indicated that many students struggle with several core calculus concepts. One of these seems to be the relationship among the characteristics of a function, its derivative and its second derivative, in particular with respect to multiple roots. The problem below, answered correctly by 28% of the students including 55% of those who earned top marks, is typical (College Board, 2003).

If $f''(x) = x(x+1)(x-2)$, then the graph of *f* has inflection points when $x =$
A) -1 only B) 2 only C) -1 and 0 only D) -1 and 2 only E) -1 0, and 2 only

To develop and support understanding, questions such as the following can be posed focus students on "contrasting cases, noticing new features and identifying important ones" (NRC, 1999): "What shape will produce a graph that has two points of inflection? Is it possible to have more than one shape that will produce such a graph?"; "What shape, if any, will produce a graph that is concave up with one point if inflection?"; "Predict the shape of the graph given an urn that is shaped like an hourglass, describe its characteristics, and defend your reasoning."

To provide students with feedback from both teachers and peers, a central feature of formative assessment (Black et al, 2004), classroom discussions should engage students in defending their solutions and sharing their graphs, preferably using a display that allows screen captures of student handhelds or computer screens (for example, the TI

Navigator). Students can be called on to look for similarities and differences in solutions, explain how another student might have been thinking about the problem, or decide which graphs are appropriate for a given question and why. The discussion can provide opportunities for students to engage in a variety of mathematical practices including:

- MP1: using visual representations to solve a problem; explain correspondences between tables and graphs; comparing approaches
- MP2: Stop and think about what the symbols represent in context
- MP3: Make conjectures; distinguish correct reasoning from that which is flawed
- MP4: interpret mathematical results in the context of the situation and reflect on whether the results make sense; identify important quantities in a practical situation
- MP5: use technology to visualize the results of varying assumptions, explore consequences, and compare predictions with data; use technological tools to explore and deepen understanding of concepts
- MP6: Communicate precisely to others.

Example 2: An Optimization Problem

An analysis of high stakes exit or end of course tests at the high school level (Dick & Burrill, 2009) suggests that students struggle with reasoning about graphs and about the mathematics. Problematic areas for students as opposed to experts include making connections among representations (Pierce, 2004; Stacey, 2005; Ramirez et al, 2005) and recognizing when to use certain techniques (NRC, 1999). The following task involves some of these elements: A sailboat has two masts, 5m and 12m. They are 24m apart. They must be secured to the same location using one continuous length of rigging. What is the least amount of rigging that can be used (figure 3)?

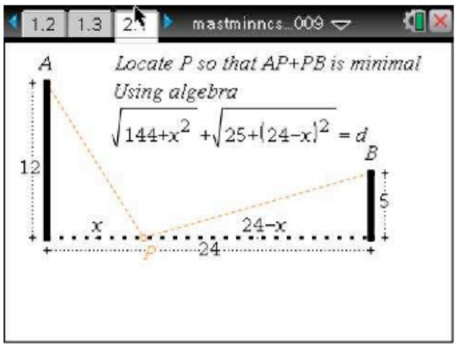

Figure 3 The Mast

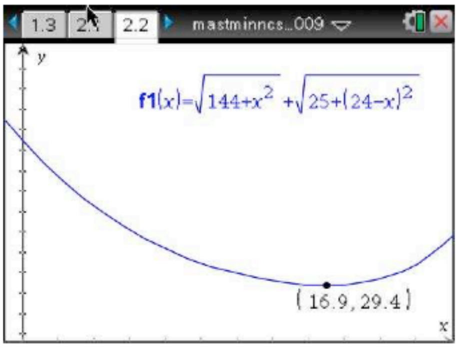

Figure 4 Using the Pythagorean Theorem

Students investigate the problem (MP1: making sense of the mathematics) by dragging point P making conjectures about possible locations for P that will minimize the amount of rigging. While different mathematical approaches to a solution are possible, fairly consistently in our work, students approach the solution algebraically using the Pythagorean theorem. Also fairly consistently, many of them graph the equation and trace to find a minimum point, which they suggest, without additional support for the claim, identifies the location of point P (figure 4). In less than half of the situations do students or teachers in workshops refer to calculus as a strategy to find the solution.

MP2 suggests mathematically proficient students should reason abstractly and quantitatively:

- make conjectures and build a logical progression of ideas
- determine domains to which an argument applies
- analyze situations by breaking them into cases
- recognize and use counterexamples
- compare effectiveness of two plausible arguments
- distinguish correct reasoning from that which is flawed and explain any flaws.

As students work through the strategies to find a solution, with the teacher as facilitator, they have the opportunity to engage in all of these mathematical practices. Graphing the equation results in a plausible suggestion for the location of P that will be the minimum but without further justification, it remains a conjecture. Students with a background in calculus should recognize the situation as one in which analyzing the derivative will yield critical points, which in conjunction with the second derivative, will lead to the solution. But it is also possible for students without a calculus background to reason from the context and implied domain for the function.

To reason about the situation means students have to recognize the difference between the *x-coordinate* of point P and the *distance* of point P from the first mast to the second (MP6: use clear definitions in discussion and in reasoning; state the meaning of symbols used), defining the total length to arrive at the same conclusion. The function defined by $L=\sqrt{x^2+25}+\sqrt{(24-x)^2+144}$ is constrained by the domain: $0 \le x \le 24$, which suggests that the length of the rigging is between the two extremes determined by the length of the deck and the length of a mast; the length is $\le 12+24=36$ and the length is $\ge$ $5+24=29$. Reasoning about the sum of the parts of the graphs of the two hyperbolas over the domain can lead to the conclusion that the sum from $5 \le x \le 24$ can have only one minimum, and thus, the one illustrated on the graph is that point.

The task can be done geometrically by reflecting one of the masts over the horizontal deck line and connecting the top of one mast to the bottom of the reflected one (figure 5). Point P lies at the intersection of this line and the deck, reasoning that the shortest distance between two points is a straight line and the two small triangles are congruent.

Configuring the problem on a grid, dynamic interactive geometry software led one student to examine the point of intersection of the two diagonals (figure 6). She

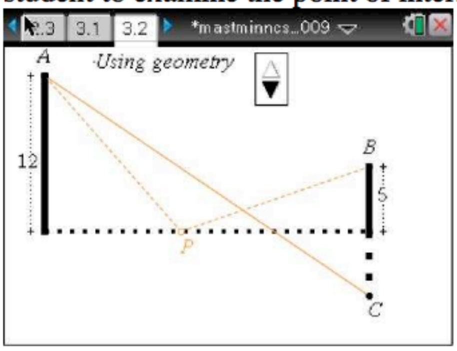

Figure 5 A transformation approach

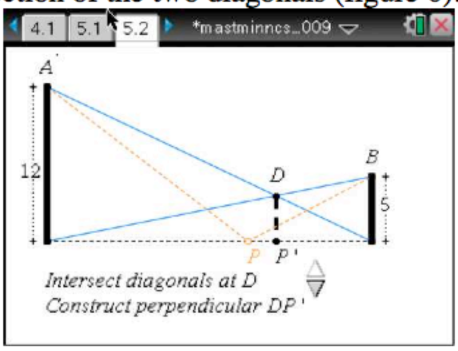

Figure 6 Diagonals

hypothesized that the *x*-value of the point of intersection might be that of point P, which has a nice proof that can be generalized (MP1: check answers by a different method; MP5: analyze graphs, functions and solutions generated by technology).

Conclusion

The two examples illustrate how mathematical "habits of mind" can emerge in a task, the way it is posed, and the way in which it is implemented, engaging students in thinking and reasoning about core mathematics. A task should be judged by the number of opportunities it affords to engage students in the mathematical practices. The role of the teacher is to frame the tasks and their implementation to maximize opportunities for learning. The questions teachers ask are central in bringing the practices – and ways of mathematical thinking and reasoning – to the foreground in the discussion. Technology is a tool that supports this work, providing students with opportunities to visualize the results of varying assumptions, explore the consequences, and compare predictions, explore and deepen understanding of concepts, and analyze graphs, functions and solutions generated by technology (MP5).

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Mathematics Teachers' Knowledge Growth in a Professional Learning Community

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Abstract
Professional learning communities are regarded as a viable teacher professional development model, but knowledge about how teachers learn in such communities is not yet well developed. This paper reports on how a conversation about ratio in a professional learning community of mathematics teachers supported the teachers' conceptual understanding of the concept. Features of a professional learning community that were found to support teacher learning include: diversity of opinion; challenging each others' ideas; voicing uncertainties; collective construction of meaning; and regarding learners' learning as the focus of the group's activities. The paper argues that professional learning communities that reflect these characteristics can support significant teacher learning.

Introduction
Contemporary teacher professional development models are mostly underpinned by the notion of professional learning communities. Professional learning communities are regarded as effective organizational structures in which teachers can learn and develop their instructional practices. Such communities enable teachers to learn from one another, and have the capacity to promote and sustain teachers' learning with the ultimate focus of improving learners' learning (Stoll & Louis, 2007). The effectiveness of professional learning communities in fostering teacher learning lies in ensuring that the nature of the collaboration can produce shared understandings, focus on problems of curriculum and instruction, and that they are of sufficient duration to ensure progressive gains in knowledge (Little, 1993). However not all professional learning communities manage to succeed in achieving these ideals. Little (1999) highlights the distinction between 'traditional communities' that coordinate to reinforce traditions, and 'teacher learning communities' where teachers collaborate to reinvent practice and share professional growth. Katz, et al. (2009) point out that "together can be worse", for example when collective activities, that may be well-intentioned, lack a clear needs-based focus and fail to address real teaching problems. These observations about the effectiveness of professional learning communities raise the need for more research-based knowledge about how professional learning communities foster real professional learning among teachers. In this paper I draw from an on-going research study to address the question: How do professional learning communities foster teachers' knowledge growth?

Learning in Professional Learning Communities
The conception of a professional learning community adopted in the study is that of 'a group of teachers sharing and critically interrogating their practice in an on-going, reflective, collaborative, inclusive, learning-oriented, growth-promoting way' (Stoll & Louis, 2007, p. 2). Learning in professional learning communities is conceptualized from a situative perspective, which regards learning as increased participation in communities of practice (Koellner-Clark & Borko, 2004; Lave & Wenger, 1991). Learning and participation are inseparable as they involve both the 'interpersonal and informational aspects of an activity' (Greeno & Gresalfi, 2008, p. 171). In professional learning communities, learning 'involves working together towards a

common understanding of concepts and practices' (Stoll & Louis, 2007, p. 3). Understanding how professional learning communities foster teacher learning therefore entails paying particular attention to the interactions within the professional learning communities, and how new understandings are created and negotiated. Such interactions and learning are enhanced if teachers view the professional learning community as a safe, non-threatening environment in which they can raise uncertainties or difficulties, with the confidence and trust that they will be supported and helped by others (Koellner-Clark & Borko, 2004). Open acknowledgement of what one does not know leads to possibilities for real new learning (Katz, et al., 2009).

Learning in a professional learning community is enhanced by: having a supportive leadership; collaborative inquiry that involves questioning and reflecting on practice (Jaworski, 2003b); and having a shared and clear learning focus that guides activities (Katz, et al., 2009). Such learning involves: challenging each others' assumptions about teaching and learning; discussion of new ideas that challenge existing knowledge; diversity of opinion; discussion of points of disagreement rather than of agreement; maintenance of the mathematical substance of the conversations; treating participants' ideas as objects of inquiry; and joint sense-making (Katz, et al., 2009). Analysing how these characteristics are manifested in the activities of a professional learning community helps determine the quality of the teacher learning that occurs. In what follows I use these concepts to show how a conversation among teachers helps them to develop a conceptual understanding of ratio and give meaning to an algorithm related to ratio.

Methodology

The qualitative case study extended over a period of nine months in 2010. Five mathematics teachers (Grades 7-10) from one school and the researcher (the author), met once a week for two hours at the school to work on activities which included: analysing the conceptual and skill demands of test items relating to ratio; interviewing learners about errors they made on the test items; analysing the errors made by the learners; planning and teaching lessons for dealing with learning needs based on the errors; and reflecting on the impact of those lessons. The activities were adopted from the Data Informed Practice Improvement Project (DIPIP), an on-going teacher professional development project based at Wits University. The researcher acted as a 'critical friend' in the group.

The activities described above supported conversations which sought to understand the errors made by learners on each test item with a view to collectively develop innovative lessons for dealing with observed errors and misconceptions. In this paper I focus on the conversations of the professional learning community in analysing learners' errors on the concept of ratio. I analyse how the conversations fostered the teachers' deeper understanding of the learners' errors, and their own subject content and pedagogical content knowledge about the concept of ratio. I also show how the teachers' developing confidence and trust in the professional learning community enabled them to voice some uncertainties; critically reflect on their knowledge about the concept of ratio; and jointly construct new understandings of the concept.

Findings: Developing Meaning for Ratio

The following two episodes drawn from the teachers' conversations illustrate some of the different ways in which the professional community led the teachers to a deeper understanding of some aspects of the concept of ratio, as well as addressing some of the uncertainties that they had. In the episodes the teachers' names are all pseudonyms, while 'Re' refers to the researcher.

Episode 1: Raising uncertainties in the group

In this episode the conversation was on the test item: 'Simplify the ratio 6:5'. In the conversation one teacher, Tsepo, raised his discomfort with the item. The following are some excerpts from the conversation:

Tsepo:	Don't you think that that ah giving learners questions like this is very much confusing because the learner will now try to work out this when it is already in its simplest form?
Re:	It depends on what we are testing. In this case I think we are testing understanding of a ratio and when a ratio is in its simplest form. But Tsepo is right, you have to think like a learner sometimes, because the moment you see simplify, in the mind you have to do something, and this is what some of the learners did.
Mandla:	And by doing that something they messing up everything.
Tsepo:	Ja, so they just have to leave it like that?
Re:	Yes, because it's already in its simplest form. How do they realize that, you can't have, six and five do not have a common factor.
Tsepo:	Uh because now the learner will think that maybe they have to give, I don't know out of how many marks is, is that, this question.
Karabo:	Two marks.
Tsepo:	So in the case like this we need to be careful with the marks also.
Karabo:	If they see the marks, they become scared the way I realize, wow, seven marks, it means I have got to work out here, whereas I am going to use a long method.
Tsepo:	I think there is a confusion with the total parts of a ratio, where you are given something like six as to five and then the total parts is six plus five which is eleven and then
Re:	Did they need to do that? They did not need to do that because they are not dividing a quantity into that ratio.

At this point the researcher pointed out that this item was testing more of the conceptual understanding of a ratio than the method of dividing a quantity in a given ratio, and marks allocated were not an issue. There was consensus that the learners who did some working as shown in the errors did not understand when a ratio is in its simplest form. It also became evident that Tsepo had a conception of ratio that was limited to dividing some quantity in a given ratio, but this became more apparent in subsequent conversations (see Episode 2 below).

In the episode Tsepo openly raised his discomfort with a test item, which could be an indication of his developing confidence and trust in the professional learning community. His public voicing of the discomfort initiated a conversation that supported an understanding that some items could test conceptual understanding only and did not necessarily involve some working or an algorithm.

Episode 2: Giving meaning to algorithms

The excerpts in this episode were drawn from an extended conversation in which the focus was on the conceptual meanings of a ratio and the algorithm for dividing a quantity in a given ratio. The conversation occurred after analyzing all the errors in the test and the professional learning community was now identifying what they thought were the critical concepts for the learners in order to inform the lesson plans.

Karabo:	Ratio is the problem.
Re:	Karabo thinks ratio is a big issue.
Kholiwe:	It's a big issue when they move to grade ten, but when they are in grade nine
Re:	What do you mean, can you explain?
Kholiwe:	They understand when they are in grade nine but when they move to grade ten
Re:	What is it that they understand about ratios in grade nine, what is a ratio? I think lets talk about this, what is a ratio?
Tsepo:	Is dividing in parts, ratio is a total. The total parts of a whole.
Kholiwe:	If you say, if maybe we say you are three, and I give you twenty rand to share it
Re:	I think that's an application of ratio in a sharing context, but what is ratio?
Karabo:	We are sharing, it's a share.
Kholiwe:	It's a way of sharing
Re:	I beg to differ when you say it's sharing because we also talk of ratio, for example what is the ratio of men to women, are we sharing?
Kholiwe:	*Laughs*. You are sharing but not in equal parts.
Mandla:	You are distributing.
Re:	What is the ratio of men to women among the educators in this school?
Kholiwe:	Ten to fifteen.
Re:	What are we doing?
Tsepo:	We are comparing.
Re:	Yes, ratio is a way of comparing quantities.

The episode shows that the teachers initially understood ratio as a way of sharing, with Tsepo specifically alluding to the algorithm for dividing quantities in a given ratio. The researcher then explained that we use ratio as a way of comparison without necessarily knowing the size of the whole, for example using a ratio of 1 teacher to 35 learners to compare teachers to learners without necessarily talking about how many teachers or learners there are altogether.
The researcher then raised the issue of dividing a given quantity in a given ratio. From the transcript below it was evident that the teachers were familiar with the algorithm for dividing a quantity in a given ratio, but could not articulate a conceptual understanding of the meaning behind the algorithm nor of the concept of ratio. Excerpts from the ensuing conversation illustrate how a conceptual understanding of the algorithm was developed.

Re:	If you are dividing a quantity in a given ratio what basically are you doing? For example divide eight hundred in the ratio five to three, why do they need to add five and three?
Tsepo:	They need to get the total.
Re:	Why, what is the meaning of that total? If you were to do it physically what would you do?
Tsepo:	The total is the meaning of a whole; the whole consists of eight parts, five and three.

Mandla:	Are you asking for the process? This is what is not clear.
Re:	Yes. If we can explain that process, that gives meaning to five as to three in a sharing context, remember we are trying to give meaning to the algorithms that we use.
Tsepo:	It doesn't mean to bring them to their equal sizes? Ja, now we are saying in terms of the denominators when we are multiplying
Re:	Remember my question, how would you do it physically?
Mandla:	I will say this one you are five, you are three, and then I give you your five hundred and you get your three hundred *(laughter from everybody)*

Mandla could not explain how or why he would do that. The researcher then explained that sharing in the ratio 5:3 meant that for every 5 given to one person, 3 is given to the other one. Using this understanding the teachers were able to work out that each time a total of 8 is given out and the process would need to be repeated 100 times in order to exhaust the 800, and one person would get 500 while the other would get 300. This led to the following remarks:

Mandla:	Mister, I can start tomorrow, delivering to the learners *(Laughter from everybody)*
Kholiwe:	But this is a nice way of teaching.
Mandla:	No, I am going to apply the very same way.

The episode shows how, in the conversation, the teachers' algorithmic understanding of ratio shifted to a more conceptual understanding. The teachers' initial understandings about ratio were used as objects of inquiry in the conversation to develop a conceptual meaning of ratio and the algorithm for dividing a quantity in a given ratio. The conversation shows how challenging the teachers to explain their initial understandings about ratio facilitated collaborative reflection on their knowledge. The conversation also illuminates how the teachers' participation supported joint sense-making in developing conceptual understanding of the concept of ratio. The two teachers' remarks at the end of the conversation could be indicators of shifts in understanding. The two teachers' reference to their teaching could be evidence of their developing confidence in teaching the topic as a result of shifts in their understanding. The remarks also allude to the teachers' conscious awareness of learners' learning as the focus of the activities.

Conclusion

From the conversations it was evident that the teachers' initial conceptions of ratio reflected a procedural understanding. This is consistent with research findings which highlight that mathematics teachers generally depend on algorithms and memorization of formulae, and do not offer conceptual explanations for those procedures (Zakaria & Zaini, 2009). Ratio as a mathematical topic is considered a difficult topic to teach and learn (Misilidou & Williams, 2002). Deepening teachers' understanding in such topics improves their subject content and pedagogical content knowledge (Shulman, 1987). The analysis shows that the teachers' conceptions of ratio shifted to a more conceptual understanding through their engagement in the conversations, and they were able to develop meaning for an algorithm. A conversation about learners' errors supported the teachers' deepening of their subject content knowledge and pedagogical content knowledge. The teachers developed the beginnings of a conceptual understanding of ratio and how to teach ratio meaningfully.

The results highlight a number of features of a professional learning community that can support teacher learning. The researcher challenged the teachers' initial understandings of ratio and these conceptions were treated as objects of inquiry in a conversation that deepened their understandings of the concept. Diversity of opinions contributed to the inquiry process in this community. The teachers' explanations and justifications of their different viewpoints supported their collective interrogation of their understanding of ratio, leading to joint sense-making and collective construction of the new understandings. The teachers' frequent reference to 'they' (learners) in the conversations could be an awareness of learners' learning as the focus of the activities. The teachers' freely expressed their uncertainties and understandings without feeling threatened, which could be an indicator of their developing confidence and trust in others. Professional learning communities which reflect these features, among others, have been found to result in shared understanding of concepts (Stoll & Louis, 2007). Although data analysis is on-going, from episodes like those cited in this paper I argue that professional learning communities as a model of professional development can support growth of teachers' professional knowledge.

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Using Online Textbooks and Homework Systems: In Particular MyMathLab and WebAssign

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Abstract
Major textbook companies are providing homework grading and testing services to educational institutions for a very nominal fee. These services can be used to provide the teacher with relief from long hours of tedium grading assignments, quizzes, and tests. They also give the student instant feedback on the correctness of their responses. The jury is still out as to whether or not these services actually improve learning.

Introduction

In an effort to meet the ever growing threat of digital pirating and illegal distribution, textbook companies are racing against time to find ways to market textbooks in a profitable way. Some of the larger textbook companies are producing extensive Internet services to try and lure teachers to adopt their books and to stay with them. These online services include e-texts, online homework programs as well as quizzes, tests, and examinations, online grade programs, and online communication options to use between teachers and students and between students and their classmates. I have used two of these services from competing companies, and this talk explores some of the features as well as frustrations I have encountered.

The first such service I tried, MyMathLab, I used in teaching hybrid classes in Elementary Algebra and College Algebra at Riverside Community College. Later I used WebAssign, a service from a different company, because of its online homework program for an Introduction to Linear Algebra at La Sierra University.

Common features of MyMathLab and WebAssign

One of the features of both MyMathLab and WebAssign is an e-text. If students are willing to use an e-text instead of a printed textbook, they can sign up for the program for approximately one-third of the cost of a printed textbook. If they choose to buy a new text, they can register for the service for a very nominal fee extra. If they have a textbook or buy a used textbook, then registration costs the same as for the e-text.

Another common feature it that the services include places for the teacher to upload syllabi and other resources for the course. There is also provision to display announcements the teacher—and the textbook company—want the student to see.

In both of these services students can do their homework, quizzes, tests, and exams online. The programs include algorithms so that each student's assignments are enough different that simply copying from another student is impossible. For example if one student has to solve the equation $3x + 4 = 7$, another student might be asked to solve $5x + 2 = 12$ for the same numbered problem.

What I did

At the community college I had the opportunity to teach both Elementary Algebra and College Algebra in an accelerated format. Accelerated meant teaching each class in 8 weeks rather than the usual 16 weeks using approximately 5 hours of lecture a week. Hybrid in theory meant I was to lecture on every second section in the text and require paper and pencil homework, and the students were expected to cover the other sections independently and submit homework electronically using the features of MyMathLab (Pearson). In actuality we found that briefly covering all of the course sections in the lecture proved better for most students. I taught each of these classes in three different semesters from August 2009 through December 2010 at Riverside Community College.

My full time work is teaching at La Sierra University. Recently, my department chair encouraged me to try using an online system for a class at LSU. Because of the multiple sections of each course offered at the elementary levels, he did not want me experimenting at those levels. So from September to December of 2010 I chose to teach a non-accelerated Introduction to Linear Algebra course using WebAssign (Cengage) electronic homework. Naturally, comparing the two systems at such different levels is not fully possible. What I can do is give an overview of the major similarities and differences between the two.

Similarities between the systems

Exploring the similarities, I noted that in both programs students had the choice of using an e-text for the course. Another feature that is the same in both systems is the ability to submit homework electronically. Students can submit some or all of their homework electronically. Since the answers are all open ended, they can submit each question electronically and get immediate feedback on their answers. I typically allowed them to make five attempts at getting their answer correct. (The teacher has the option of setting how many attempts students are allowed.)

There are plusses and minuses for allowing multiple attempts. On the positive side, this encourages a student to stick with the question until he or she gets it right. Since the responses are open-ended, a particular response might not be in the format that the website was expecting. Multiple attempts mean students can experiment with getting their format to match the one the website is expecting. On the negative side, this allows students to simply guess at the right answer until they get it right.

Another similarity between the two systems was the computer enforced submission deadlines for assignments, which I found particularly helpful. While I might have a soft heart toward a particular student's hardships, the computer program enforces deadlines impartially. I set the due dates to correspond to the beginning of the next class. This meant that trying to finish the homework was not an excuse for being tardy.

Each system has a very limited palette or toolbar of symbols that allows users to submit, fractions, radicals, exponents, matrices, etc. Yet both are quite exacting about how answers are entered. This proved frustrating at times and became the reason why I allowed up to five attempts to submit an answer. The toolbar in WebAssign seemed a little bit more intuitive than the one in MyMathLab, although both provided enough tools so the student could write an appropriate answer (see Figure 1).

Figure 1. Tool bars for MyMathLab and WebAssign.

The MyMathLab toolbar is on the left and that of WebAssign is on the right. Each named bar on the WebAssign toolbar opened further mathematical symbols, as did the "More" button on the MyMathLab toolbar on the left.

In both systems students appreciated the immediate feedback. When given a choice, the vast majority of students opted for--and in fact begged for--the opportunity to submit their work electronically. On the other hand, they were often frustrated because the computer did not accept some equivalent forms of a correct answer. For example if the question asks for the equation of a line it might accept $y = 3/2\,x + 7$ but would not accept $y = 1.5\,x + 7$.

In terms of support, sales representatives from both companies were initially very helpful. Yet when problems arose or when we discovered errors in the program, it was almost totally impossible to get any response out of the textbook company. I finally I quit trying to phone or e-mail them because I could never get a person on the phone nor any response to my messages.

Both textbook companies advertised that the student could print out the portions of the textbook they needed. In practice however, these companies made it very difficult do anything more than simply view the text on the screen.

When I was lecturing on a portion of the book, I liked to project it onto a screen since most students did not possess a printed copy of the text, nor was there sufficient room in the classroom for all students to bring their laptops. However, it was impossible to enlarge the text to make it easily readable from the back of the classroom. I found this seriously restrictive. Another disadvantage connected to this is that in most of the classrooms where I used the computer, the screen covered a portion of the white board. This limited the amount of writing I could have in front of the students at any one time.

Related to the homework systems, both seemed to do much the same thing. They created individualized homework assignments and quizzes. They kept a grade book that was limited because one couldn't upload scores into it. On the other hand, one could download the scores from the online grade book into a spreadsheet such as Microsoft Excel.

Differences between MyMathLab (by Pearson) and WebAssign (by Cengage)

When looking at the differences between the two services, MyMathLab came across as cumbersome. The interface is uninspiring and very non-intuitive to me. I could easily "paint myself into a corner." For example, I would be working with the e-text, and then when I wanted to switch to the Homework/Test Manager. There was no way I could seem to get there short of going back almost to the login screen. Probably this is because everything is done with pop-ups, which I normally keep switched off. Yet an advantage it offered was that as a teacher I could easily see what the students see when they log in.

To me WebAssign seemed simpler to operate. It was not driven by pop-ups. I could also enlarge the font sufficiently so it could be read from anywhere in the classroom. However, in neither program can the view of the textbook be widened enough to see the full width of the page once it has been enlarged so that students can read it. Probably the most annoying feature with WebAssign was that the graph "paper" on which the student had to draw graphs was fixed for values of x and y between -11 and 11 (see Figure 2). But the random question generator had no such limitations. Therefore, if a graph was part of the answer, it would often be impossible for the student to get it correct. This is a feature I kept trying to bring to the attention of the sales rep or one of the tech's, to no avail.

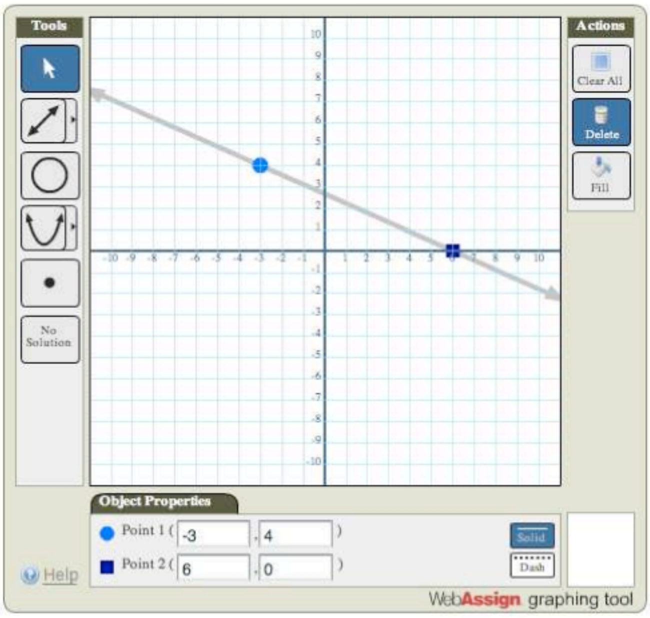

Figure 2. WebAssign graph template

Both programs gave hints as to how to do exercises. When I used it, the WebAssign version for my linear algebra text, however, must have still been in the programming stage because, after we got past the first chapter, the hints were non-existent. They would simply refer the student back to the beginning of that chapter in the text. MyMathLab, on the other hand, provided hints for completing exercises throughout the text.

Pedagogical Concerns

Looking back, I see that allowing students multiple attempts at getting the right answer may have been counterproductive. On tests, which were all pencil and paper, they had only one chance, of course, to get the correct answer.

While immediate feedback is very helpful for the students, I am concerned that it might encourage sloppy thinking. Rather than learning to actually critically evaluate their own thinking, they may only wait to see what the omniscient computer gives as a reply. They become willing to accept whatever the computer tells them as actually being true.

The literature I found about how well students learn using this type of service is divided in its evaluation. Some scholars indicate that, as far as the students are concerned, on-line homework does not hurt them. LaRose writes, "We find that students working homework on-line appear to do no worse in the course than those with pencil-and-paper homework, and may do better."[i] Allain and Williams concur: "Results show that there are no significant differences in conceptual understanding or test scores."[ii] Their study was conducted in a science class. Brewer also found little difference between traditional and online students: "The results of the study found that while the treatment group generally scored higher on the final exam, no significant difference existed between the mathematical achievement of the control and treatment groups."[iii] In a study conducted on "web-based versus paper-based homework" done for physics classes, Demirci states, "In general, statistically no significant differences were found."[iv] Lenz[v], Bonham[vi], and Hauk and Sequalla[vii] also all weigh in on the side of there being no significant difference in performance between students who submit web-based homework and those who turn in paper-and-pencil homework.

On the other hand, some researchers found that the online students did not do as well. Moosavi, for example, states "Regardless of whether achievement is measured in terms of a single semester test, comprehensive final exam, course average, or test performance across the semester the results presented here indicate that students perform better in traditional classes than in CAI classes regardless of the CAI curriculum used."[viii] Furthermore, he goes on to indicate that students using Thinkwell Computer Aided Instruction (CAI) did better than those using MyMathLab.

One would be surprised, of course, if there were no findings supporting online homework. Affouf finds a strong correlation between achievement on the web-based homework assignments and achievement in the final examination.[ix] Mendicino likewise reports that with a "group of 28 students, students learned significantly more when given computer feedback than when doing traditional paper-and-pencil homework."[x]

So the jury is still out as to whether or not there is any real benefit to the students in using online homework systems. Why then, should we spend the extra time it takes to set up the system for our courses? The answer is because of the much larger amount of time, especially in subsequent courses, we as teachers can save when it comes to grading homework and quizzes.

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Hearing the teacher voice: teachers' views of their needs for professional development

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Abstract
Research literature (Little, 1993; Goodall et al, 2005; Joubert and Sutherland, 2008) suggests that for professional development (PD) to be effective the aims of the PD should fit the perceived needs of the teacher. This paper reports on a study, which investigated teachers' views of their needs for PD and how these needs were being addressed when partaking in PD. Data consisted of responses to semi-structured interviews with 38 teachers who were involved in different types of PD initiatives. The data was analysed using a process of constant comparison (grounded theory). The analysis offers descriptive categories of teachers' perceived needs for PD which seem to resonate with Perry's (1999) descriptions of learner development. The findings corroborate a model of teacher change as growth or learning where teachers are themselves learners who work in a learning community and support models of empowerment for professional development, not models of deficiency (Clarke and Hollingsworth, 2002).
Keywords: teachers' needs, professional development, teacher voice

Introduction
Several authors in the research literature (Little, 1993; Goodall, 2005; Joubert and Sutherland, 2008) have reported that for PD to be effective the aims of the PD should fit the perceived needs of the teacher. Joubert and Sutherland (2008) report in their review of recent literature on professional development of mathematics teachers that formal professional development can arise from changes in government policy and innovation, from identified student-performance and from under-qualification of teachers. This suggests a deficiency of skills and knowledge leading to the adaptation of a deficiency model of PD with the needs of the teachers perceived as needs for prescribed knowledge and skills. At the same time, research indicates that PD initiatives based on deficiency models are ineffective (for example Clarke and Hollingworth, 2002; Guskey 1986) questioning the rationale for supporting such models of PD.

The study
The study reported on in this paper was part of the Researching Effective CPD in Mathematics Education (RECME) project, a large research project funded by the National Centre for Excellence in the Teaching of Mathematics (NCETM) and explored factors of effective PD for mathematics teachers. It was a short term (15 months) non-interventionist project investigating 30 ongoing PD initiatives representing different models of PD for teachers of mathematics in England. Overall, about 250 teachers in pre-primary, primary, secondary, further and adult education settings were involved in these initiatives. The project adopted the theoretical framework that all human activity, including the learning of teachers, is historically, socially, culturally and temporally situated (Vygotsky 1978). This suggests that the experiences and contexts of teachers will have a major influence on their learning and implies a need to pay attention not only to the situation, the opportunities and the context of sites of learning (in this case initiatives of professional development), but also to the individuals taking part in professional

development. Importantly, the philosophical underpinning of the project was one of co-constructing meaning with teachers, researchers and other stakeholders. The data we obtained for the study we report on in this paper reflects this and contains self-reported data from teachers. Details and descriptions of the full data set, the research design and case studies can be found in the final RECME report and other publications (NCETM 2009; De Geest *et al* 2008).
Joubert and Sutherland (2008) report in their review that the voice of the teacher is under-represented in most research on PD of teachers. The RECME project offered us the opportunity to hear the teacher's voice on what they perceived to be their needs for professional development. We addressed the following research questions:

1. How do teachers perceive their needs to be addresses in the PD they are involved in?
2. Are there common characteristics in their perceived needs, independent of the model of PD?

The study was conducted using a constant comparison approach from grounded theory looked at from a social constructivist theoretical perspective (Vygotsky 1978). Insights into the perceived needs are given through descriptive categorisation of empirical evidence. Data consisted of 38 responses from interviews with case study teachers representing the participating PD initiatives in the RECME study. We asked the questions: "Does this PD in which you are currently involved address your needs? Could you please explain why?"

Findings
Of the 38 teachers, two teachers replied 'no', four said 'partly' and 32 said 'yes' to the question "Does this PD address your needs? Could you please explain why?"

Some of the interviewed teachers voiced that they were not sure what their needs were, or made a distinction between professional and personal needs, for example:
"In some ways the PD addresses my needs in that I like meeting up with others in the field. However, I do not feel that I have direct needs concerning my immediate work and I am not looking to the PD to fill any particular needs at the moment. However, applying standards unit principles to Key Skills does have a useful role in validating my ongoing classroom practices in a college environment where learner training is sometimes seen as more important than educating – the network is also an excellent place to get support if needed"
"Yes – my NQT targets were all in maths as area for development. Ironically this is where I have made the most success. All my professional targets relate to raising attainment in mathematics with specific improvement grades for my children. My personal need for PD is that I want to be inspired. I feel a bit like a groupie when involved in all this academic debate – I am hanging on to every word. I find the talk so interesting. "

The PD addressed the needs of the teacher
Teachers reported factors or a combination of factors of how the PD *had* addressed their needs (n=32). Through constant comparison of the responses of the teachers who considered their PD to address their needs, we identified six descriptive categories:

1. The teachers felt enabled to respond, and at times find solutions, to issues they had identified as problematic, or not knowing how to do address, in their classroom practice. For example:
"It was clear that there was also a need for a good assessment system and for recording assessment for learning and to see where support is needed on a day to day basis and that has come from [the PD initiative] as well"
2. The PD made them *think* at a high level, had *challenged* them which had made it interesting. For example:
"It seems a long time since I last engaged in research at this level – reading about what current thinking is in maths. It is interesting. [the PD organiser] fed a few articles that started me going"
3. The PD made them look afresh at things and had inspired them with new ideas. For example:
"Really inspired to do lots of things] "
"It challenges me, makes me think, makes me look afresh"
4. They had been able to follow/satisfy their own interests within the PD. For example:
"Yes we have the opportunity to say if there are things we'd like to do so we can shape it. I think it's invaluable"
"I am really pursuing my own interests"
5. They felt strengthened in their views, their opinions, their thinking because the input into the PD had been theirs, based in their needs. Some teachers reported this had made them feel stronger, enabled them to argue their views better. For example:
"Yes because we run it ourselves and it follows the needs of the group like the problem solving" "The more we've had to stand up for ourselves the more we feel we're not alone"
6. They knew how to put the theory of their professional development into their classrooms by having concrete examples and practical skills, and had experienced positive responses from the students. For example:
"Yes because you are seeing someone work in a practical context and seeing someone work in your class and that is amazing. Because a lot of training you have is theoretical rather than practical. We all want ideas that you can apply straight away and that's what [the PD organizer] provides really."
"Yes exactly so you are using that as well as in action so you are using the theory and you know how to apply it because you have been shown that".

The PD *partly* addressed the needs of the teacher
The teachers (n=4) who said the PD was only *partly* addressing their needs lacked some of the factors identified above, and these missing factors were considered *important* by these teachers. They included some, but not necessarily all of:
The teachers could not see a pragmatic use of what was learned for the teacher's specific classroom in *some* parts of the PD, and as a result had no interest in that aspect of the PD. This referred to the learning of subject knowledge. For example:
"Some of the things we don't even touch on well I can't really use any of this and I'm not interested in learning that. It depends how interested in maths you are yourself. A lot of us just wanted to find different ways of presenting things to children and resources and so on. Some of the earlier stuff like the algebra we couldn't really use with our children"

The teachers acknowledged a personal enriching of new pedagogies and tools for teaching, but experienced the mathematics learned as too advanced for him/her. The PD also did create an awareness of what alternatives might have worked better. Again, this concerned PD on subject knowledge. For example:
"It has certainly given me lots of tools and allowed me to consider different strategies for teaching. I was not confident going in, I maybe could have used a refresher course in GCSE maths. I know you can do level 3 numeracy courses and they are more on personal skills, so I wondered whether that would have been good for me to do that first. Then do this higher level course …I think it is just confidence for me, I know that for some I can remember how to do it, but for some I was reading it before the lesson trying to work out how do I re-arrange the formulae. That is not always the best position to be in"
One teacher considered the PD very useful, but felt nothing new was learned because he/she had worked on the same issues at university the year before:
"I think it has been very useful but I think, as I am new out of university quite recently I think it backs up what I have already learnt at university".
One teacher was missing seeing what was worked on at the PD in a pragmatic setting of the classroom:
"I need to see other people using the resources and have the chance to speak with some of the kids about how they experience it or would they rather have the book back?"

The PD did *not* address the needs of the teacher
The teachers (n=2) who said the PD was *not* addressing their needs lacked some of the factors identified, and these missing factors were considered *crucial* by these teachers:
One teacher reported that the course on subject knowledge did "its best to cater for what they saw as our needs", however did not address the teacher's need because the teacher could not perceive a pragmatic use of what was learned in the classroom.
"I am teacher of less able children and I'm doing proof theory but practically it is of no use to me at all. Maybe something to go on for the hands on, nitty gritty, in the classroom ideas and resources. Whereas modulo arithmetic is of less use".
The other teacher acknowledged that she liked the discussions that were taking place during the PD, however she personally would like to do a higher degree
"I am thinking of doing something else – maybe a doctorate. We've got, what I like is having a discussion about the mathematics and we have two good NQTs and I can have really good discussions"

Discussion and conclusion
The needs the teachers perceived and reported as being met in the PD seem to concern one or more of: finding answers and solutions to issues they had identified as problematic, made to think at a high level, being inspired with new ideas, being able to follow their own interests, feeling strengthened in their thinking and views on teaching and learning, being able to put theory into practice. The teachers who felt the PD had met their needs only partly or not at all identified a shortcoming in one or more of the above factors.
Several researchers consider the professional development of teachers as professional learning, and see the process of developing professionally similar to a process of learning (Eraut, 2007; Franke et al.,1998; Clark and Hollingworth, 2002).

What struck us is how the descriptive categories of this study seem to resonate with ideas of learner development as described by Perry (1999/1968) confirming views that PD is similar to the process of learning and development. Perry developed a framework for characterizing student development at university level.

Our findings seem to resonate with his ideas of student development in terms of self-trust and commitment. To think, be challenged, be inspired and find solutions to problems, teachers are exposed to views and interpretations, which can new, different or a re-phrasing of existing views, allowing for learning. This resonates Perry's description of his 'relativism' phase, when there is *"a plurability of points of view, interpretations, frames of reference, value systems, and contingencies in which the structural properties of contexts and forms allow of various sorts of analysis, comparison, and evaluation of multiplicity"* [Perry, in glossary]. The views of the teachers in our study were strengthened by following their own needs and interests, using their input, and being able to apply what was learned into their own classrooms. Their own personal values and choices were affirmed.

This study wanted to hear the teacher voice on what they perceived to be their needs for professional development. Findings and analysis suggest their needs are similar to those in adult learner development. We suggest professional development should not be based on a model of deficiency seeking to address perceived inadequacies but on a model of professional growth, which builds on existing knowledge, and expertise of the practitioner.

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Using A Computer Pen to Investigate Students' Use of Metacognition during Mathematical Problem-Solving

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Abstract

The major aim of this paper is to present the computerized Smartpen as a tool for capturing and exploring students' metacognitive processes as they solve mathematical problems. After sharing their thinking through self-talk or group-talk students worked with others to share their strategies and reflect upon the process. An additional aim for this paper is to share any generalizations that may be helpful for teachers who are helping students strengthen their metacognitive and mathematical problem-solving skills. In this study the Smartpen was used to listen in on undergraduate preservice teachers' problem solving as they explored the Problem of Points, a classic probability problem. Capturing each stroke of the pen as the students wrote while simultaneously engaged in self-/group-talk, each Smartpen session produced a pdf document and accompanying audio for further analysis. Problem solvers who lacked confidence in their problem-solving ability were more reserved in their self-talk and often solved problems unrecorded in advance to appear more successful later during the recorded problem-solving session. It would appear that direct, real-time monitoring by the teacher is needed to capture significant information about students' metacognitive reasoning.

Introduction

The more successful mathematical problem solvers are those who think more deeply *during* the problem-solving process and *after* they have achieved a solution (Pugalee, 2001; Teong, 2003). The term metacognition best characterizes this phenomenon (Legg & Locker Jr., 2009). When teachers attempt to help students become better problem-solvers by employing questioning strategies to tease out critical steps in their thinking verbal or written communication is often a barrier. Students find it difficult to articulate what they were thinking previously when attempting to solve a problem (Hennessey, 2003). The major aims of this paper are to present the Smartpen as a tool for capturing and exploring students' metacognitive processes as they solve mathematical problems and to share generalizations about students' mathematical processing of a classic probability problem known as the Problem of Points.

A computer Smartpen (hereafter simply referred to as a Smartpen) may be useful in helping students to think out-loud or engage in self-talk while they solve problems. The Smartpen stores the written strokes—the image of a student's writing in real-time—and any oral communication that occurs simultaneously during the recording session. Consequently, the communication barrier between the student and the teacher, or between the student and him/herself could become less of a problem. When a student's written work in solving a mathematical problem is completed while using a Smartpen, his/her metacognitive work can receive a boost as the student listens in on any self-talk or group-talk, monitors the solution as it unfolds, and reflects on the process to determine decisions that should be made. Teachers can listen, make observations, note strengths, and determine weaknesses to support intervention as needed.

This paper will share highlights of specific phenomena associated with problem solving for which the Smartpen may assist in monitoring and improving students' metacognitive and mathematical problem solving skills. Specifically we will share a brief introduction to metacognition and problem-solving then move to the context of the classic probability problem—the Problem of Points (Berlinghoff & Gouvêa, 2004). We will also share our findings and generalizations from the investigation of the problem-solving attempts of preservice teachers who worked on the Problem of Points.

Metacognition and Problem Solving: In A Nutshell
The process of thinking about one's own thinking takes place internally—in one's mind—yet the results should also be manifested externally to lead to productive problem solving and to support sharpening of this skill through personal reflection and external intervention by teachers, peers, and learned experiences (Hacker, 2009; Lee, Teo, & Bergin, 2009; Legg & Locker Jr., 2009; Ponnusamy, 2009; Pugalee, 2001; Steif, Lobue, Kara, & Fay, 2010). As early researchers highlighted the importance of metacognition for student learning four key components of the process were identified: (a) verbal reports as data (telling what you know); (b) executive control (directing/managing what you know); (c) self-regulation (monitoring and making decisions about how you will use what you know); and (d) other regulation (internalizing feedback from interactions with others and external experiences) (Brown, Bransford, Ferrara & Campione, 1983 as cited in McKeown & Beck, 2009).

Metacognition has actually been identified as an essential skill in reading and in problem solving (Lee et al., 2009). Problem solving is one of the mathematical process standards of the National Council of Teachers of Mathematics (2000) and an essential component of the Standards of Mathematical Practice of the new Common Core State Standards (2010) in the United States of America. Stronger problem solvers "start by explaining to themselves the meaning of a problem…analyze givens, constraints, relationships, and goals… make conjectures…monitor and evaluate their progress and change course if necessary" (Common Core State Standards Initiative, 2010, p. 6).

Students are often taught specific strategies for problem-solving such as Polya's four-step problem solving model: understand the problem; devise a plan; implement the plan; and looking back (Taylor & McDonald, 2006). However, actual next-steps in the problem-solving process are not easily tracked when students are solving complex problems. Thus the role of metacognition along with the associated skill of communication (NCTM, 2000) can be particularly helpful. Artz and Armour-Thomas (1992) recognized this connection between problem-solving, metacognition, and communication (e.g., thinking out-loud) when they suggested an eight-step problem solving process (with the predominant cognitive levels shown in parentheses): "read (cognitive); understand (metacognitive); analyze (metacognitive); explore (cognitive or metacognitive); plan (metacognitive); implement (cognitive or metacognitive); verify (cognitive or metacognitive); and watch and listen" (p. 142).

Probability and the Problem of Points
One of the main implications for instruction in probability is that students must be engaged in activity-based, experimental investigations to support their conceptual understanding. The prevalence of misconceptions with regards to probability and the persistence of those misconceptions despite traditional instruction are well-documented (Fischbein & Schnarch, 1997; Khazanov, 2005; Polaki, Lefoka, & Jones, 2000; Shaughnessy, 1992). Metacognitive processes, combined with effective instruction, should help to increase students' awareness and

understanding of their misconceptions as well as help to identify ways to overcome them (Fiscbhein & Schnarch, 1997).

Using the probabilistic thinking framework developed by Polaki, Lefoka, and Jones (2000) one might investigate students' understanding of probability with regards to five major constructs (sample space, probability of an event, probability comparisons, conditional probability, and independence) at four different levels, numbered from 1 to 4 (subjective, transitional, informal quantitative, and numeric), respectively. Students operating at the lower level (Level 1) have difficulty appropriately operating in probability situations with anything more than subjective judgments. However, on the high end of the scale (Level 4) where deeper metacognitive thought is applicable, students are able to identify a sample space, predict and assign numerical values for the probability of events, and engage in numerically-supported probability comparisons and generalizations with conditional as well as independent events. The Problem of Points was specifically chosen to engage students in this higher end of the framework.

Subjects

During the spring semesters of 2010 and 2011, convenient samples of preservice teachers preparing to teach children of ages 9-14, consented to participate in this study. The preservice teachers were enrolled in a combined mathematics history-technology course in a medium-sized university (approximately 16,000 students) in the Midwest (USA). The group included 42 students in their second to fourth year of undergraduate study: 23 in spring of 2010 (22 females, 1 male) and 19 in spring of 2011 (14 females, 5 males).

Method

During a four-week period of 12 hours of in-class instruction, with additional instructional assistance provided in an online learning environment, supported by Smartpen pencasts, students explored various probability problems in various multicultural and historical contexts. In 2010 a Pulse™ Smartpen was only distributed to groups of students when engaged in hands-on probability activities in class. However, the need to be better acquainted with the technology dictated distribution of a Smartpen to each student for at least eight weeks (including four weeks prior to the study). Practice engaging in self-talk while writing and doing mathematical problem solving was suggested in specific instructions to students to communicate their inner thoughts as if tutoring a peer or sharing their procedures with the classroom teacher.

The Problem of Points was posed in class (as shared in brief below).

> Our story begins around 1654 as the Chevalier de Méré, a wealthy gambler, requested that Blaise Pascal provide a mathematical validation for a specific dice problem. As Pascal collaborated with Fermat on the solution the study of probability as a field of mathematics began! … Actually there is some confusion about the actual dice problem that the Chevalier first proposed, so the problem we will explore is a simple version of one of his problems that became known as the **Problem of Points**.
>
> > Xavier and Yvon staked $10 each on a coin-tossing game. Each player tosses the coin in turn. If it lands heads up, the player tossing the coin gets a point; if not, the other player gets a point. The first player to get three points wins the $20. Now suppose the game has to be called off when Xavier has 2 points, Yvon has 1 point, and Xavier is about to toss the coin. What is a fair way to divide the $20? (Berlinghoff & Gouvêa, 2004, p. 207)

After giving some initial thought to understanding the problem [while engaging in self-talk] and determining a plan for a possible solution you will join a team of four to provide solutions to Question 1 taken directly from of our course text.

a. Let's call the interrupted-game case described there *(X2, Y1)*, meaning that Xavier has two points and Yvon has one. How many other cases are there? What are they?
b. Analyze the rest of the cases from part (a). (Hint: Only two others are "interesting;" you might begin by disposing of the rest quickly.)
c. State your answers for part (b) as fractions or percentages that apply to any amount of money at stake.
d. Suppose the game required 4 points to win. What unfinished case is analogous to the *(X2, Y1)* case of the 3-point game? Generalize your answer to describe cases that result in a $\frac{3}{4}$-to-$\frac{1}{4}$ division of stakes for an unfinished *n*-point game.

... Berlinghoff & Gouvêa (2004, p. 213)

After approximately 10 minutes, students moved into groups to share their individual perspectives about the problem and become informed or inform others about a possible solution. The instructor monitored the groups as they engaged in group-talk, with one member of the group serving as a recorder while writing notes using the Smartpen. Once consensus was reached each group member asked questions of the group as needed to build his/her confidence in sharing the solution to another group or to the entire class.

All pencasts (i.e., the oral recordings and the accompanying pdf documents) were uploaded to a special class email address for analysis by the instructor. A mixed qualitative-quantitative design was used to analyze the data gathered. A comparison of the means of the pre-/post-assessment on probability was conducted using a *t*-test (in 2010, but a post-assessment was not given in 2011). Constant comparative and content analysis qualitative methods were used to investigate the categories of responses regarding metacognitive thought and the alignment of comments with the probability framework. The findings are shared in brief here.

Results

Self-talk sessions indicated that additional targeted training would be needed to help students truly share the details of their thoughts as they solved a problem. Most students appeared to lack confidence in sharing a "work in progress," and preferred to share their more polished results in a recording *after* attempting to solve the problem one or more times without the pen. Students with better than average scores on the pre-assessment of probability content knowledge were more likely to share more details of their thinking process or engage in the verbal reports of telling with such statements as "What I did was...I assumed that… The approach I'm taking is… I came up with …I thought there were 3 coin tosses at first rather than 3 [points] to win. That's the way I understood it. … I always looked at what the next toss would be." However, students who shared such comments often engaged in self-talk much more than they wrote. (See Figure 1.) For example, one student shared her metacognitive thoughts quite eloquently for almost 6 minutes before writing anything with the pen!

Group conversations featured much better sharing of metacognitive thoughts as students explained their positions, questioned the thoughts of others, and contributed to the solution. Since each student had expectations for later sharing with the class, each student clearly wanted to be prepared and often sought to confirm their understanding of the problem and the solution more than once. For example, one student who shared her thinking quite well with the group often asked at interim steps: "Do you understand that?" Another student, who often responded affirmatively to those questions as she indicated that she understood, waited until the last step of the solution and asked again, "Is it [i.e., the goal] the first to get to 3, or is it three (3) rolls?"

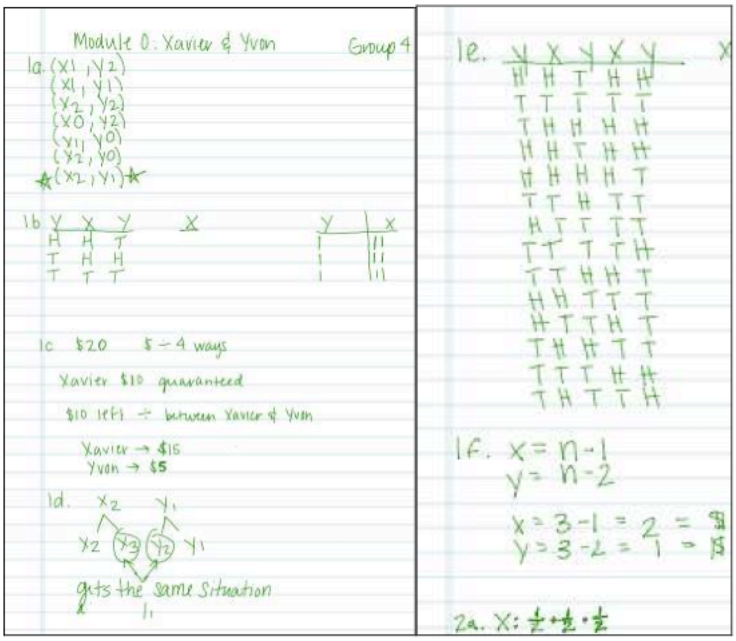

Figure 1. Two-page copy of pdf document resulting from Group 4's pencast.

Conclusion

It appears that using a Smartpen to support students' metacognitive thought while engaging in mathematical problem-solving may increase their awareness of metacognitive processes, but will do less to share it with others. Many of the students we encountered were often too self-conscious to share their unpolished thoughts. Perhaps they believe that the classroom teacher or the high achievers in class do not have moments of uncertainty during their attempts to solve mathematical problems. Perhaps we should be sure to have students share portions of their decision making process as they report their solutions to class. Of the four historical roots of metacognition, we believe we approached three: the verbal reports of telling, a hint of self-regulation, and metacognition that comes from interacting with other sources and experiences. However, most difficult to capture—and perhaps a source of greatest insight for helping novice problem solvers—was the executive control (directing/managing thoughts). Our informal experiences with classroom teachers, undergraduates, and high school students seems to indicate support our findings in this study: a pattern of solving problems "off the record" and sharing only best attempts with others. Most of the decisions about what to do and think next is still happening in the minds of students—virtually uncaptured, for the most part—because of fear of imperfection or appearing cognitively disheveled. We would like to suggest that students need many more experiences working in groups to share and defend solutions (through each step of the problem-solving process) to increase the awareness of the many moments of uncertainty during a problem-solving process that may eventually lead to a beautiful solution to a problem. To tap into the metacognitive thought that occurs throughout the problem-solving process we would suggest teachers or researchers sit and observe/question students while they are solving a problem and students should be encouraged to prepare to share a solution—along with accompanying decisions (executive control).

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Conceptualization – a necessity for effective learning of mathematics at school

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Abstract

This paper examined reasons why effective learning does not always materialize in mathematics and more specific in algebra at school level. In an attempt to identify possible reasons why effective learning evades learners a qualitative investigation was performed on students enrolled for mathematics education courses as well as on teachers furthering their studies in mathematics education. The outcomes were compared to possible reasons as portrayed in literature. In this paper the responses of the participants are discussed by analysing their responses to some of the questions posed to them.

Introduction

Teaching mathematics for effective learning was and still is a big challenge to mathematics teachers. Various reasons contribute to this phenomenon. It can be the way in which teachers execute their roles; it can be the confidence systems of learners based on their perspectives of mathematics; it can be the teaching methods used in the mathematics classroom; it can be misplaced outcomes; it can be the text books (incorrect content; way of unpacking the content; etc.) used to teach mathematics.

At first, effective learning and how it fits into the paradigm of social constructivism will be discussed. An investigation into the dichotomy between algorithms and heuristics; procedural knowledge and conceptual knowledge; inductive and deductive reasoning and concept definition and concept image will be address.

There is a saying that states that teachers must research their teaching and then teach what they have researched. This contribution can serve as an example of this saying.

Theoretical background

- **Effective learning**

De Corte and Weinert (1996) identified a series of characteristics of effective and meaningful learning processes which emerged from research that constitute building blocks that can serve as an educational learning theory. Those characteristics about which there is a rather broad consensus in the literature can be summarized in the following definition of learning:

> Learning is a constructive, cumulative, self-regulated, goal-directed, situated, collaborative, and individually different process of meaning construction and knowledge building (De Corte & Weinert 1996: 35-37).

The characteristics of this definition relates to the principles of constructivist teaching as discussed by Muijs and Reynolds (2005). It thus fits perfectly into the framework of reference of social constructivism.

Various authors (Cobb 1988; Hiebert & Wearne 1988; Nieuwoudt 1989; Schoenfeld 1988) stated that research has shown that teachers can formulate good goals, but despite of that there were still core problems that existed in the teaching of Mathematics at school. Learners are not seen by educators as constructors of their own knowledge; Learners cannot relate procedures of manipulating symbols with reality; Learners accept methods taught by educators without any criticism and apply it just like that; The over emphasizing of the answer; The teaching suppresses divergent thinking activities and creativity and problem-solving strategies are not established; are some of the core problems highlighted in the literature.

If this is true, then no effective learning took place if measured against the definition of effective learning by de Corte and Weinert. The dominant role of the teacher, the perspective of learners, misplaced objectives and the teaching methodology used to teach mathematics were identified by these authors as possible reasons that contributed to the existence of the mentioned core problems.

- **Algorithms and heuristics**

Researchers (Suydam 1980) distinguish between two methods of problem solving, namely the algorithmic and heuristic methods. An algorithm is defined as "...a recursive specification of a procedure by which a given type of problem can be solved in a finite number of mechanical steps" (Borowski & Borwein 1989:13). The aim of heuristic is to study the methods and rules of discovery and invention (Polya 1985:112-113). It is evident that self-discovery plays an important role in this method (Schultze 1982:44-45).

Heuristic methods are not rigid frameworks of fixed procedures which provide a guarantee for the obtaining of a solution. The purpose of value thereof lies mainly therein that you search purposefully and systematically for a solution (De Villiers 1986).

These two methods of problem solving clearly differ from one another. For instance, an algorithm ensures success if it is used correctly and also if the correct algorithm is selected and used. Algorithms are problem-specific, while the heuristic method is not problem-specific, because it is normally a combination of strategies. This leads to the fact that a heuristic method is applicable to all types of problems. A heuristic method provides the "road map", a blue print, which leads a person to the solution of a certain problem situation. In contrast to algorithms the heuristic method does not necessarily lead to immediate success (Krulik & Rudnick 1984).

It is important to note that, although the heuristic method could serve as guideline in the solution of relatively unknown problems, it cannot replace knowledge of subject content. Quite often the successful implementation of a heuristic strategy is based on the fixed foundations of subject-specific knowledge (Schoenfeld 1985).

The heuristic way of doing problem solving should play an ever-increasingly important role in the teaching learning situation where problem solving is the focus of teaching. Algorithms, on the other hand, form part of the subject content and are therefore also important. What is of cardinal importance, however, is that an algorithm should be part of the package of knowledge only after it was constructed in a heuristic manner.

Students will empower themselves if they are capable to apply a range of problem solving strategies when confronted with a problem (Schoenfeld 1988). The heuristic method should not be viewed as a goal in itself but must rather be seen as a way in which a certain goal is achieved. Drawing diagrams, for example, should not be taught as a unit in the mathematics classroom, but must rather be used to solve problems where applicable.

Groves and Stacey (1988) consider the strategies as important, especially at the beginning when actual problems are tackled. These strategies give the pupils a degree of control in the process of problem solving and it is important that they should be able to apply it spontaneously without being dependent on the teacher's support (see also Roux 2009).

- **Procedural and conceptual knowledge**

Students and even some teachers have a limited conceptual knowledge span of algebra and it was further found that there conceptual knowledge does not correlate with their procedural knowledge (O'Callaghan 1998; Hollar & Norwood 1999; Roux 2009). Procedural knowledge focuses on the development of skills and can it therefore be deduced that it relates more to the use and application of algorithms (O'Callaghan 1998). Conceptual knowledge on the other hand is characterised by knowledge that is rich in relationships between variables and also including the ability to convert between various forms of presenting functions, i.e. in table format or in graph format, etc. (Hiebert & Lefevre 1986). Conceptual knowledge lends it more to self discovery which relates more to the use of heuristics and inductive and deductive strategies.

Developmental, reinforcement, drill and practice as well as problem solving activities are generally used in the teaching and learning of mathematics (Troutman and Lichternberg 1995). Developmental and problem solving activities lends it more towards the development of conceptual knowledge whereas reinforcement and drill and practice activities lend it more towards the development of procedural knowledge. The advantage to first expose learners to developmental and problem solving activities is that they are challenged to develop conceptual knowledge before being exposed to procedural knowledge (Davis 2005).

RESEARCH QUESTIONS

The following research questions were investigated:

- Is the procedural knowledge which teachers use the outcome of conceptual knowledge?
- Is the procedural knowledge which student teachers use the outcome of conceptual knowledge?

RESEARCH METHODOLOGY

Qualitative research methods were used to find answers to the mentioned research questions. At first mathematics school textbooks and examination papers were analysed to determine to what extend the focus was placed on procedural and/or conceptual knowledge. A questionnaire consisting of mainly algebraic statements was designed based on these findings. These questionnaires were administered by the researcher. The target population consist of different groups of fourth year mathematics education students over the period 2005-2009 as well as practicing teachers who have enrolled for an advanced certificate in mathematics education and/or who have participated in mathematics education workshops (2005-2009). This was thus not a longitudinal study because each year different groups of students were involved in the study. The questionnaire consisted of ten questions. The respondents completed the questionnaire in class and it took them more or less 15 minutes to do so. The responses to each of the questions were either true or false. These responses were noted and the questionnaire was thereafter discussed and debated which contributed towards the reliability and validity of the questions posed in the questionnaire. During these discussions the researcher continually posed questions, obtained answers, and critiques the answers, to obtain a deeper understanding of the thought processes of the respondents.

RESULTS AND DISCUSSION

The outcome of two of the questions will briefly be discussed in the following paragraphs.

- Question 1: If $x^2 = 4$, then $x = 2$

Most of the respondents over the years indicated that $x = \pm 2$ should actually be the answer. The answer $x = 2$ was also indicated as the correct answer by quite a few participants. They arrived at this answer by substituting $x = 2$ into the equation and then found 4 as answer. It was also clear from the discussions that the procedure 'taking roots on both sides' was applied by most of the respondents in solving the equation. This procedure is also advocated by some text books. The participants did not realise that this procedure are actually lowering the grade of the equation from a quadratic equation to a linear equation and by doing that there can only be one answer, namely $x = 2$. The concept of solving $x^2 = 4$ was visualized by representing $y = x^2$(using excel) as a graph. The two values $x = \pm 2$ were identified as the solutions to the equation. The algebraic solution of the quadratic equation $x^2 = 4$ was discussed by solving the equation by means of factorization.

- Question 2: If $x^{\frac{2}{3}} = 4$, then $x = \pm 8$

Most of the respondents guessed the answer, but a few substituted $x = \pm 8$ into

$x^{\frac{2}{3}} = 4$ and concluded that the statement is true. In solving the equation algebraically – as is done in some text books - the statement was found to be true, but when represented as a graph, it was clear that $x = 8$ was the only solution. This once more was an indication that learners are in a framework of mind to follow procedures rather than conceptualise the problem. In question 1 they saw that the solving of the equation provide the solution and therefore argued that it must work in this instance as well. The solving of the equation does provide the solution if the restriction $x > 0$ is applied.

Over the mentioned period (2005-2009) only one student (mathematics on third year level) demonstrated a clear correlation between his procedural and conceptual knowledge. He applied algorithms where applicable but work heuristically when confronted with a situation that seemed unfamiliar to him. The majority of the students applied rules mechanically without reflecting on their answers. Rare evidence of conceptual knowledge was noted. It can thus be concluded that the students focused mainly on concept definitions and did not demonstrated concept images. Conceptualised knowledge was thus not the outcome of the application of procedural knowledge. The demonstration of subject content knowledge was not on a desired level and the same applied to their professional pedagogical knowledge.

The situation was even worse in the case of the teachers. Each group clearly demonstrated that they apply rules mechanically without applying problem solving strategies at all. They in other words did not work heuristically or inductively. A possible reason for this phenomenon could be that these teachers did not receive any training in mathematics at post grade 12 level. Their training in mathematical content was restricted to grade 12 level because they were all trained at Colleges of Education. Their mathematical factual knowledge was at a substandard. It is evident that they will not be able to apply their professional pedagogical knowledge in full when teaching mathematics due to the lack of mathematical content knowledge. It can thus also be concluded that conceptualised knowledge was not the outcome of the application of procedural knowledge for these groups of teachers.

CONCLUSION

Teachers and mathematics education students who participated in this research apply mainly algorithms when solving problems involving algebra. They work deductively and demonstrate procedural knowledge. It can be concluded that the same core problems discussed previously still exist in the teaching of mathematics and more particularly in the teaching of algebra.

It was evident that students and teachers who have participated in this endeavour have a limited conceptual knowledge span of algebra and it was further found that there conceptual knowledge is not in line with their procedural knowledge.

The questionnaire used in this investigation was not discussed in full in this written report because it is the intention to actively involve those who will attend this presentation by exposing them to the questionnaire. In the discussion of each of the questions the aspects as discussed above will be unpacked and debated.

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Meeting under the "Omei" Tree in the Torres Strait Islands: Networks and Funds of Knowledge of Mathematical Ideas

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This paper focuses on the turning point experiences that worked to transform the researcher during a preliminary consultation process to seek permission to conduct of a small pilot project on one Torres Strait Island. The project aimed to learn from parents how they support their children in their mathematics learning. Drawing on a community research design, a consultative meeting was held with one Torres Strait Islander community to discuss the possibility of piloting a small project that focused on working with parents and children to learn about early mathematics processes. Preliminary data indicated that parents use networks in their community. It highlighted the funds of knowledge of mathematics that exist in the community and which are used to teach their children. Such knowledges are situated within a community's unique histories, culture and the voices of the people. "Omei" tree means the Tree of Wisdom in the Island community.

Background

A recurring theme in government policy and literature on Indigenous parents engagement in education is the importance of parents actively engaging in their children's learning (Department of Education and Training, 2010; Lester, 2004). Today, an important shift is occurring. A critical factor in this shift has been the establishment of community network spaces in Aboriginal and Torres Strait Islander communities where this engagement can occur. Communally, this has resulted in knowledge and learning that is situated within community networks that can be used to support Indigenous parents and their children. Through networks Indigenous parents are wanting to develop new discourses about learning and engaging in education that is community-based but which is also linked to contemporary environments (see for example, O'Connor, 2009). They have used community spaces as a way of locating themselves in their community to learn the knowledge, languages and protocols of their culture.

Indigenous Networks

The idea of networks has not previously been examined extensively in the literature (see for example, McIntyre, McGaw & McGlew, 2008) but is important to investigate. This is because Indigenous parents and networking are interconnected and rely on networks. That is, Indigenous parents belong to community networks, and, in turn, community networks are comprised of parents (Ewing, 2009; Martin, 2007).

Community networks are not static homogenous entities. They reflect values, beliefs, as well as hopes and accomplishments, through the people that members associate and those who are avoided. They are complex and can be mobilizing forces for social justice and the redistribution of power and material advantage (Sefa Dei, 2005). They can exist in a range of combinations: 1) as spatial localised settings that are defined for the pursuit of socially meaningful interactions; 2) as affective and relational communities where members draw on bonds of affinity, and shared experiences of values, attitudes, beliefs, concerns and aspirations and; 3) as moral communities where participation and belonging in a citizenry work to achieve common goals defined as a collective good. Within these communities, tensions, struggles, ambiguities and contradictions are captured; however, the integrity of a collective membership is maintained. Local knowledges are nurtured and made relevant for daily life (Lahn, 2006; Sefa Dei, 2005).

Networks as Funds of Knowledge of Mathematics

Networks build capacity in Indigenous parents (Makuwira, 2007). They validate the parents' own definitions of maths as it exists in their communities—"funds of knowledges" that have been historically and culturally accumulated into a body of knowledge and skills essential for people's functioning and well-being (Moll, 1992, p. 133). The idea of funds of knowledge views that people are competent and have knowledge that has been grown and developed through their life experiences that have given them that knowledge. The claim in this paper is that through working with communities, such knowledge and competence leads to possibilities for Indigenous parents engaging in teaching and learning to support their children. A funds of knowledge approach provides a powerful and rich way to learn about communities in terms of their resources, competence, the wherewithal they possess and the way they utilise these resources to support them with the education of their children.

If funds of knowledge of mathematics are those that reflect the unique histories and culture of Indigenous communities, they provide an effective entry point into engagement in learning because it is connected with and situated in communities and the voices of the people. The process of learning then is owned and framed by the community and its people which then works to build a sense of pride and self worth in individuals (Khamaganova, 2005). Indigenous parents' identities are shaped in their distinct ways in a distinct physical space such as networks, with their knowledges, stories and relationships an integral part and tied to physical locations (Khamaganova, 2005). This relationship ensures that maths is emerging from communities, their networks and funds of knowledge because they are taught and learned in such contexts.

Methodology

The project adopted a qualitative design approach: community research (Smith, 1999). Community research is described as an approach that "conveys a much more intimate, human and self-defined space" (Smith, 1999, p. 127). It relies upon and validates the community's own definitions. As the project is informed by the social at a community level, it is described as "community action research or emancipatory research", that is, it seeks to demonstrate benefit to the community, making positive differences in the lives of one Torres Strait Islander community. The researcher established strong working relationships with the parents and community members over time. This paper focuses on the illuminative moments that worked to transform the researcher and provide turning point experiences that resulted in a cumulative sense of awareness before consent to conduct the project was given. Embarking on this preliminary process in close collaboration with the community was a challenge intellectually, linguistically and geographically.

A geographic excursion

The Torres Strait Islands consist of eighteen island and two Northern Peninsula Area communities (Torres Strait Regional Authority, 2010). They are geographically situated from the tip of Cape York north to the borders of Papua New Guinea and Indonesia and scattered over an area of 48, 000 square kilometres. There are five traditional island clusters in the Torres Strait: top western, western, central, eastern and inner islands. The research project was conducted at one site in the eastern cluster.

Participants

Twenty adults and eight children took part in the community consultation meeting. All live at the site where the meeting was held. For ethical reasons pseudonyms have been used to protect the identity of participants. The location is referred to as the Island.

Data collection

For the purposes of this paper, the data collection techniques included: digital photography, field notes and email documents. Digital photography as a non-written source of data allowed for the capturing of visual images that were central to the preliminary process and which served as a reminder for the researcher (Stringer, 2004). They also assist audiences to more clearly visualise settings and events. Field notes provide descriptions of places and events as they occur. They provide ongoing records of important elements of the setting and assist with reporting and reflecting back over events. Email documents provided an efficient and easy form of community between participants who then networked with their community. Each technique afforded the value of insight into the important preliminary planning of the project.

Analysis and discussion: learning about the community's networks and funds of knowledge

In recent years, building on what communities bring to particular contexts and on their strengths has been shown to be effective with engaging with communities (GonzÃ¡lez & Moll, 2002). How does this occur? A way to engage community was to draw them in with knowledge that was already familiar to them, and which then served as a basis for further discussion and learning. However, with this process there was the challenge and dilemma. How did the researcher know about the knowledge that they brought to the meeting without falling into stereotyping their cultural practices? How did the researcher address the dynamic process of the lived experiences of the community? The responses to these questions have emerged from community-based research that relies on the community's definitions.

The Community Meeting

Prior to the official commencement of the project, preliminary community meetings were held to discuss the project's intention. The process of networking within the community developed through several steps: a) discussion of the project with school campus leader, b) discussion with the Island Councillor and to seek permission to meet under the Omei Tree, c) a chance meeting with the Radio Announcer for the Island radio that resulted in an interview that was broadcast to the Island community, d) with support from one Senior Woman, Denise, and a parent from the community, a paper-based flyer was delivered in person to the homes of Island parents to inform them face-to-face about a proposed community meeting (Figure 1) and, d) community meeting held under the Omei Tree (Figure 2).

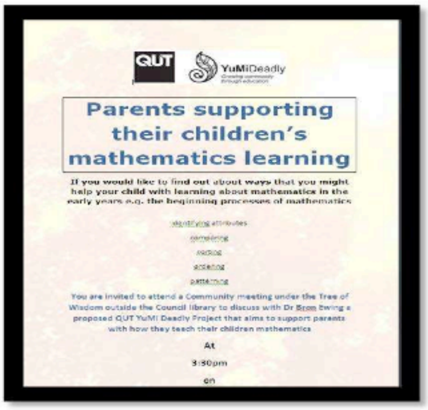

Figure 1: Proposed community meeting flyer

Figure 2: Community meeting held under the Omei Tree

The content of the flyer was brief and aimed to provide succinct information. The tree is a large fig tree believed to be over 100 years old and has been a significant meeting place for the Island community.

During the meeting the researcher explained the project and how participants might be involved. Gaining consent was respectful of the community's place and environment as also was that as a visitor, the researcher needed to be mindful of her actions and presence in the community and conduct herself in an ethical manner. She explained some of the early number ideas using shells, sticks, leaves, and poinciana pods gathered from the community. Verbal permission was sought from one senior woman, Julia, in the community to collect some shells from the beach front near her home to be used in the meeting.

As the meeting progressed, early number ideas were discussed, for example the beginning process of sorting or classification. Children learn to sort objects into sets, such as shells, from their environments into groups. They learn to identify sameness that defines the characteristics of groupings (Sperry Smith, 2009). In the meeting, Denise volunteered to sort shells into groups (see Figure 3). The larger community group and the researcher had to identify what criteria were used for the groupings.

The idea of creating and naming sets continues throughout life and is a way of creating and organising information and making connections with peoples' experiences. Before young children can learn to count sets, they begin the process of defining a collection using the objects in their daily lives (Baroody & Benson, 2001). Denise established the features of each of the sets of shells. If the criteria for membership to a set are vague, it is more challenging to decide whether the shells belong to a particular set. Meeting members talked amongst themselves, with the Denise allowing them time to identify the features of the sets.

Figure 3: Sorting shells into sets

The researcher could not identify the criteria that defined the sets; however, there was consensus amongst community members that criteria had been established—edible and non-edible creatures that live in the shells. In this example, the community used their daily lives and activities as an opportunity to talk about sorting using their home language—Creole, and English. When asked when children learn about edible and non-edible shells there was consensus that this occurs very young, for example, one to two years of age, and during times when families walk along the shores of the Island and when fishing or playing in the water. This example reinforces what Nakata (2007) and Moll (2002) state, that learning can be rich and purposeful when it is situated within that which already exists— the culture, community and home-language of the group.

As the meeting progressed, discussions lead by the researcher focused on an introduction to early algebra—patterning. Patterns are a way for people to recognise and organise their lives. In the early years, two particular pattern types are explored: repeating patterns and growing patterns. They are used to find generalisations within the elements themselves (Warren & Cooper, 2006). What comes next? Which part is repeating? Which part is missing? Repeating patterns are patterns where the core elements are repeated as the pattern extends. Young children recognise patterns when singing songs, when dancing, learning how to weave and when playing. Some examples of repeating patterns include:

ABABAB

AABAAB

Repeating patterns can be represented with actions: jump, hop, jump, hop; as sounds: bell ring, clap hands, bell ring, clap hands; as geometric shapes: triangle, square, triangle, square; and as feel: soft, hard, soft, hard. Generally children explore patterns in a sequence: copy a pattern, continue the pattern, identify the elements repeating, complete the pattern, translate the pattern to a different medium.

Using two poinciana pods, the researcher tapped them together to create a repeating sound pattern. The community was then asked to continue the pattern and identify the repeating elements, using clapping. They were then asked if they would like to offer a repeating pattern. One community member clapped a pattern to which the remaining members responded. The community were then asked where they might see or use patterns in their communities. Responses included: the seed pattern inside the poinciana pod (ABABAB), the weaving pattern used to weave coconut leaves together (ABABAB), when singing songs to the children, the seasons of the year and how the winds, seas, sea life, plant life, bird life work in a repeating cycle with many core elements. It was this final response that reminded the researcher of her place within the community, a visitor who had a great deal to learn from community about patterns and how they are evident in their natural environment. It also reminded the researcher that the community have extensive knowledge of patterns because they exist in their everyday lives in very rich ways—funds of knowledge, knowledge that researcher did not possess.

In concluding the meeting, the community were asked if they would like the researcher to plan and trial some early mathematics workshops with the parents and children. In doing so, she also stated that she had a lot to learn from community, their funds of knowledge—learning was to become two-ways for the researcher as well as the community. Of importance was that the community needed time to network and discuss whether they wanted the researcher to return and work with parents and children on the Island.

Conclusion

As outlined in this paper, the process of networks within the Island community has been significant for the researcher's engagement with the community and to respond to research as described by Smith (1999). The researcher found this imperative to be somewhat of a challenge because she had to rely on the community's networks which the researcher did not belong. However, what did become evident was that the networks comprise of parents and they use such networks for communication. What was also evident was the networks were used as a valuable source for nurturing local knowledges and how these are used in the Islanders' daily lives. At the meeting, the community validated their own definitions of knowledge—sorting and patterning. In doing so, this process provided a rich way to represent their knowledge, competence, and wherewithal that they possess to support them with educating their children.

At this stage, the researcher can report that after providing time for the community to network and consult about whether to permit her to conduct the parents and children mathematics workshops, the process has allowed her to:

1. Have confidence in the way that the researcher has consulted with community and the community with the researcher;
2. Continue to work with community;
3. Continue to work with the materials similar to those gathered for the meeting;

4. Frame the project's agenda that situates research as reflexive engagement with the real—the community's funds of knowledge as well as her own;
5. Understand how pluralism is about respect for diversity and a willingness to explore and change in ways that continues to remain diverse for situated learning.

These points relate to the processes of engagement and community consultation and research which are envisaged as continuing once the project officially commences.

Acknowledgment

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Problem solving: A psycho-pragmatic approach

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Abstract
The aim of this paper is to present an alternative approach to problem solving in general and mathematics in particular. Problem solving has become very prominent especially in the last two decades due to a number of reasons: the move from an industrial society to a knowledge society; globalisation; complexity in management systems; and technological innovations, to name just a few (Halpern, 1997; Pascarella & Terenzini, 1991; Bérubé & Nelson, 1995). An analysis of the various definitions and research indicates that problem solving: involves a number of cognitive processes (e.g. analysis, synthesis, comprehension and so on); it requires a certain mode of thinking for different problematic situations; and it is a skill that needs to be developed over a period of time (Green & Gillhooly, 2005:347; Halpern, 1997:219).

Behaviourism and cognitivism learning theories have been used in the teaching and learning situation but with mixed results. Pragmatism on the other hand has started 're-emerging" due to a number of reasons. The most important reason that was "rejected": Emphasis on the practical application of knowledge. Mathematics, unlike other subjects, by its nature is a subject whereby problem solving forms its essence but paradoxically is "accused" of being too abstract. However, there can be no mathematics without problem solving. A generic model is developed where by the psychological theories as well as pragmatism are used in the teaching and learning situation.

Introduction
As the world economy moved from an industrial economy to a knowledge economy, it can be argued that the nature of many problems also changed and new problems have arisen which may require a different approach to overcome them. Certain problems remained unchanged for the human being. For example, human beings that are born, they have to learn how to walk, run, speak and so on. Educational institutions and governments have recognised long ago the importance of problem solving and volumes of research have been written about problem solving (Pushkin, 2007; Astin, 1993; Halpern, 1997; Pascarella & Terenzini, 1991; National Education Goals Panel, 1991; South African Qualifications Authority (SAQA), 1998; National Council of Teachers of Mathematics [NCTM], 2005). Universities and other higher learning institutions are entrusted with the task of producing graduates that have such higher order thinking skills among other skills (Pushkin, 2007; Astin, 1993).
Theories about learning in general and problem solving in particular have also taken cognizance of these changes and old theories have been revisited and modified if necessary while new ones have come to existence. Behaviourism, cognitivism and their variations dominated and still dominate education where depending on the style of the teacher he or she uses more than one and less of the other. Since learning is about knowledge, the difference between the two theories lies on the fact that the one views learning as acquisition of knowledge while the other as knowledge construction. Both theories recognise that problem solving is essential for all learners (Johansen, 2003) especially complex problems (DeCorte,

Verchaffel & Masui, 2004; Gareth, Weiner & Lesgold, 1993 cited by Sigler & Talent-Runnels, 2006). However, the 'how' to teach and 'what to teach' to achieve this is still debatable.
Pragmatism on the other hand has started 're-emerging" due to a number of reasons. The most important one being the main reason that was "rejected": Emphasis on the practical application of knowledge. Mathematics, unlike other subjects, by its nature is a subject whereby problem solving forms its essence but paradoxically is "accused" of being too abstract. Howeve,r there can be no mathematics without problem solving. There is evidence (Luneta, 2008; Pushkin, 2007; Williams, 2005) that success in mathematics is directly related to academic success as mathematics is contained in other subjects to a greater (e.g. physics, strength of materials, thermodynamics and so on) or a lesser extent (e.g. history, geography, psychology and so on). A generic model is developed where by the psychological theories as well as pragmatism are used in the teaching and learning situation to improve problem solving and academic success.

Learning theories

For the last twenty years behaviourism, cognitivism and variations of them have shed some light to the phenomenon of learning in general and in mathematics in particular. Hergenhahn & Olson (1997) discuss the various learning theories in detail. Briefly, according to behaviourism learning brings change in behaviour as we learn through experiencing the world. By responding to the environment we learn (stimulus-response phenomenon). However, there are internal processes that are involved in learning. It is cognitive psychology that is concerned with these various mental activities (such as perception, thinking, knowledge representations and memory) related to human information processing and problem solving (Shuell, 1986 cited in Hergenhahn & Olson, 1997).
Cognitive psychology like behavioural psychology has given rise to a number of different perspectives such as constructivism and variations of it such as, personal (Kelly (1955) and Piaget (1972), radical (Von Glasersfeld, 1985, 1987a, 2002), social (Vygotsky (1978), critical (Taylor) and contextual (Cobern) (all cited in Venter, 2003). Parsons, Hinson & Sardo-Brown, (2001:431) define constructivism as a "cognitive theory emphasising learner interest in and accountability for their own learning which manifests in student self-questioning and discovery." In a way it was a reaction to the traditional way of teaching that gave rise to it. Constructivism is a theory of how learning occurs (Henson, 1996 cited by Parsons et al., 2001) rather than the product of such learning. This learning theory is student-centred where learners are actively involved in constructing their own knowledge and making use of past experience or prior learning or pre-existing schemas.
However, John Dewey saw existing theories to be too theoretical and lacked practicality, applicability, usefulness. So pragmatism was born. It endorses practical theory (theory that informs effective practice; praxis). And according to pragmatism knowledge is validated by its usefulness: What can we do with it! Of late pragmatism (Johnson & Onwuegbuzie, 2004; Schaffler, 1999) has started gaining ground again as it is felt by some educationists we have become too theoretical again. The pragmatic view stresses the experimental character of the empirical science, emphasising the active phases of the experimentation. It promotes an inquiring mind with respect to physical laws. "Inquiry itself is action, but action regulated by logic, sparked by theory, and issuing answers to motivating problems of practice" states (Scheffler, 1999:4). For pragmatism knowledge is viewed as being both constructed *and* based on the reality of the world we experience and live in. Learning from experiences is an active process. The mind is viewed as a capacity for active generation of ideas whose function is to resolve the problems imposed to the organism (the human being) by the environment. Pragmatism encourages imaginative

theorising by the student but at the same time insists upon control of such theorizing by the outcomes of active experimentation (Scheffler, 1999).

The psycho-pragmatic approach to teaching and learning in problem solving

This approach combines behaviourism and cognitivism (neocognitivism, the psychological aspect) and pragmatism. Pragmatism could be seen as the missing link between the other two theories as it promotes learning from experiences which is an active process. For this reason, this new paradigm gets its name "Act of learning" (see Figure 1), which gives rise to 'thinking while doing' and 'doing while thinking'. Thus the constructs 'thinking' and 'doing' form the two pillars of the learning. So the first step in the teaching – learning situation is the promotion of these two actions. But thinking can be of low (concrete) or high (abstract) order. Through thinking and doing, concepts are formed in a conscious as well as subconscious way. But concepts can be primary or secondary. Primary concepts give rise also to other secondary concepts. Concepts can be concrete or abstract and as concepts are connected and form a web of connections, principles are formed. This is the second step in teaching – learning situation, concept formation, a very important prerequisite to problem solving. Various principles make up the structures of knowledge as knowledge is either acquired and assimilated in the existing cognitive structure or knowledge is constructed and a re-structuring of the cognitive structure takes place. Experience adds a new dimension to existing knowledge as a result tacit knowledge also becomes part of the cognitive structure. Knowledge can be of different types such as procedural (knowing how), declarative or conceptual (knowing 'that'), schematic (knowing 'why'), and strategic (knowing 'where, when and how') (Hiebert & Lefevre, 1986; Shavelson, Ruiz-Primo & Wiley, 2005; Stolovitch & Keeps, 2002).

It can be said knowledge remains dormant in the cognitive structure and it comes to life when a problem is encountered, when the knowledge has to be applied into a real situation, the pragmatic aspect. If a problem is defined as an 'obstacle' on the path of an individual to a goal, then problem solving is finding a way out of a difficulty, a way around an obstacle, attaining an aim that was not immediately understandable (Polya, 1973). Green and Gilhooly (2005:347) state that 'problem solving in all its manifestations is an activity that structures everyday life in a meaningful way.'

Knowledge can be viewed as a 'tool' used to solve problems. But possessing any tool (be it a drill, a spade, a welding machine and so on) is not sufficient. One has to have the skill to use it. A skill is defined as expertise developed in the course of training and development (Malone, 2003). "A skill is an ability to do something well, to competently perform certain tasks" and "they (skills) consist, in part, of methods and strategies that have been incorporated into a performance routine" (Smith, 2002). Ferrett (2008:467) associates skills with capabilities that have been learned and developed. Skills include trade or craft skills, professional skills, social and sporting skills or more broadly motor skills and cognitive skills. And one of the most important cognitive skills is use of critical thinking (see Figure 1) which precedes problem solving. This is the third step in the teaching – learning situation. A literature review though on critical thinking and problem solving revealed that there is no conclusive evidence that critical thinking is a prerequisite to problem solving or vice versa (Papastephanou & Angeli, 2007). From a mathematics perspective, irrespective of the type of problem to be solved, in every step of the solution, given information needs to be critically examined.

The pragmatic approach then to problem solving creates a new approach to problem solving (see Figure 2).

Problem solving then is an outcome of a number of co-ordinated cognitive processes. Existing knowledge combined with critical thinking skills are applied to a real problematic situation. Irrespective of the problem solving method to be used, from the simplistic Polya's approach (understanding the problem, devising of a plan, carrying out the plan, looking back (evaluating)) to a more comprehensive model of Sternberg (2007) (see Figure 3) every step requires a certain type of knowledge and information to be critically examined.

Figure 1 *The act of learning*

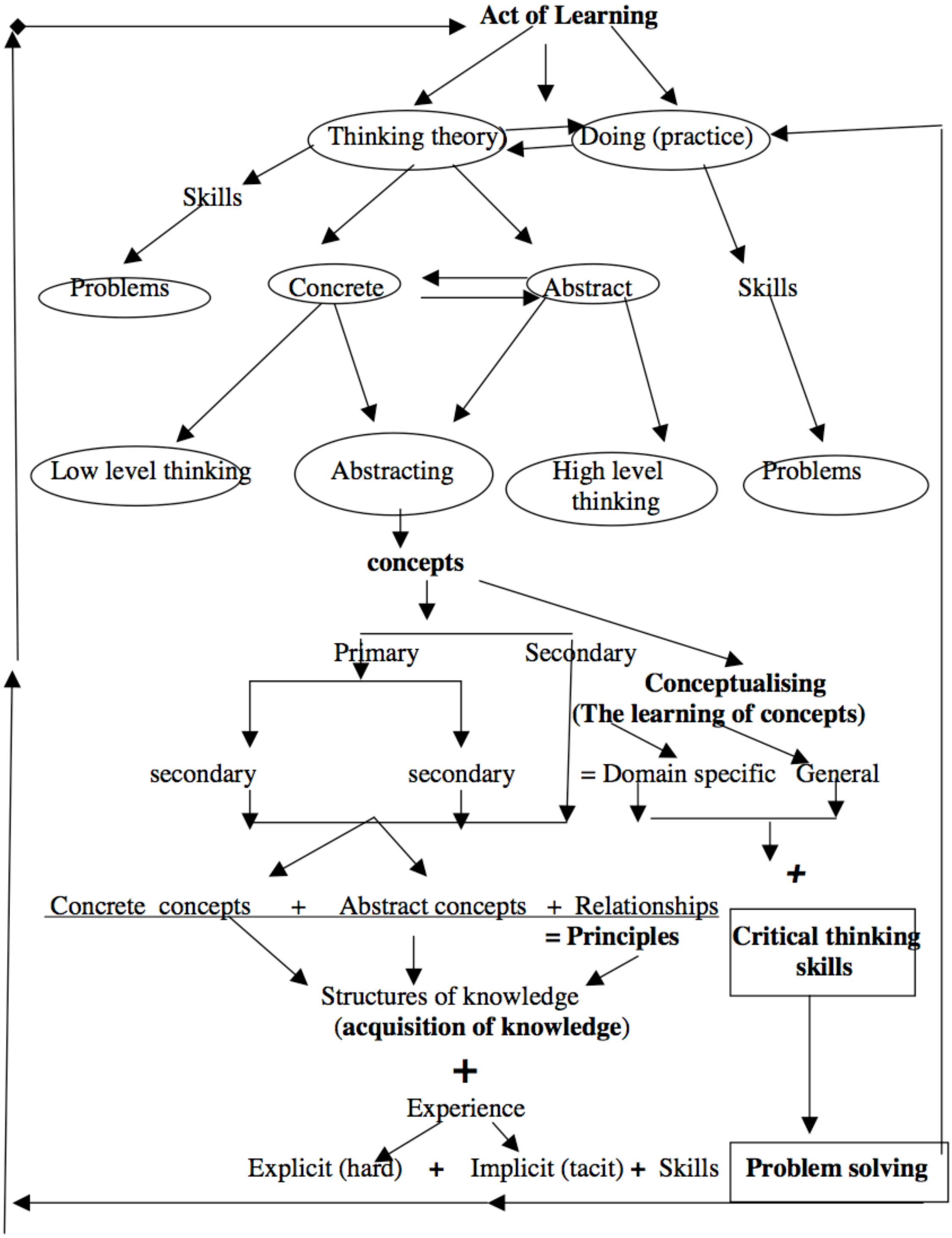
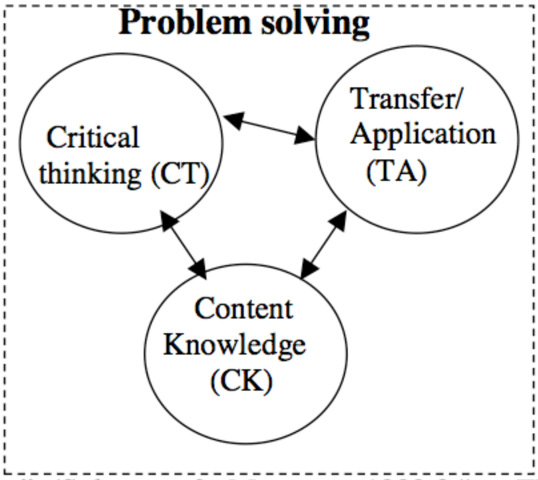


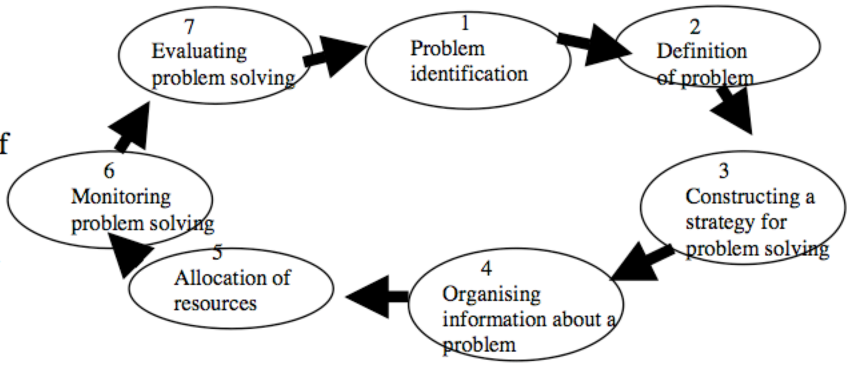
Figure 2. *A problem solving conceptual model*



However, the success of implementing such teaching-learning approach depends on the two main role players, the teacher and the learner. Briefly speaking, the teacher has to become what Solomon and Morocco (1999) call a diagnostic teacher. Diagnosing goes beyond finding out what the learners know. It is about understanding students' "*particular* thinking patterns, current understandings, or misconceptions" (Solomon & Morocco, 1999:234). A teacher who concentrates on diagnosing focuses on the individual rather than the class. Diagnosing is non judgemental. It concentrates on trying to understand student's understanding and "assessment is part of a recursive cycle of observation, selection of teaching strategies, reflection, and readjustment of one's strategies" (Solomon & Morocco, 1999:34). The learner, has to be willing to learn and if necessary to restructure his or her cognitive structure (Vosniadou, 1999). Furthermore the teacher has to become a researcher. Structural equation modelling (SEM) (as discussed in detail by Ullman and Bentler (2004) can be of great help when he or she evaluates their teaching.

Finally, the 'Act of learning' is of cyclic as well as of a spiral nature. Cyclic, by the virtue that the learning of concepts and problem solving go through the various steps; spiral, due to the fact that concepts are acquired in higher levels in each cycle.

Figure 3: *Problem solving model, Sternberg(2003: 361)*



Conclusion

The above exposition introduced a new approach to problem solving, which can be applied in any situation, any subject. This approach complements the behaviourist and the cognitivist approach to learning by combining them with pragmatism. Pragmatism brings problem solving to life by applying knowledge to real situations. However, the success of it depends mainly on the teacher becoming a diagnostic teacher.

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Reflecting Problem Orientation in Mathematics Education within Teacher Education

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Summary: Problem orientation is an important aspect within mathematics education we all know. But we also know that problem orientated mathematics teaching is practices in school reality rarely. To get a change the paradigm of mathematics teaching need a transformation.
I think besides theoretical discussions within the didactical community and the presentation of interesting proposals for different classroom conditions - which are important - first of all the teachers must be familiar with and get a positive belief about problem orientation in mathematics education.
In my presentation I will report about experiences in respect to problem orientation I did make with teacher students at our university in Bielefeld.

Problem orientation is an important aspect within mathematics education we all know. It is demanded since more then fifty years. The students first can learn a lot of mathematics on this way. Secondly they learn mathematics as a process. But most important is that they can train several general goals as
- how to handle a problem, how to make investigations,
- ability of problem solving and finding analogies
- why and how to order and systematize a given situation
- positive belief about mathematics and self-confidence .

But in most countries its realisation in school leaves a great deal to be desired. Therefore it is important to support a teaching which contains a lot of acting with problems.
For this - besides the necessary theoretical discussions within the didactical community and the presentation of interesting proposals for different classroom conditions - ***first of all the teachers must be familiar with and get a positive belief about problem orientation in mathematics education***. The teachers must have own experiences with working on single problems as well as problem fields. Moreover they must know about and reflect upon fundamental ideas about heuristics, problem solving and problem finding or variation of problems. Moreover they should be able to find different ways of working with a problem and be open for new approaches used by the students.
Here I will report about experiences in respect to problem orientation I did make with teacher students at our university in Bielefeld.

Survey of four seminars

Besides integrating some theoretical and practical aspects about problem orientation into all my lectures and seminars in the last time I offered four special seminars concentrating on problem orientation in mathematics education. Three of these seminars have been seminars for preparing teacher students for writing a final Bachelor paper. For all of these about 20 students per seminar I determined that the theme of each paper should refer to problem orientation. The other seminar was a normal one for senior teacher students within their Bachelor study. In all of these three seminars on one hand the students had to work by themselves with different problems and find new problems within the discussed problem field – I here already will mention that the last task was very hard for the students – and on the other hand I made some inputs with copies out of literature. Of course they also had to reflect on mathematics teaching with special concern to problem orientation.

Before going into details I would like to give an overlook of the topics we discussed. In all of the seminars in the first session (lasting one and a half hour) I presented *three different problems to work on by themselves* (together with their neighbours) without giving any help or hint. I will report on the results of this session later on.

Lateron we discussed with help of literature and internet the following themes: *History of problem orientation as well as learning by discovery, learning by doing and self regulation, definitions of "What is a problem", methological hints for problem solving, heuristics and problem orientation* and also *types of tasks, types of problems and ways of developing tasks by your own, self-activity and self-regulation in the discussion of didactics of mathematics within the last ten years* (cf. e.g. Pehkonen & Graumann 2007, Büchter & Leuders 2005 and Polya 1949).

We also deepened some theoretical aspects like *variation of taks* (according to Schupp 2004), "*logic of failure*" (according to Dörner 1989), *mathematical learning from constructive view*, *beliefs about problem orientation*, *barriers in respect to changing mathematics teaching*, *aims of and motivation for problem orientation*, *statements according to problem orientation in official guidelines*.

Mathematical topics considering the aspect of problem orientation have been e.g.:

- *Figured numbers and special sums, Partition of sets and representations of numbers as sums in grade 1, sequences and chains of numbers, number trains, number walls* (cf. Graumann 2009)
- *Magic squares and sudoku - a topic for grade 2 and 4, Polyominos – a geometrical problem field for grade 3, distribution of prime numbers in grade 3, division with rest in grade 4*
- *Triangles with integers as side length, regular polygons and polygons in space, Pythagorean triples,*
- *problems from PISA, Fermi tasks*

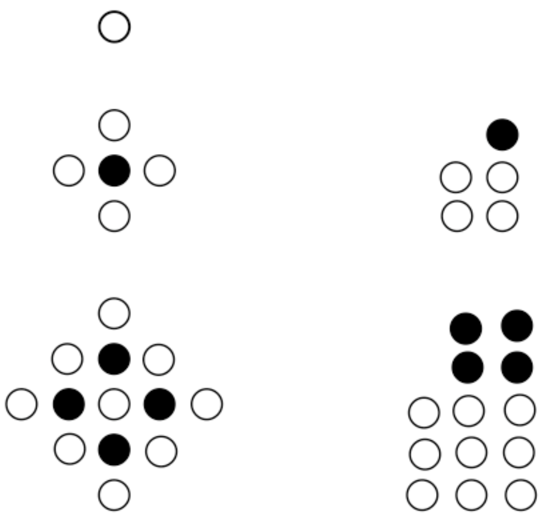
The "Mason problem" as inspirer

Some years ago I did hear from a seminar in Debrecen where John Mason as guest was present. In this seminar John Mason asked the participants (mathematics teacher students and secondary mathematics teacher) to solve the following problem.

$1 =$

$1 + 3 + 1 =$

$1 + 3 + 5 + 3 + 1 =$

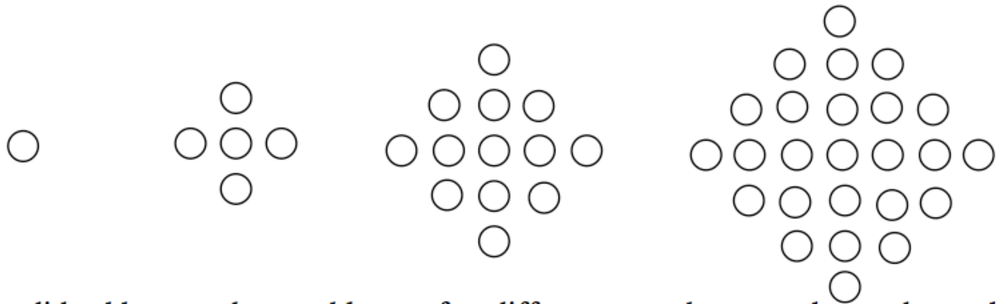


Task: "*Formulate a general question for this problem! Try to formulate a conjecture to your question! Prove the conjecture!*"

After 10 to 15 minutes it was clear that such open problems are very uncommon to Hungarian students and teacher, most of them could not do anything. This caused me to present the "Mason problem" to my students at the beginning of the seminar so that they are not influenced by discussions within the seminar (even though the title of the seminar may have influence and in their first semester they have already seen simple figured numbers like square numbers and triangle numbers). I wanted to know how they will act on this problem.

I varied the presentation of the Mason problem a little bit in that way not giving symbolic hints in respect to sums of numbers and not painting the little circles different because I wanted to see how the students will do it by themselves and whether they will discover different structures.

Two rows of the figures shown below have been given with the following text: "*Draw the next two figures of this sequence. Look out for partial figures and mark them. Which arithmetical representation do you can find on this way?*"



I also did add two other problems of a different type because the students should make experiences with different types of problems too. These two other problems have been given in text in the following way.

Problem concerning a ghost of a river: *A ghost of a river says to a walker who just will cross a bridge: "If you cross the bridge I will double the money you have in your pocket; but if you go back across the bridge I will take 8 Euros away from your pocket." When the walker came back the third time the money in his pocket was gone away (exactly 0 Euros).*

Problem concerning small animals: *Grandfather Miller has in his yard hens and rabbits. Once upon a time he counted 7 heads and 20 legs. (Variation: He did count only 20 legs).*

Different results of these two problems given in text

First of all I can tell that all of the students except one in the discussion at the end of the first double hour reported that an open problem like that from Mason was very new for them. But all of them (at least together with a neighbour) started to work on that problem and most of them got at least one special result. And it could be noticed that in any of the seminars There appeared different ways of working with these three problems.
In the following I will present all different ways of working with these problems; in doing so I will start with the two problems given in text.

Problem concerning a ghost of a river

1. Method of trial and error with variation: We start with 6. The transformation from this are $6 \rightarrow 12 \rightarrow 4 \rightarrow 8 \rightarrow 0 \rightarrow 0 \rightarrow$ not possible. We see that we have to get 6 after the first crossing and the way back. Thus we try it with two more Euros and get $8 \rightarrow 16 \rightarrow 8$

$\rightarrow 16 \rightarrow 8 \rightarrow 16$ and find an endless sequence. Now we try the number between and get the solution $7 \rightarrow 14 \rightarrow 6 \rightarrow 12 \rightarrow 4 \rightarrow 8 \rightarrow 0$.

(By varying our thoughts we could start with 5 or 4 and see that all numbers of the sequence decrease just like starting with 9, 10, … will let increase all numbers of the sequence. That means we may find a functional relation.)

2. Method of working backwards: $0 \leftarrow 8 \leftarrow 4 \leftarrow 12 \leftarrow 6 \leftarrow 14 \leftarrow 7$.
3. Method with algebraic formula: $2 \cdot (2 \cdot (2 \cdot x - 8) - 8) - 8 = 0$ or

$x \rightarrow 2x \rightarrow 2x\text{-}8 \rightarrow 2\cdot(2x\text{-}8) \rightarrow 2\cdot(2x\text{-}8)\text{-}8 \rightarrow 2\cdot(2\cdot(2x\text{-}8)\text{-}8) \rightarrow 2\cdot(2\cdot(2x\text{-}8)\text{-}8)\text{-}8$

From $2\cdot(2\cdot(2x-8)-8)-8 = 0$ we will get $x = 7$.

We can get a generalisation via this method with $2 \rightarrow a$, $8 \rightarrow b$ and $3 \rightarrow n$:

$x \rightarrow ax \rightarrow ax\text{-}b \rightarrow a\cdot(ax\text{-}b) \rightarrow a\cdot(ax\text{-}b)\text{-}b \rightarrow a\cdot(a\cdot(ax\text{-}b)\text{-}b) \rightarrow a\cdot(a\cdot(ax\text{-}b)\text{-}b)\text{-}b$

$\rightarrow a\cdot(a\cdot(a\cdot(ax\text{-}b)\text{-}b)\text{-}b) \rightarrow a\cdot(a\cdot(a\cdot(ax\text{-}b)\text{-}b)\text{-}b)\text{-}b \ldots \rightarrow a^{n\cdot}x - (a^{n-1} + a^{n-2} + \ldots + 1)\cdot b$

Problem concerning small animals

1. Method of trial and error with variation: We try 4 rabbits → 16 feet, with the left 4 feet we get 2 hens; that make together 6 heads. Because one head is undercharged we have to increase the number of hens respectively decrease the number of rabbits. 3 rabbits → 12 feet and 4 hens → 8 feet gives 7 heads and 20 feet as desired.
2. Method of working backward from the heads: Any animal has at least 2 feet, so with 7 heads we have at least 14 feet. The rest of 6 feet is going in pairs to 3 rabbits, so we get 3 rabbits and 4 hens.
3. Method with algebraic formula: x = number of rabbits, y = number of hens. $4x + 2y = 20$ (number of feet) and $x + y = 7$ (number of heads) . Then solving with algebraic instruments gives $x = 3$, $y = 4$.

Variation of this Problem

The variation shall show the students that we also can get a problem that has more than one solution and we have to find a systematic for finding all solutions.

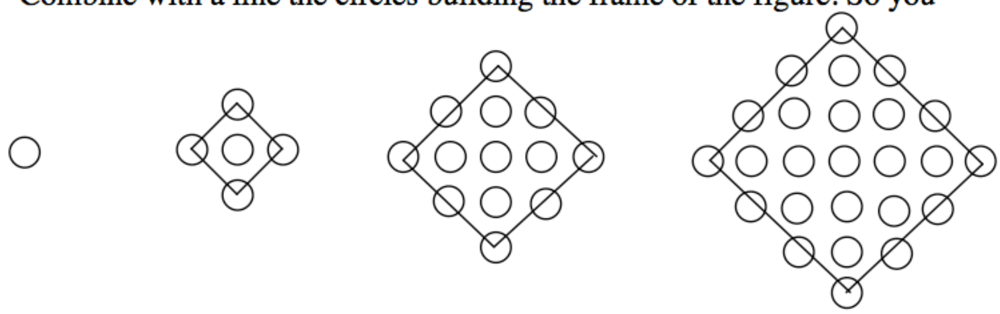
Here the minimal number of heads is 5 because 5 rabbits and 0 hens makes 20 feet. Reducing the number of rabbits step by step you will get 6 heads (4 rabbits and 2 hens), 7 heads (3 rabbits and 4 hens), 8 heads (2 rabbits and 6 hens), 9 heads (1 rabbit and 8 hens), 10 heads (0 rabbits and 10 hens). The solution with 0 hen and that one with 0 rabbit probably does not fit to the text and thus these solutions have to be erased.

In addition we can make investigations in respect to functional relations like “Reducing the number of rabbits by one causes increasing the number of hens with two” or “Reducing the number of heads by one causes decreasing the number of hens with two”.

Different ways the students worked with the “Mason Problem”

The following different groupings by colouring some circles or combining some circles with a line and symbolic descriptions have been the following:

1. Combine with a line the circles building the frame of the figure. So you

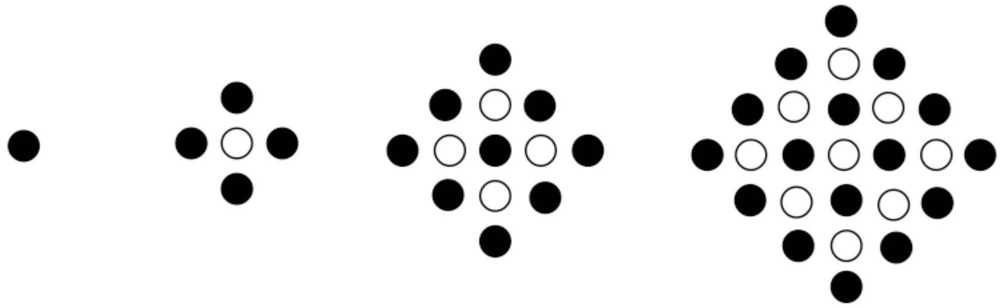


get the description 1, 1+4, 1+4+8, 1+4+8+12, … and in general $1 + 4 \cdot [1+2+3+\ldots+(n-1)]$.

If we already know that $1+2+3+\ldots+(n-1) = \frac{1}{2} \cdot (n-1) \cdot n$ we will get the general symbolic description $1 + 2 \cdot (n^2-n)$ [resp. $2n^2-2n+1$].

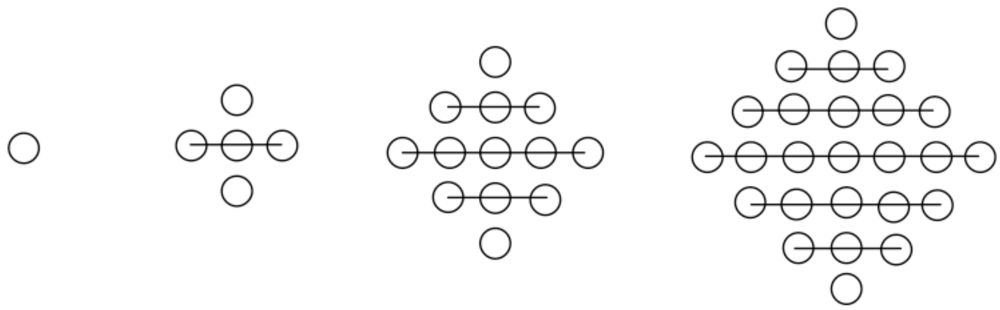
2. In a more arithmetical view on these figures some students looked at the respective number of circles: 1, 5, 13, 25, … . From this they detected that the difference sequence is built by the multiple of 4. On this way they got the same general description as above.

3. Some students coloured the circles in the frame together with inner circles for getting a squared number. The non-coloured circles then built a squared number too but a smaller one, more precisely the length of the side is one less than the length of the side of the coloured square.



The symbolic description thus came out as 1^2, 2^2+1^2, 3^2+2^2, 4^2+3^2, … or in general $(n-1)^2 + n^2$ [respectively $2n^2 - 2n + 1$].

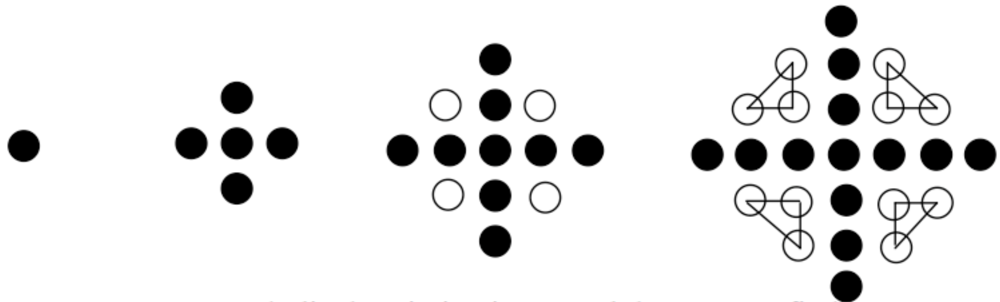
4. A fourth group of students looked at the horizontal (or vertical) rows and got the symbolic description 1, 1+3+1, 1+3+5+3+1, 1+3+5+7+5+3+1, … .



If we already know that the numbers 1, 1+3, 1+3+5, 1+3+5+7, … describe a square number we can see the identicalness with the symbolic descriptions above.

5. One student coloured the vertical and horizontal middle lines building a cross. The non-coloured circles then build four triangle figures and we get 1, $1 + 4 \cdot 1$, $1 + 4 \cdot 2 + 4 \cdot 1$, $1 + 4 \cdot 3 + 4 \cdot [1+2]$, … .

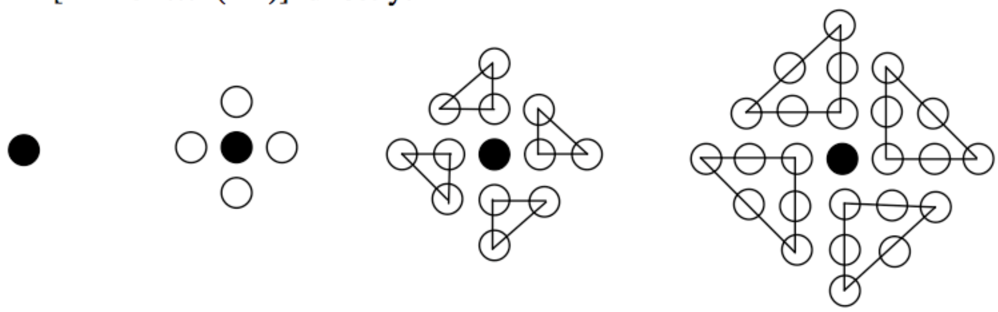
6.



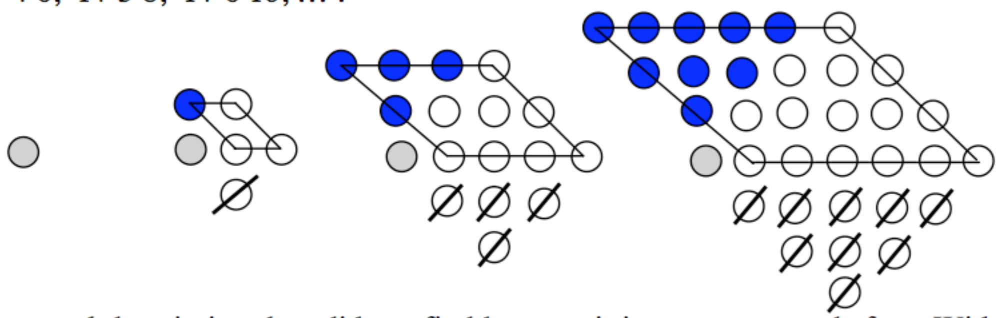
If we want to get a symbolic description in general then we can find

$1+4\cdot(n-1)+4\cdot[1+2+3+\ldots+(n-2)]$ resp. $1+4\cdot[1+2+3+\ldots+(n-1)]$.

7. Two students also saw four triangle numbers but including always one branch of the cross so that only the middle point is extra standing. This leads to the formula $1+4\cdot[1+2+3+\ldots+(n-1)]$ directly.



8. Another group of students resorted the figures of circles by erasing the rows below the horizontal middle line and then added these circles on the left side of the remaining circles so that after that all horizontal line have the same length without the bottom line which has one circle more. So they got the sequence 1, $1+2\cdot2$, $1+3\cdot4$, $1+4\cdot6$, $1+5\cdot8$, $1+6\cdot10$, … .



A general description they did not find because it is no so easy as before. With some considerations you can find the formula $1+n\cdot(2\cdot(n-1))$ resp. $1+2n^2-2n$.

In the discussion with the whole group the different solutions were presented with adding the missing symbolic descriptions. For teacher students this is very important because they can see the large variety of working on such a problem. Later on as teacher in school on one hand it is important to be open for different ideas of pupils and on the other hand the arrangement for working with problems should include working in small groups as well as reflecting different approaches and their connections.

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A Good Instruction in Mathematics Education should be Open but Structured

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That we have to change the teaching method away from the traditional cramming school was demanded already about hundred years ago and is uncontested in pedagogy today. But the reality in school often is different from the theory. Even though the conception “Self Directed Learning” / “Independend Study” / “Minimally Invasive Education” (Offener Unterricht) is practiced a lot in primary school in Germany the pedagogical cardinal points are not understood by many of the teachers; they often think one can let work the pupils by themselves without those instructions the pupils need to become successful.

Self Directed Learning is amongst others characterized by free work, week-plan, project work and differentiating teaching. These teaching methods trace back to conceptions of reform pedagogy arising from different theories in different times and different countries (cf. e.g. Montessori, Petersen, Dewey, Kilpatrick). Therefore Self Directed Learning is defined by an assortment of several elements of different theories. Thus one runs the risk to lose sight of the theoretical background which can lead to a frivolous simplification and non-reflected acting.

Some critical remarks about a reduced setting of open instruction are the following:

- One assumes in respect to Self Directed Learning that children of today need more possibilities of own decisions and more free space for self-activity and creativity. But do become children more autonomous if they get the choice to take one working sheet or another one? And don’t have children of today already too much stress of decisions in their every day life so that it would be better to relieve them from nonsensical stress in school?
- The change from commanding style to bargaining style in household escalates today and also captures school. Many children already start with bargaining instead of looking at the task. This keeps energies away from struggling with the objects. Shall we strengthen this trend?
- Self Directed Learning can mislead a teacher to relieve him/her from handling and responsibility in course processing. In this situation children can feel left alone which may underline their experiences from at home.
- In underprivileged families and families with migration background the bargain style often is not infiltrated in household. Most decisions there will be made by the parents or the personal living condition. Children of such families will get problems with open instruction if they do not get help from the teacher or a classmate.
- Children of underprivileged families and families with migration background as well as children with mental handicaps often do not have the ability for motivation and creativity. Can we see here the limitation of Self Directed Learning?

Another question is: How shall we react on the existing contradictions, which are caused besides others from the ascending demands by the big heterogeneity of the learning groups and the achievements ask from society with parallel necessity of differentiated and individual learning. To make plain this I first will paint a situation as example.

***Jonas** is in grade 3 of a primary school. He has two sisters and lives in a one family house. His father is a technician and his mother is a teacher. He has an own Computer. When starting school he already could read and count until thousand.*
***Simon** in the same class. He has several brothers and sisters and lives in a flat of a high-rise building. His father and mother both go for working so that the children on the afternoon are mostly by themselves. Simon has interest for general science.*
*After Christmas holydays the children of this class first discuss their experience during their holydays. **Jonas** was skiing with his father. **Simon** did not have any special adventures. On Christmas Eve his parents did have a struggle with each other.*
In the second part of this lesson the children get two different work sheets about symmetry – the subject they did deal with before Christmas holyday. They can decide by themselves with which one they will start. One sheet bears reference to a Christmas tree. The children shall put different figures- they can decide by themselves - on the tree so that the tree is symmetric still.
In the third part of this lesson the children get a working sheet with tasks for a week-plan covering exercises of arithmetic, an essay about holiday adventures and a plan to find different connections for an electric circuit. In the classroom there is a corner the children can go to read something or to make experiences with material like electric circuits.
This described situation in school has a conceptual design of an Self Directed Learning. But you may imagine that Simon and Jonas can learn in this situation less and more.

School cannot afford to have so much idle time as in the example described above. What did **Simon** learn in this lesson? The working sheet with the Christmas tree for him is reminiscent of the bad situation at home on Christmas Eve so that he did not find figures to put at the tree. With the arithmetic tasks on the week-plan he did have had problems already before Christmas. Because there was nobody in the class to help him it again came out that he is not able to overcome given tasks. The essay also was nothing for his situation. And the electric circuit he already knew before.
And what did **Jonas** learn in this lesson? The given tasks he could carry out very fast because they did include nothing new for him. He was used to do arts and handicrafts at home. And about his adventures on Christmas holydays he reported on his classmate in the talking circle in the morning. So he did overcome the time somehow or other but did not learn anything new.
Self Directed Learning runs the risk to degenerate to a mere actionism if not each step, each task is prepared for different previous knowledge as well as different situations of life of the children.

In a good Self Directed Learning there must be given possibilities for each child to find an access to any of the topics dealt with in class. There must be a basis for each child to take a step forward in his/her development.

This is not a simple task for the teacher and often it is not possible to realize this in total. But during preparing a lesson the teacher has to look out for different possibilities of access and take account of the different situations of life of his/her children in class.

There is a second point of good instruction we have to bear in mind. Children do not only learn in school. Therefore it is an important task of school to help children to order and structure their diffuse and unstructured knowledge and experience which deliver the children in school.

Week-plan is besides of free work and project work the most popular form of Self Directed Learning. But it cannot be put on the level as differentiated teaching. An empirical research of Huf (2000) e.g. has shown that children see week-plans not as possibility for individual work but more as a set task which should be fulfilled as fast as possible with using time-saving tricks. This means that undifferentiated week-plans can be seen only as a little variation of tradition exercise courses. Jürgens (1996, 1965) for example could find out by an inspection of teaching research concerning Self Directed Learning that Self Directed Learning cannot do without a clear structured and task orientated arrangement of learning. Hereby it is important to find a balance between structured openness and course orientated closeness, between spontaneity and checking, between distance and closeness, etc. In principle also for Self Directed Learning we have to respect the findings from Weinert and Helmke (1997) in their SCHOLASTIK-study that for an achievement promoting education clearness of demonstration, efficient management, control of the class and positive social climate in the class is important.

In this way I will speak about “Self Directed Learning” or "Developmentally Appropriate Teaching" or “Minimally Invasive Education“. ***I would define “Self Directed Learning” as a teaching which is doing what the child does need.*** If the child is able to learn by themselves so it must get the possibility to do so. If the child is not able to learn by themselves the teacher must give him the help which the child does need. Self Directed Learning means in my comprehension that the teacher always is ready to help or to go in the background. Self Directed Learning in my comprehension first and foremost does not mean openness in respect to the methods and social forms but ***openness for the child*** with its special circumstances.

Very important in this way of thinking is that the teachers must be capable to diagnose the learning ability of each child. This is very difficult and demands knowledge, endurance and attentiveness. Often the teachers do not learn this in the field of study and so they can not help the children really. Thus in the preserves as well as in the in-service study of teachers the diagnosis of children and the main points of Self Directed Learning in the above named way have to be discussed – and this means in the pedagogical part as well as in the part of didactics of mathematics.

In the following the necessity of structuring Self Directed Learning will be made clear with some different points of view.

1. A clear structure of courses of events at school is required as already seen by Weinert and Helmke. This is no contradiction to Self Directed Learning. Clear structures, a noticeable thread (developed by the teacher or/and the pupils) make instruction efficient and comprehensible for all pupils. Clear structures reduce incertitude on which children are suffering on often and they bring calmness to the courses of events.
   The pupils need guidance for systematic learning. Systematic thinking and acting is for the future of the children important more than ever. The learning with a special systematic makes it possible new contents of learning to integrate into the existing knowledge and understanding in a new wholeness. This is necessary for children with problems of learning in particular but also the others need help to sort and integrate the plenitude of knowledge and experiences.
   Also several children sometimes need a course orientated and teacher centred instruction. In an Self Directed Learning the teacher can work with one child a while whereas the others work by themselves / in groups if the structure of working is clear for all of the pupils.

2. A solid preparation and postprocessing of instruction is necessary for open instruction. During scheduling the Self Directed Learning the teachers have to look into the object of instruction as well as discuss different learning proposals, ways of structuring approaches to solve the possibly appearing tasks and solutions. If the teacher in our situation named above did ask him/herself which conceptions, hints, situations, detections, narrations, experiments, models etc (cf. Klafki 1963, 141) he/she would have found out that for Simon and Jonas then his/her given tasks would be different. Self Directed Learning requires a good preparation with reflection on aims and topics as well as social and institutional conditions. This is not in contradiction to Self Directed Learning because once during the performance you can let go the pupils their own ways and another time a good preparation avoids excessive demands as well as under-challenge, idle time and blind actionism. Nevertheless the pupils will not be restrained in their freedom to look out for own focuses, planning of time to work on a special problem, to develop own ideas, to manage team working, etc.
3. Self Directed Learning needs a personal responsibility of the teacher more than ever. The teacher can not just give a task and then let work the children. The teacher can not behave laissez faire. He/She has to bring in him/herself much more than in traditional teaching namely not so intensive in respect to the object but more intensive in respect to the subjective demands. Thus a teacher in Self Directed Learning must have the ability of noticing and making a diagnosis.
4. Self Directed Learning is no grab box where you can pull out this or that. Moreover Self Directed Learning does not mean "take any working sheet you want" out of a pile of different working sheets concerning one topic. Unfortunately often this will be mixed up with Self Directed Learning. Do you really call the decision between some trivial working sheets a free decision whereby the children can learn about decision doing?
   The Question how self-determined a child can work on a course can be answered only individual. One child may open oneself for a special topic only after the teacher did give guidance for a small walk on this way and another child does see an access immediately. Learning self-determined does not mean without adults.
   Self Directed Learning can not be measured by the extending of guidance through the teacher. The degree depends on the amount of alternatives the children do have to develop themselves.

If one bears in mind all named conditions of Self Directed Learning then Self Directed Learning leads to a good instruction with intellectual, emotional and social development of all children. But of course it is not easy to perform such a good open instruction and not at all times a teacher can perform a good instruction. But most important is that at any time the teacher goes to great length with such good Self Directed Learning. This does include that the teacher reflects on his method and social conditions.

This in particular demands a teacher training with reflection on these points as well as good mentoring while making experiences in school. Let me demonstrate this with an example about a "poor" teacher.
A teacher-student was watching a class while the children did handicrafts with paper. She asked the teacher why the children should do so. After a while the teacher answered that the children like to do that.

If the teacher would have answered that he/she did not have time for preparation (of course an answer which would not be good with an inspection) so it would have been a candid answer which shows that he/she exactly knows what he/she performs and why he/she performs like this. No teacher is free to do anything anytime anyhow.

Return to our example:
What should the teacher do with Jonas and Simon? He/She could give Jonas new special tasks about symmetry to challenge him but not overtax him. And Jonas could write a fairy tale that is playing in Christmas Eve to show his fantasy. On the other hand the teacher could take time to explain Simon the arithmetic tasks while the other children are working by themselves. Later on Jonas could discuss with Simon a plan to construct an electric equipment in a little house. So both boys can win new knowledge and social competences

As summing up I would say:
For any instruction - open or not open - it is important that pedagogical and didactical acting does have a good basis with passable aims as well as a thread and a clear structure.

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Do South African Mathematics teachers need narrative therapy?

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Abstract This paper will argue that ubiquitous stories of mathematics teachers as the root cause of the crisis in mathematics education shuts down the space for meaningful teacher learning. There are many statistics used to 'back up' these stories, for example: South African learners performed worst on the TIMMS 1999 and TIMSS 2003 study. Newspapers are quick to pick up on this in headlines such as: "Teachers flunk maths"(Mail and Guardian, 03/08/08); "Teachers battle with Maths" (news.africa.com 05/04/11 – from SACMEC111).These stories create a cycle of ongoing failure. Of course there are always many other stories that don't get told such as "mathematics teachers have the experience and classroom knowledge needed to inform curriculum change". In this paper I argue that a primary concern of in-service mathematics teacher learning should be 'narrative therapy' – that is a focus on supporting teachers to actively construct preferred realities (Freedman and Combs, 1996). Such construction requires the formation of supportive communities of practice (with a focus on inquiry into mathematics teaching and learning through partnership between 'teacher educators' and teachers) where teachers are supported through active participation in the community to challenge negative stories and to develop and foreground new stories.

Introduction

Do mathematics teachers need teacher educators or do we as teacher educators need to construct teachers as 'needing' through deficit discourses in order to justify our work? This is indeed a challenging question and one teacher educators need to reflect on. Breen (1999) illuminates the interdependence between teacher educators and teachers in his referenc e to 'fix it' approaches - teacher educators need someone to fix and teachers need fixing. He highlights that such approaches tend to ignore what teachers are actually doing and look for solutions outside of the practice of teaching. The problematic nature of this relationship seems clear. Yet the dominance of teacher development models in which 'teacher educators' (e.g. Department of Education (DoE) district officers, NGO or university employees) position themselves as knowledge-authorities bringing knowledge to less knowledgeable teachers, would suggest that teacher educators have not reflected sufficiently on the nature of their own need in the relationship. Indeed in order to understand the nature of mathematics teaching and learning and to advance this field of knowledge we (teacher educators and researchers) need access to the world of teachers and their classrooms. The right to access is sometimes taken for granted as if participation in what we have to offer will automatically provide rewards for teachers. Yet in service 'development' is often experienced by teachers as disempowering and teachers complain of unprofessional treatment (OECD, 2008).

I would like to argue that this relationship must be changed and that as teacher educators we need to change our stories from 'development on teachers' to forming supportive communities of inquiry into mathematics teaching and learning in South African classrooms where 'equal' partnerships with teachers are established. The negative stories that teachers themselves often buy into need to be re-narrated as stories that foreground teachers as experienced and of teachers as life- long learners, willing and able to partner with policy makers, the department of education, teacher educators and so forth to find solutions to the challenges faced in mathematics education. Of course 'equal' partnerships are not simply the result of naming the partnership equal but will involve practices that lead to the experience of the partnership as equal and this will take conscious work and time to develop. Equality of course does not mean that the knowledge each partner brings is the same – indeed the reason for the partnership is precisely because each has knowledge (through their participation in differing professional practices and landscapes) that the other does not. The knowledge should however have equal status (at least within the practices of the evolving partnership).

Drawing on Sfard and Prusak's (2005) definition of identity I will argue that deficit stories of Mathematics teachers result in self- fulfilling prophecies and that teacher educators need to become the significant narrators that narrate teachers as experienced professionals, critical partners and life-long learners.

Providing an operational definition of identity

In previous work (Graven 2003, 2004, 2005) I built on Wenger's (1998) notion of identity to analyse teacher learning, but the work of Sfard & Prusak (2005)goes further to operationalise the definition. In doing so, they equate identity with reifying, endorsable and significant stories about a person. While Sfard and Prusak (2005) concur with Wenger's work in terms of linking learning with the construction of identities they argue that the 'notion of identity cannot become truly useful unless it is provided an operational definition.' (15). They highlight that notions of identity as being a kind of person "sound timeless and agentless"; and therefore reject such definitions as "potentially harmful because the reified version of one's former actions that comes in the form of nouns and adjectives describing the person's "identity" acts as a self fulfilling prophecy" (Sfard & Prusak, 2005, 16). Sfard and Prusak (2005, 16) thus choose to define identities as "collections of stories about persons or, more specifically, as those narratives about individuals that are reifying, endorsable, and significant". Reification comes with verbs such as 'have'. (E.g. "Teachers have mathematical weaknesses". Stories are considered endorsable if the identity builder can answer to them being a faithful reflection of a state of affairs (E.g. we use the headline"teachers flunk maths" because a study showed…). Stories are significant if a change in the story changes the storyteller's feelings about the identified person. (E.g. 'teachers are unqualified' to 'teachers are experienced').

Within their definition identities are human made, collectively shaped by authors and recipients. They explicitly highlight that their definition presents identities as the discursive counterparts of lived experiences whereas Wenger (1998, p151) sees such words as only a part of "the full, lived experience of engagement in practice". Sfard and Prusak thus stress "No, no mistake here: We did not say that identities were finding their expression in stories – we said they were stories" (p.14).

This definition gives increased agency to the learner as it opens the space for the re-authoring of identities. It also opens the space for significant narrators, such as mathematics teacher educators, to deliberately challenge existing negative stories and to reflect on their own authoring of mathematics/ numeracy teacher identities. Reflection should lead to the re-authoring of negative stories that may be obstacles to learning into stories that enhance teacher learning. My assumption here is that every negative story can be countered with a different story that is more conducive to stimulating learning. For example: 'Teachers are poorly trained to cope with the new curriculum' can be countered with 'Teachers, with their wealth of teaching experience, are best placed to make sense of the curriculum and provide feedback'. It is this space for re-authoring that appealed to me.

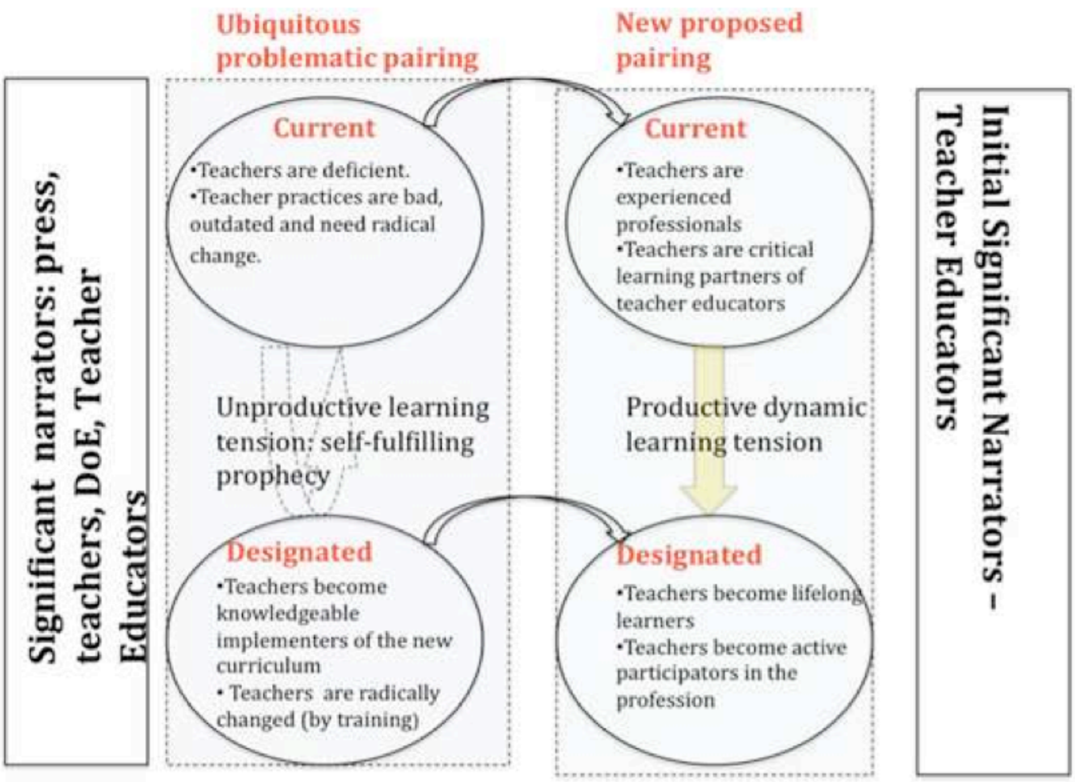
Sfard & Prusak (2005) continue to identify two sub categories of stories: *current* identities (e-mail correspondence with Anna Sfard (2009) suggests a move away from the term 'actual' identities to 'current'), told in the present tense and formulated as actual assertions, and *designated* identities (narratives expected to be the case – now or in the future). Learning is then conceptualized as closing the gap between *current* and *designated* identities. With this definition of identity as discursive counterparts of one's lived experiences, the re-authoring of identities is not only possible but could enable and give momentum to learning. This is especially important in cases where identities have been negatively constructed. Indeed this is precisely what narrative therapists enable people to do:

> A key to this therapy is that in any life there are always more events that don't get "storied" than there are ones that do… this means that when life narratives carry hurtful meanings or seem to offer only unpleasant choices, they can be changed by highlighting different previously un-storied events, thereby constructing new narratives. Or when dominant cultures carry stories that are oppressive, people can resist their dictates and find support in subcultures that are living different stories (Freedman & Combs, 1996, p32-33).

The above quote highlights that narrative therapy is not restricted to the domain of individuals and their therapists but extends the opportunity to groups of people in supportive communities or 'communities of practice' (Wenger, 1998) which enable 'living different stories'. In a similar vein Sfard & Prusak (2005, 18) note that "A person may be led to endorse certain narratives about herself without realizing that these are "just stories" and that there are alternatives". A supportive community of practice, such as those formed in in-service teacher education programs can and should open up these alternatives especially when existing stories 'carry hurtful meanings', undermine professional identities or impede learning.

Thus part of what drew me to Sfard & Prusak's definition of both identity and learning is the increased agency afforded to learners and the opportunity for both learners and significant narrators to deliberately reject negative stories (thus breaking down the stumbling blocks to learning) and re-author more productive stories which will give momentum to learning. In my experience with in-service mathematics teacher education over the past 15 years, 'substantial' teacher learning requires re-authoring of certain negative current teacher identities and counter productive designated teacher identities. It requires the creation of supportive communities which can provide the space and the 'subculture' where teachers can challenge these stories and live out more productive stories.

The pairing of 'deficient' *current* identities with designated 'curriculum knowledge authority' leaves teachers trapped between two conflicting and imposed identities. The hypothesis here is that widespread low morale and retention (of South African mathematics teachers in particular) are partly a result of teachers feeling trapped between two conflicting identities where one's history and experiences are negated and one's designated future is unattainable (especially in the context of constantly changing curricula). This gap results in the removal of stimulus for learning. In contexts where teacher morale is low and negative stories predominate, teacher education must involve deliberate re-authoring so as to construct a productive learning tension (gap) between teachers' current and designated identities. Of course these current and designated identities and the gap between them are dynamic and changes will emerge through the learning process. That is stories become modified, new stories emerge, negative stories become a stimulus for learning (e.g. I have little knowledge of probability and I need to learn about it to teach it), and new designated identities might be added (e.g. I'm a lifelong learner constantly making sense of curriculum initiatives'). A diagram is useful to highlight key *deficiency* narratives that need to be re-authored as *proficiency* narratives by teacher educators:



This re-authored pairing requires mathematics teacher educators to narrate teacher *current* identities as experienced teachers and learners (acknowledging the value of bringing experience and existing knowledge to the learning process) and critical partners in the process of making sense of and reviewing the curriculum. Teacher educators narrate the *designated* identities as life long reflective learners. Linked to this emphasis on life long learning and the value of teachers' experience is the designation as active participators in a range of professional activities such as: engaging with others, providing feedback on the curriculum, attending professional conferences and so on. While teacher educators are the initial significant narrators tasked with the job of challenging previous stories of teachers, with time and through successful learning within in-service programs, other significant narrators (fellow teachers, community members, learners, parents) are likely to reinforce these stories.

From what has been discussed above it is argued that the way forward in working with teachers is to form supportive inquiry communities where teachers and teacher educators partner to both reflect on and learn from teaching practices and to look towards finding innovative ways to strengthen teaching and learning in Mathematics classrooms. The South African Numeracy Chair, Rhodes University is aimed at improving the quality of learning and teaching of numeracy at primary level. This development aspect of the chair is dialectically connected to the aims of researching sustainable and practical solutions to the challenges of improving numeracy in schools as laid down in the concept document of the Chairs Initiative. In my work as the South African Numeracy Chair, Rhodes University, it has been important to develop a conceptualization of 'teacher development' that challenges deficit discourses of teachers and works with a conceptualization of teachers as critical partners. While this was always the intention the need for this became increasingly clear when my colleague, Zonia Jooste, and I visited schools in the Grahamstown area to invite numeracy teacher participation. Teacher histories of 'teacher development' and 'workshops' had not led to a 'we want more' response but rather a skepticism of the value of participation. Explanations that we wanted to partner with them and that in this partnership we would not tell teachers what to do were well received but skepticism persisted. Our launch and orientation focused on what working as partners might mean and the importance that the path for the way forward must be carved from the perspective of numeracy classroom practices in collaboration with researchers/teacher educators rather than the other way round.

Thus we named our partnership with teachers the Numeracy Inquiry Community of Leader Educators (NICLE) where all participants, teacher educators, professors, and researchers would be learners in the community and through participation would provide leadership within their sphere of influence and their overlapping communities. Thus NICLE was conceptualized as a community of practice based community of inquirers where teachers, lecturers, researchers and professors partnered to inquire into looking towards finding solutions to challenges faced in primary mathematics education from a classroom based perspective. In this partnership all are co-learners. By working together each brings different experiences and expertise to share in the community. Through active participation each member of the community will increasingly take on leader roles in primary mathematics education relating to their sphere of influence. For example teachers will run workshops or mathematics competitions in their community of schools, researchers will publish and engage in panels as conferences, teachers and researchers will present their work at teacher and research conferences, teachers will publish their classroom reflections in teacher focused journals and so forth.

The start of NICLE

Fifteen school were initially invited to participate in NICLE. Six schools and 19 teachers participated in this launch held on the 26th March 2011. Following this word seems to have spread that this might indeed be a different type of learning endeavor and some schools that had declined participation have since committed participation. By the second NICLE session (12th April 2011) fifteen schools and 51 teachers attended indicating willingness of teachers to become life-long learners provided their views and experiences are taken seriously. Indeed there will be the challenge of sustaining teacher involvement as well as developing practices that truly support equal partnerships where learning is led from the basis of teacher experiences. Such practices do not follow automatically from the naming of a learning community in this way nor from the removal of deficit discourses. The notion of partnership must come alive in the practices of NICLE. Further research into the nature of learning evolving within this community for all participants will hopefully reveal key elements of NICLE practices that enable or constrain learning so that these might inform future endeavors with numeracy teachers.

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Horizontal and Vertical Concept Transitions

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Abstract:
Transfer of concepts, ideas and procedures learned in mathematics to a new and unanticipated situation or domain is one of the biggest challenges for teachers to communicate and for students to learn because it involves high cognitive skills. This study is an attempt to find ways for driving students to generalize and expand mathematical results from one domain to another in a natural way, and to promote that mathematics is not a collection of isolated facts by providing meaningful ways for students to construct, explain, describe, manipulate or predict patterns and regularities associated with a given system of theorems and mathematical behavior. One would wish there were a universal genetic decomposition for generalization and for the abstraction of properties from a given structure and applying it to a new domain. In this study I plan to focus on particular cases of generalizations in calculus and distinguish between two different types of examples.
Keywords: genetic decomposition, abstraction, generalization, Calculus, RME

Many instructors choose to informally display concept transfer by just drawing the students' attention to the connection between the initial result and the new transferred result so as to make the extension seem natural in retrospect. This helps the learner gain ownership over the mathematical content. One could say that even if the students are not able to deduce the generalized concepts on their own, at least they are convinced that those mathematical facts are the result of a human activity (Streefland, 1991, p. 15). To facilitate this, it helps to view effective learning as a series of processes of horizontal and vertical mathematization that together result in the reinvention of mathematical ideas. Realistic Mathematics Education (RME) is about how mathematics is learned and at the same time about how it should be taught: students must be guided and encouraged to create their own systems or internalize the process of such creations; because by doing that they learn best or even reinvent mathematics. RME distinguishes between two types of "mathematization": horizontal and vertical mathematization. Horizontal mathematization involves set patterns, rules and models that should be learned and applied with given principles. Vertical mathematization, on the other hand, requires flexibility and allows students to shape and manipulate mathematical results. The first type relies primarily on memorization; whereas the second requires higher cognitive skills and activities. Students will develop attitudes to mathematics more in line with those preferred by mathematicians while standard mathematics lectures designed to "get through the material" may force them into rote-learning habits that mathematicians hate.

Unfortunately, many teachers still choose to teach mathematics "as a set of rules of processing or…algorithms" because "it is the way they learned it themselves" (Freudenthal, 1991, p. 3). Those instructors, I believe, are suspected to be the ones who usually tend to overemphasize details and conditions earlier on, at the expense of suppressing mathematical intuition and free "guessing".

In the examples below I distinguish between two types of generalizations:

1. The chain rule for the case of $y = f(u(x))$ is $\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$. In the case of higher dimensions it translates into $\frac{dz}{dt} = f_x \frac{dx}{dt} + f_y \frac{dy}{dt}$, when $z = f(x, y)$ and $x = x(t), y = y(t)$.

2. Total differential: $dy = f'(x)dx$ for a function of one variable translates into: $dz = f_x dx + f_y dy$. For the case of a function of two variables. A plausible question would be: why not the average of the last two addends? Specially that the error's upper bound involves an average. $|E| \leq \frac{1}{2} Max(f_{xx}, f_{xy}, f_{yy})$, where the maximum is taken over a certain domain.

3. Taylor theorem as an extension of linearization.

4. Independence of path in the case of line integrals as an expansion of the Fundamental theorem of calculus (into line integrals).

5. Fourier series as an inspiration from Taylor theorem: while Taylor theorem models certain functions as power series, Fourier's theorem models certain periodic functions as trigonometric series.

There is an obvious distinction between two different types of generalizations in the examples above: when a result A is a generalization of a result B, then A could be seen as a special case of B. However when a result D is an abstraction of a result C, then it means that they both share properties. Or that are applied to different fields altogether, or that the properties themselves are extensions of one another. I refer to this distinction by vertical and horizontal transition:

- The chain rule for the case of a function of a single variable can be seen as a special case of the chain rule for a function of several variables; likewise, linearization can be perceived as a special case of Taylors theorem for $n = 1$. Moreover, the Fundamental Theorem of Calculus $\int_a^b f'(x)dx = f(b) - f(a)$ can be perceived as a special case of the path independence theorem in the case of line integral.
  According to Niss (1999) one of the major findings of research in mathematics education is the key role of domain specificity. The student's conception of a mathematical concept is determined by the set of specific domains in which that concept has been introduced for the student. By expanding the domain the problem of domain specificity would be transcended.
- On the other hand, Taylor's theorem or Taylor approximation may not be thought of as a special case of Fourier Theorem. But the similarity is clear in that both of