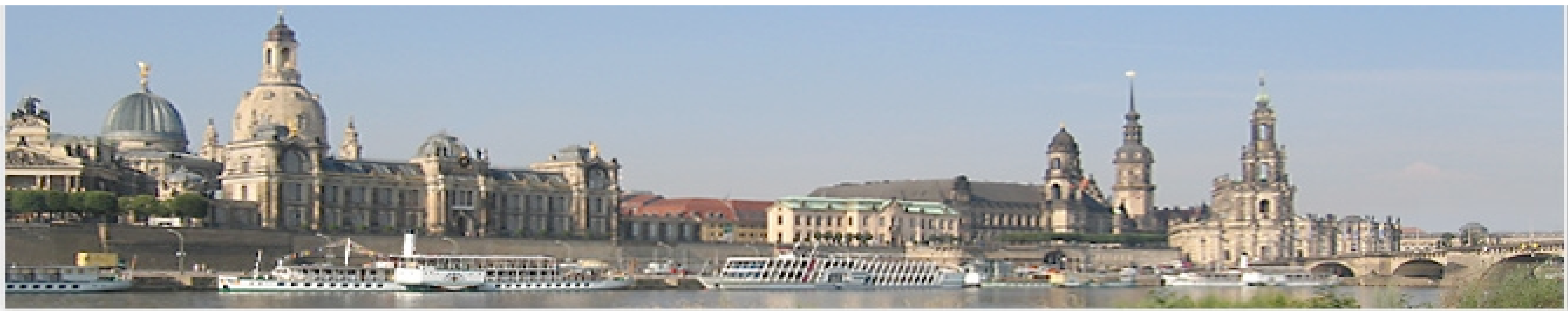
Using the ClassPad300Plus in Analysis to Solve a System of Linear Differential Equations

Ludwig Paditz, University of Applied Sciences Dresden (FH), Germany



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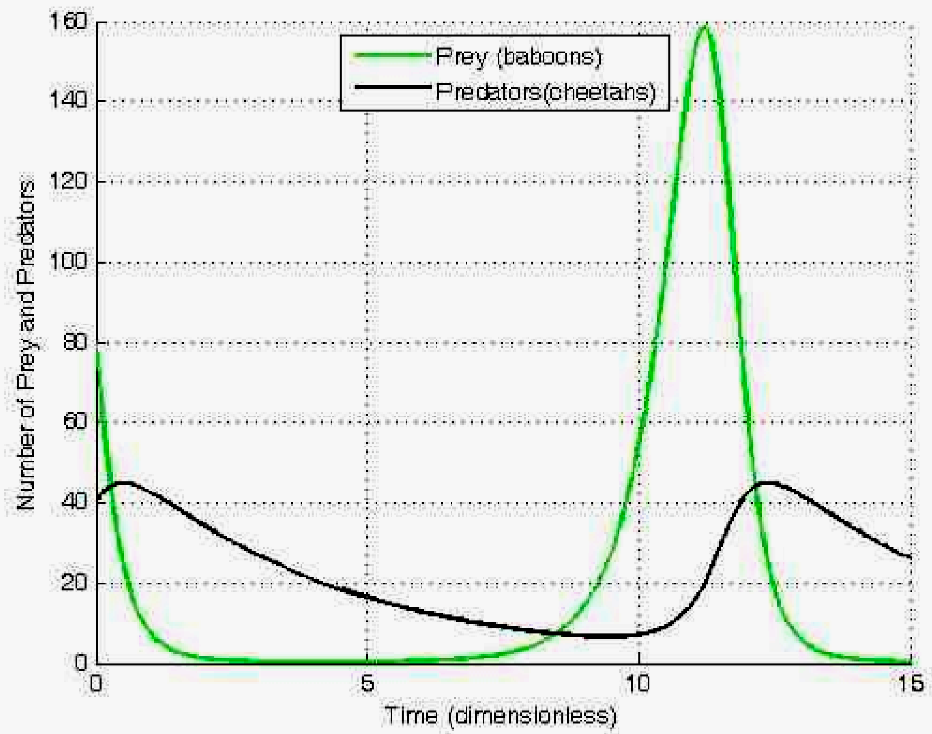
Abstract:

- In real life situations quantities and their rate of change depend on more than one variable. For example, the rabbit population, though it may be represented by a single number, depends on the size of predator populations and the availability of food. In order to represent and study such complicated problems we need to use more than one dependent variable and more than one equation. **Systems of differential equations are the models to use.**

[edit] An example problem

Suppose there are two species of animals, a baboon (prey) and a cheetah (predator). If the initial conditions are 80 baboons and 40 cheetahs, one can plot the progression of the two species over time. Time is dimensionless.



One can also plot a solution which corresponds to the oscillatory nature of the population of the two species. At any given time, the solution is somewhere on the inside of these elliptical solutions.

- **The nonlinear systems are very hard to solve explicitly,** but qualitative and numerical techniques may help us to get some information on the behaviour of the solutions.

- Let us consider the **ClassPad300Plus** (with the new operating system **OS 03.02**) and discuss on some new exercises in analysis, **e.g. solving a linear system of differential equations.**

- We know several ways to get a solution. The techniques for studying systems fall into the following three categories:

  analytic, ***graphic*** and ***numeric***.

- We can transform a **system of equations in one equation of higher order**
- and we have for

  linear systems with initial conditions the possibility to use the

  Laplace transformation.

- On the other hand we can transform a **system of differential equations** in a **system of difference equations**, i.e. sequences of numbers given by the help of **recursive equations**. These sequences are used as a **discrete mathematical model** for differential equations.

- The **ClassPad300** has the **dSolve**- and the **rSolve**-function to study systems of differential and difference equations respectively and additionally the **Laplace** and **inverse Laplace** transformation.

- Finally we have the possibility to generate large **dSolve-** or **rSolve-**terms by the help of commands for strings and characters.

- Thus the calculator can generate the large syntax for the used **dSolve**- and **rSolve**-function.

- This is a convenient method to input a long command row not manually but by the help of a program.

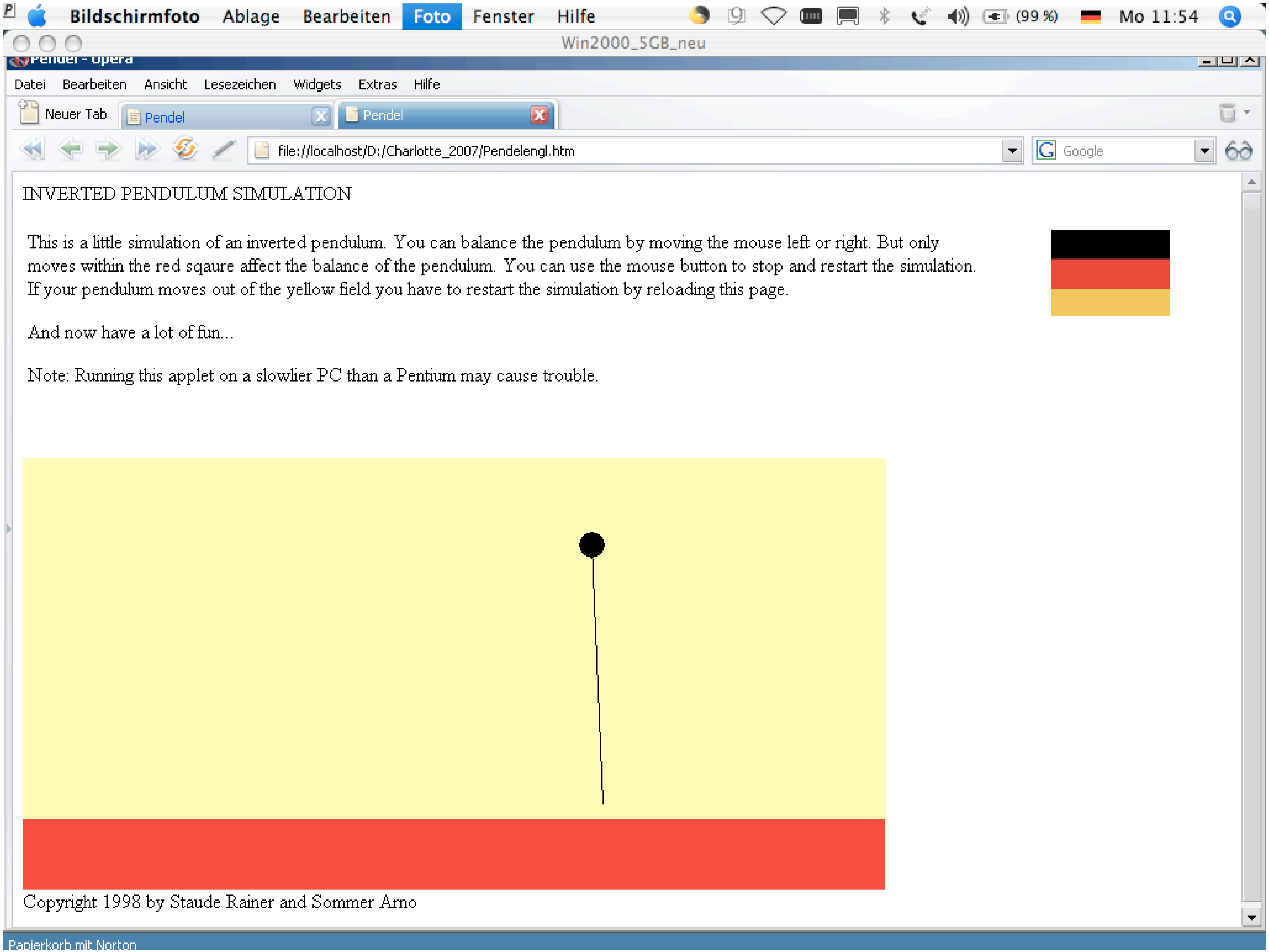
- By the help of several examples **the interactive work with the ClassPad300Plus** is considered.

- The student can solve difficult exercises of practical applications **step by step** using the symbolic calculation and the graphic possibilities of the calculator.

- Sometimes several fields of mathematics are combined to solve a problem.

Example of finding the mathematical model and several ways of solution:

- The following mathematical model due to an **inverted pendulum**, cp.
  http://www.fh-kempten.de/deu/hochschule/fachbereiche/fbe/labore/digital/homepage/swpr/ss98/Staude_Sommer/Pendel/Pendelengl.htm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INVERTED PENDULUM SIMULATION
This is a little simulation of an inverted pendulum. You can balance the pendulum by moving the mouse left or right. But only moves within the red sqaure affect the balance of the pendulum. You can use the mouse button to stop and restart the simulation. If your pendulum moves out of the yellow field you have to restart the simulation by reloading this page.
And now have a lot of fun...
Note: Running this applet on a slowlier PC than a Pentium may cause trouble.
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Papierkorb mit Norton

INVERTED PENDULUM SIMULATION

This is a little simulation of an inverted pendulum.

You can balance the pendulum by moving the mouse left or right. But only moves within the red square affect the balance of the pendulum. You can use the mouse button to stop and restart the simulation. If your pendulum moves out of the yellow field you have to restart the simulation by reloading this page.

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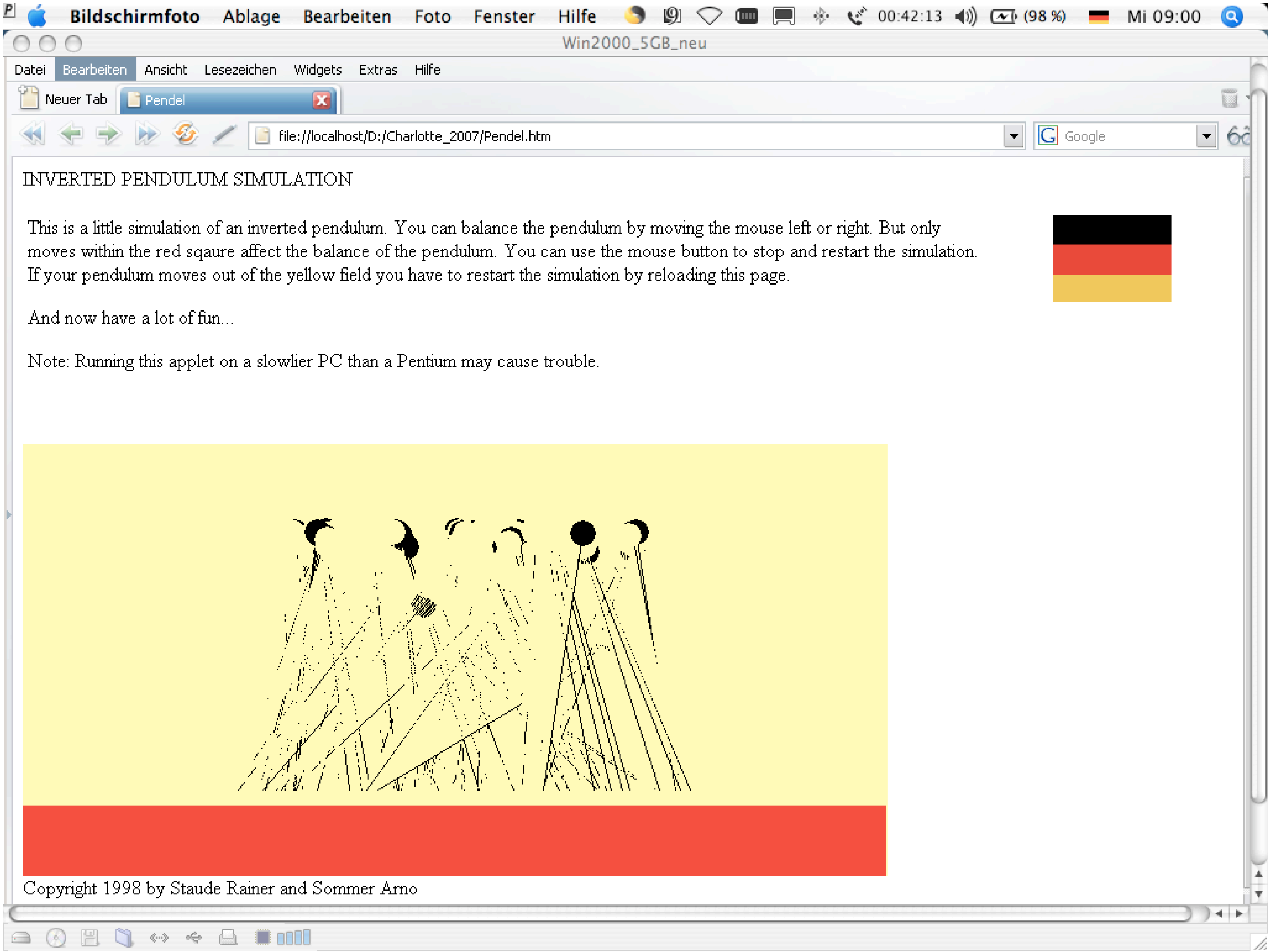
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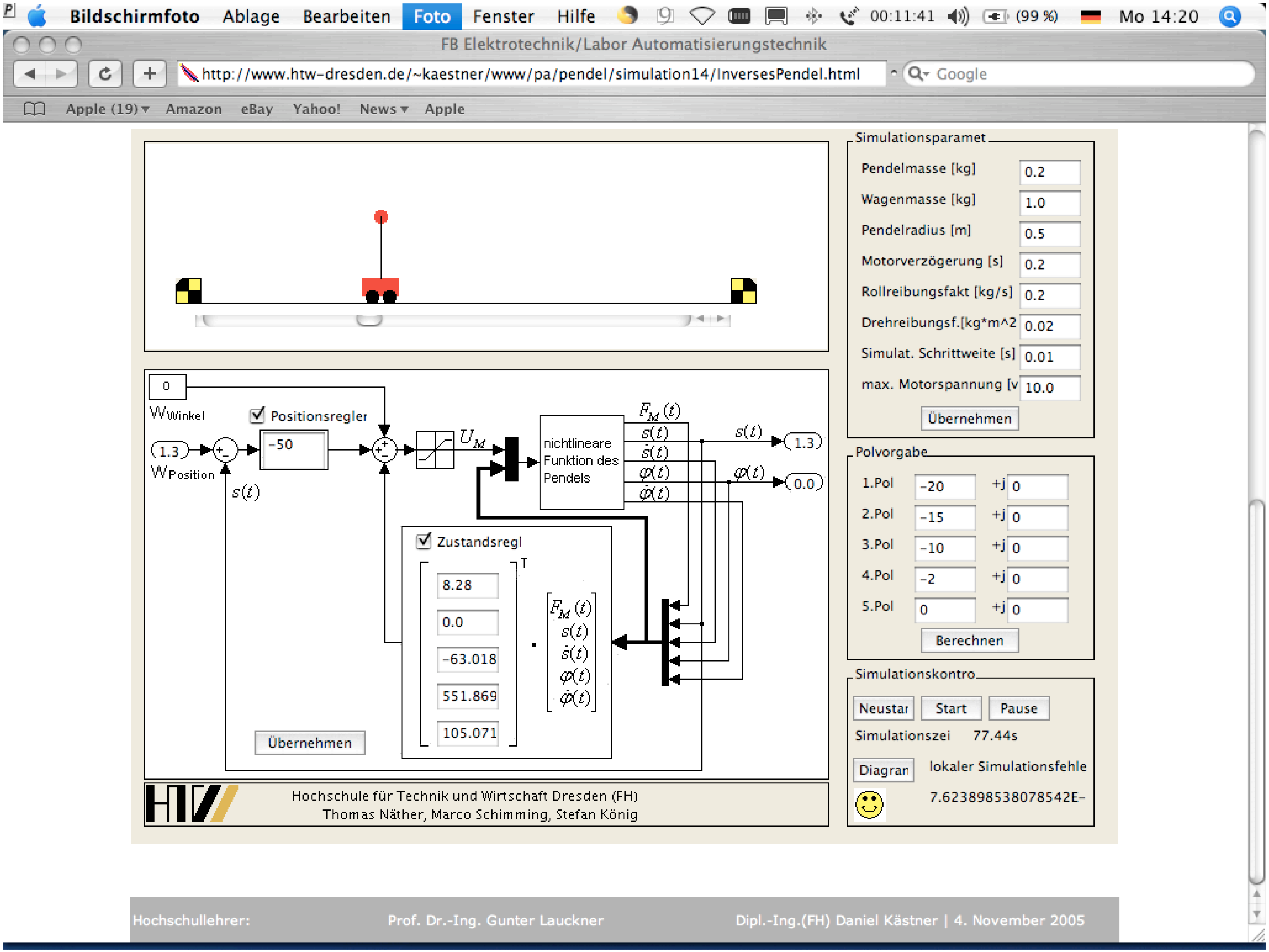
http://instruct1.cit.cornell.edu/courses/ee476/FinalProjects/s2003/es89kh98/es89kh98/Inverted_Pendulum_Balancer.mov

http://instruct1.cit.cornell.edu/courses/ee476/FinalProjects/s2003/es89kh98/es89kh98/

- **Inverted Pendulum Balancer**
- The goal of this project was to build and implement an inverted pendulum balancer, **in the vertical two dimensional plane,** using Proportional-Integral-Derivative (PID) feedback control.
- Motivated by the School of Mechanical & Aerospace Engineering's Feedback Control Systems course at Cornell University, the desire was to integrate the knowledge of stabilizing an unstable system using feedback control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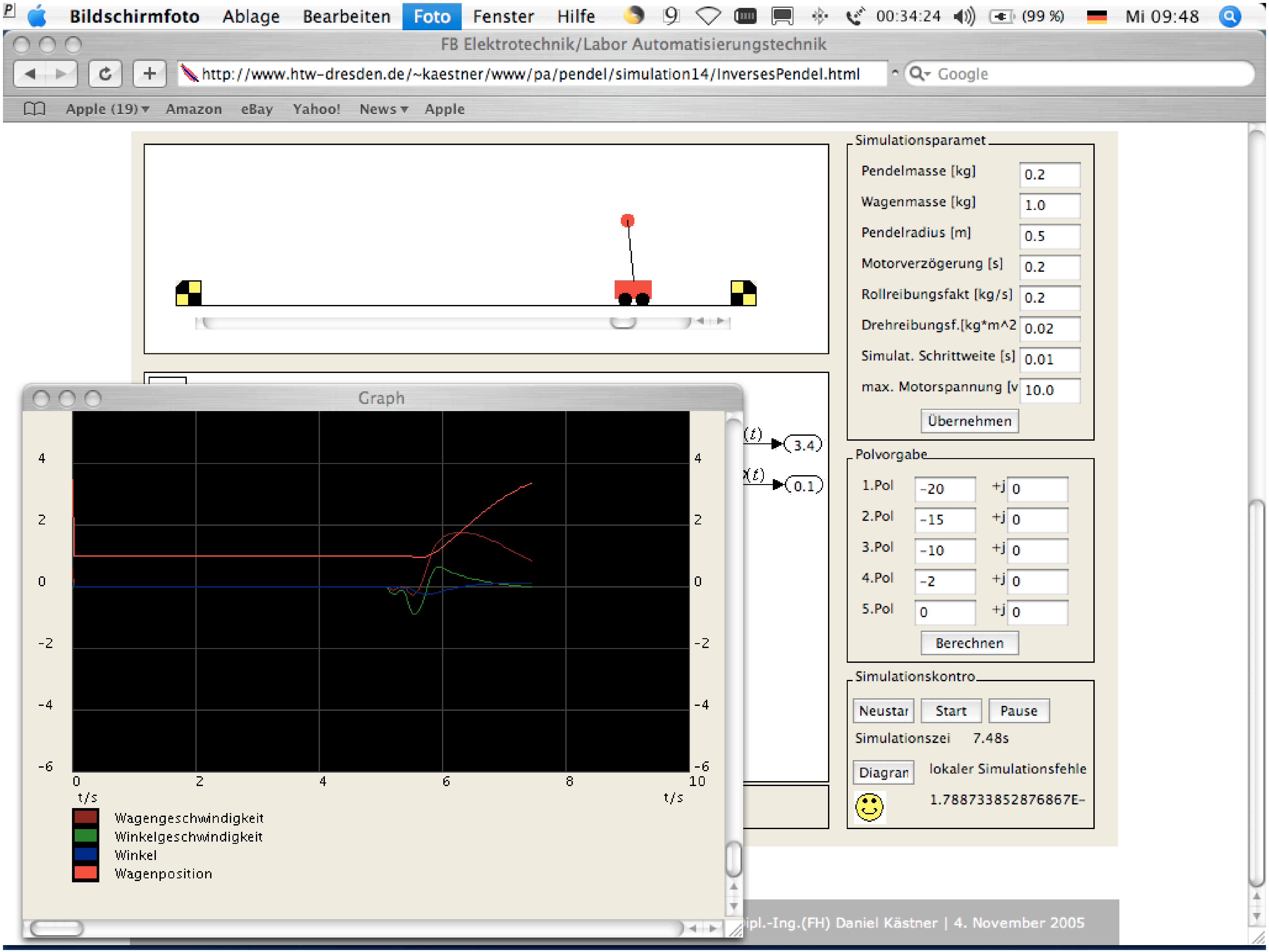
http://www.htw-dresden.de/~kaestner/www/pa/pendel/simulation14/InversesPendel.html
Simulationsparamet
Pendelmasse [kg] 0.2
Wagenmasse [kg] 1.0
Pendelradius [m] 0.5
Motorverzögerung [s] 0.2
Rollreibungsfakt [kg/s] 0.2
Drehreibungsf.[kg*m^2 0.02
Simulat. Schrittweite [s] 0.01
max. Motorspannung [v 10.0
Übernehmen
Polvorgabe
1.Pol -20 +j 0
2.Pol -15 +j 0
3.Pol -10 +j 0
4.Pol -2 +j 0
5.Pol 0 +j 0
Berechnen
Simulationskontro
Neustar Start Pause
Simulationszei 77.44s
Diagran
lokaler Simulationsfehle
7.623898538078542E-
Positionsregler
-50
nichtlineare Funktion des Pendels
Zustandsregl
8.28
0.0
-63.018
551.869
105.071
Übernehmen
Hochschule für Technik und Wirtschaft Dresden (FH)
Thomas Näther, Marco Schimming, Stefan König
Hochschullehrer: Prof. Dr.-Ing. Gunter Lauckner
Dipl.-Ing.(FH) Daniel Kästner | 4. November 2005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http://www.htw-dresden.de/~kaestner/www/pa/pendel/simulation14/InversesPendel.html
Google
Apple (19) Amazon eBay Yahoo! News Apple
Simulationsparamet
Pendelmasse [kg] 0.2
Wagenmasse [kg] 1.0
Pendelradius [m] 0.5
Motorverzögerung [s] 0.2
Rollreibungsfakt [kg/s] 0.2
Drehreibungsf.[kg*m^2 0.02
Simulat. Schrittweite [s] 0.01
max. Motorspannung [v 10.0
Übernehmen
Polvorgabe
1.Pol -20 +j 0
2.Pol -15 +j 0
3.Pol -10 +j 0
4.Pol -2 +j 0
5.Pol 0 +j 0
Berechnen
Simulationskontro
Neustar Start Pause
Simulationszei 7.48s
Diagran
lokaler Simulationsfehle
1.788733852876867E-
Graph
3.4
0.1
4
2
0
-2
-4
-6
0 2 4 6 8 10
t/s
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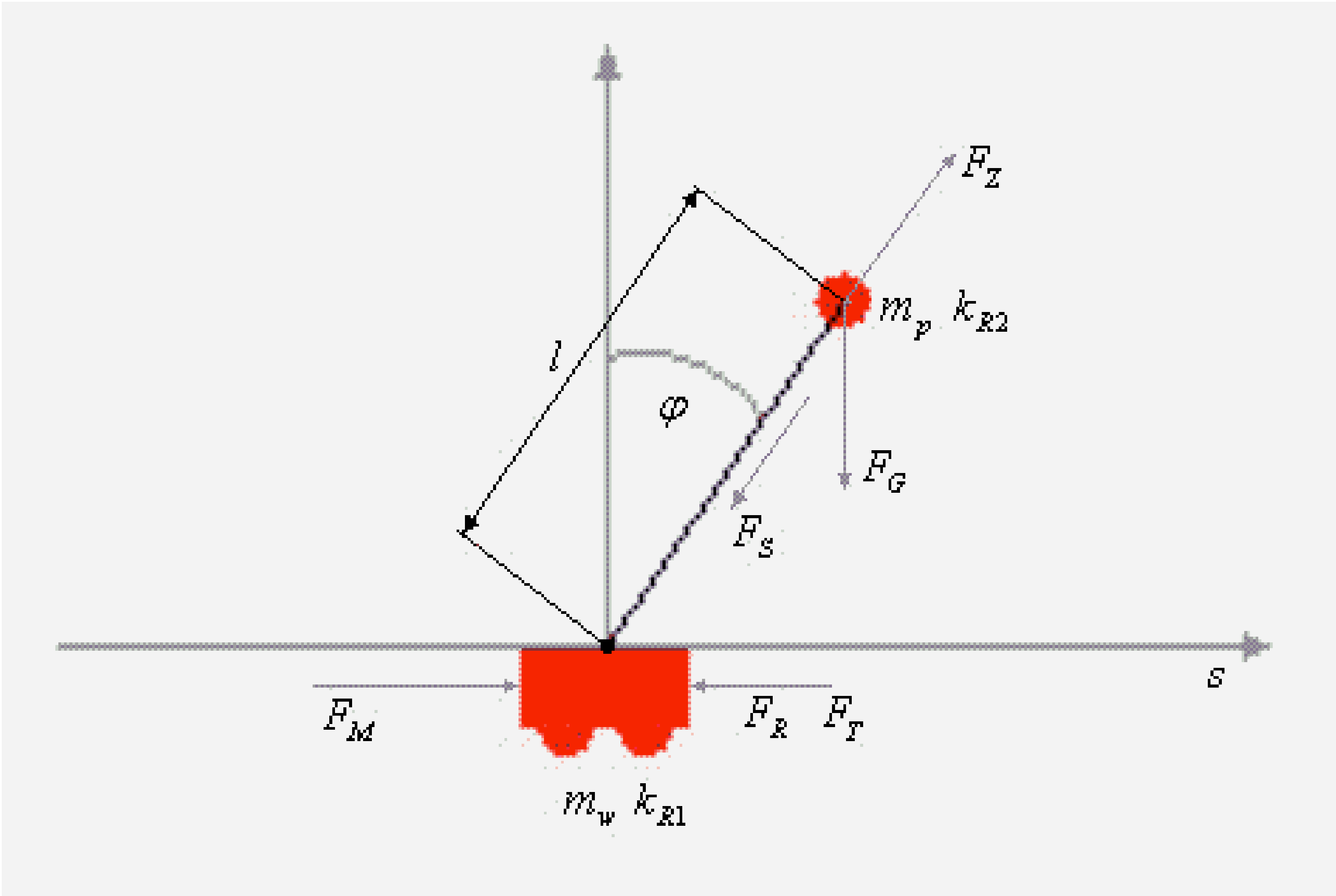
Winkelgeschwindigkeit
Winkel
Wagenposition
ipl.-Ing.(FH) Daniel Kästner | 4. November 2005



F_Z
m_p k_{R2}
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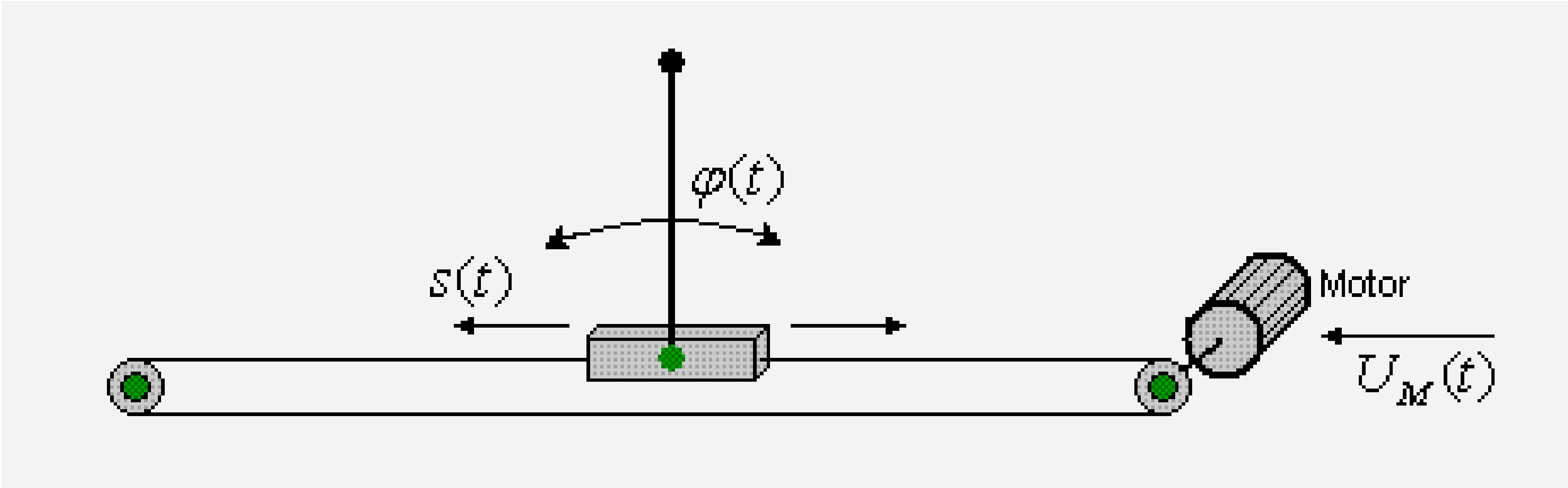
φ
F_G
F_S
s
F_M
F_R F_T
m_w k_{R1}



A complete analytic model of the inverted pendulum controlled by a DC motor is derived in three parts, the pendulum-cart dynamics, the friction model, and the motor dynamics.

Here we will study the dynamics of the DC motor by the following equations:

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Solve a System of Linear Differential Equations (Inverted Pendulum)

Consider the linear state space model for the armature-controlled DC motor

$\frac{d}{dt}(\phi(t))=\frac{d}{dt}(\phi(t)) \Rightarrow Equ1$

$$\frac{d}{dt}(\phi(t))=\frac{d}{dt}(\phi(t))$$

$\frac{d^2}{dt^2}(\phi(t))=-\frac{B_m}{J_m+r^2\times M}\times\frac{d}{dt}(\phi(t))+\frac{K_m}{J_m+r^2\times M}\times Ia(t) \Rightarrow Equ2$

$$\frac{d^2}{dt^2}(\phi(t))=\frac{K_m\cdot Ia(t)}{J_m+M\cdot r^2}-\frac{B_m\cdot\frac{d}{dt}(\phi(t))}{J_m+M\cdot r^2}$$

$\frac{d}{dt}(Ia(t))=-\frac{Kb}{La}\times\frac{d}{dt}(\phi(t))-\frac{Ra}{La}\times Ia(t)+\frac{1}{La}\times Va(t) \Rightarrow Equ3$

$$\frac{d}{dt}(Ia(t))=-\frac{Ra\cdot Ia(t)}{La}+\frac{Va(t)}{La}-\frac{Kb\cdot\frac{d}{dt}(\phi(t))}{La}$$

matrix form

$$\begin{bmatrix}\frac{d}{dt}(\phi(t))\\ \frac{d^2}{dt^2}(\phi(t))\\ \frac{d}{dt}(Ia(t))\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & -\frac{B_m}{J_m+r^2\times M} & \frac{K_m}{J_m+r^2\times M}\\ 0 & -\frac{Kb}{La} & -\frac{Ra}{La}\end{bmatrix}\times\begin{bmatrix}\phi(t)\\ \frac{d}{dt}(\phi(t))\\ Ia(t)\end{bmatrix}+\begin{bmatrix}0\\ 0\\ \frac{1}{La}\end{bmatrix}\times u(t)$$

technical parameters, e.g.: $\begin{bmatrix}1[kg]\\ 0.025[m]\\ 0.001[Nms^2/rad]\\ 0.006[Vs]\\ 0.00625[Nms/rad]\\ 0.006[Vs]\\ 1[V/A]\\ 0.001[H]\end{bmatrix} \Rightarrow \begin{bmatrix}M\\ r\\ J_m\\ K_m\\ B_m\\ Kb\\ Ra\\ La\end{bmatrix}$

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Define $x(t)=\begin{bmatrix}\phi(t)\\ \frac{d}{dt}(\phi(t))\\ Ia(t)\end{bmatrix}$

done

system is now: $\frac{d}{dt}(x(t))=\mathbf{A}\times\mathbf{x(t)}+\mathbf{B}\times\mathbf{u(t)}$ and $\mathbf{y(t)}=\mathbf{C}\times\mathbf{x(t)}$ with **C=[1,0,0]**

$$\begin{bmatrix}1\\0.025\\0.000\\0.006\\0.00625\\0.006\\1\\0.001\end{bmatrix}\Rightarrow\begin{bmatrix}M\\r\\J_m\\K_m\\B_m\\Kb\\Ra\\La\end{bmatrix}$$

$$\begin{bmatrix}1\\0.025\\0\\0.006\\0.00625\\0.006\\1\\0.001\end{bmatrix}$$

$$\begin{bmatrix}0 & 1 & 0\\ 0 & -\frac{B_m}{J_m+r^2\times M} & \frac{K_m}{J_m+r^2\times M}\\ 0 & -\frac{Kb}{La} & -\frac{Ra}{La}\end{bmatrix}\Rightarrow A$$

$$\begin{bmatrix}0 & 1 & 0\\ 0 & -10 & 9.6\\ 0 & -6 & -1000\end{bmatrix}$$

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[1 0 0]⇒C

[1 0 0]

create the **controllability matrix Ss:**

augment(augment(B,A×B),A^2×B)⇒Ss

$$\begin{bmatrix} 0 & 0 & 9600 \\ 0 & 9600 & -9696000 \\ 1000 & -1000000 & 999942400 \end{bmatrix}$$

controllability matrix **Ss with full rank** (Ss^{-1} exists), i.e. **linear model is controllable:**

Ss^{-1}

$$\begin{bmatrix} 1.047666667 & 0.1041666667 & 0.001 \\ 0.1052083333 & 0.00010416666 & 0 \\ 0.00010416666 & 0 & 0 \end{bmatrix}$$

using the **eigenvalues** (-.4±.3j and -10) for **Ackermann's formula** to get the **feedback gain matrix K:**

Define q(λ)=(λ-(-.4+.3j))×(λ-(-.4-.3j))×(λ-(-10))

done

cExpand(q(λ))

$2.5+8.25\cdot\lambda+10.8\cdot\lambda^2+\lambda^3$

Alg Decimal Real Rad

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the feedback gain matrix K:

[0 0 1]×Ss^{-1}×(2.5·I+8.25·A^1+10.8·A^2+A^3)⇒K

[0.00026041666 -0.00597395833 -0.9992]

approx(A-B×K)⇒matAK

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -10 & 9.6 \\ -0.2604166667 & -0.02604166667 & -0.8 \end{bmatrix}$$

check the **eigenvalues of matAK** using eigVl or solve function:

eigVl(matAK)

{0.3·j-0.4,-0.3·j-0.4,-10}

solve(det(matAK-λ×I)=0,λ)

{λ=-0.3·j-0.4,λ=0.3·j-0.4,λ=-10}

==

solution of $\frac{d}{dt}$(x(t))=**matAK×x(t)** is (with unknown coefficients)

$$\mathbf{x(t)}=\begin{bmatrix} A1 \\ A2 \\ A3 \end{bmatrix}\times e^{-.4t}\mathbf{cos(.3t)}+\begin{bmatrix} B1 \\ B2 \\ B3 \end{bmatrix}\times e^{-.4t}\mathbf{sin(.3t)}+\begin{bmatrix} C1 \\ C2 \\ C3 \end{bmatrix}\times e^{-10t}$$

==

now consider the components of x(t) with the argument x:
(at first delete the vector x)

DelVar x

done

Define y1(x)=A1$e^{-.4x}$cos(.3x)+B1$e^{-.4x}$sin(.3x)+C1e^{-10x}

done

Define y2(x)=A2$e^{-.4x}$cos(.3x)+B2$e^{-.4x}$sin(.3x)+C2e^{-10x}

done

Define y3(x)=A3$e^{-.4x}$cos(.3x)+B3$e^{-.4x}$sin(.3x)+C3e^{-10x}

done

$\frac{d}{dx}$(y1(x))=dotP([1 0 0]×matAK,[y1(x) y2(x) y3(x)])⇒equ1

-2·x -2·x -2·x -2·x

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$\frac{d}{dx}(y1(x))$=dotP([1 0 0]×matAK,[y1(x) y2(x) y3(x)])⇒equ1

$$-10\cdot C1\cdot e^{-10\cdot x}-\frac{2\cdot B1\cdot \sin\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{5}+\frac{3\cdot B1\cdot \cos\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{10}-\frac{3\cdot A1\cdot \sin\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{10}-\frac{2\cdot A1\cdot \cos\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{5}=co$$

$\frac{d}{dx}(y2(x))$=dotP([0 1 0]×matAK,[y1(x) y2(x) y3(x)])⇒equ2

$$-10\cdot C2\cdot e^{-10\cdot x}-\frac{2\cdot B2\cdot \sin\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{5}+\frac{3\cdot B2\cdot \cos\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{10}-\frac{3\cdot A2\cdot \sin\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{10}-\frac{2\cdot A2\cdot \cos\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{5}=\frac{4\ldots}{}$$

$\frac{d}{dx}(y3(x))$=dotP([0 0 1]×matAK,[y1(x) y2(x) y3(x)])⇒equ3

$$-10\cdot C3\cdot e^{-10\cdot x}-\frac{2\cdot B3\cdot \sin\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{5}+\frac{3\cdot B3\cdot \cos\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{10}-\frac{3\cdot A3\cdot \sin\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{10}-\frac{2\cdot A3\cdot \cos\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{5}=\frac{-4\ldots}{}$$

DelVar A1,A2,A3,B1,B2,B3,C1,C2,C3

done

Now solution of the **linear system** $\begin{bmatrix}\frac{d}{dx}(y1(x))\\ \frac{d}{dx}(y2(x))\\ \frac{d}{dx}(y3(x))\end{bmatrix}$=**matAK**×$\begin{bmatrix}y1(x)\\ y2(x)\\ y3(x)\end{bmatrix}$ with unknown coefficients A, A2, ..., C2, C3

for the **initial conditions y1=1, y2=-1, y3=1 for x=0:**

approx(equ1|x=0)
approx(equ2|x=0)
approx(equ3|x=0)
approx(equ1|x=.1)

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```
⎧ approx(equ1|x=0)
⎪ approx(equ2|x=0)
⎪ approx(equ3|x=0)
⎪ approx(equ1|x=.1)
⎨ approx(equ2|x=.1)
⎪ approx(equ3|x=.1)
⎪ approx(y1(0)=1)
⎪ approx(y2(0)=-1)
⎩ approx(y3(0)=1)    |A1,A2,A3,B1,B2,B3,C1,C2,C3
                                                        ERROR:Overflow
```

change in **real mode** now!

```
DelVar A1,A2,A3,B1,B2,B3,C1,C2,C3
                                                        done

⎧ approx(equ1|x=0)
⎪ approx(equ2|x=0)
⎪ approx(equ3|x=0)
⎪ approx(equ1|x=.1)
⎨ approx(equ2|x=.1)
⎪ approx(equ3|x=.1)
⎪ approx(y1(0)=1)
⎪ approx(y2(0)=-1)
⎩ approx(y3(0)=1)    |A1,A2,A3,B1,B2,B3,C1,C2,C3
{A1=0.793495935,A2=1.06504065,A3=1,B1=4.608130081,B2=-2.081300813,B3=-2.114583333,C1=0.206504065,C2=-2.▸

                                        ⎡A1⎤
                                        ⎢A2⎥
                                        ⎢A3⎥
                                        ⎢B1⎥
listToMat(getRight(approx(ans)))⇒       ⎢B2⎥
                                        ⎢B3⎥
                                        ⎢C1
```

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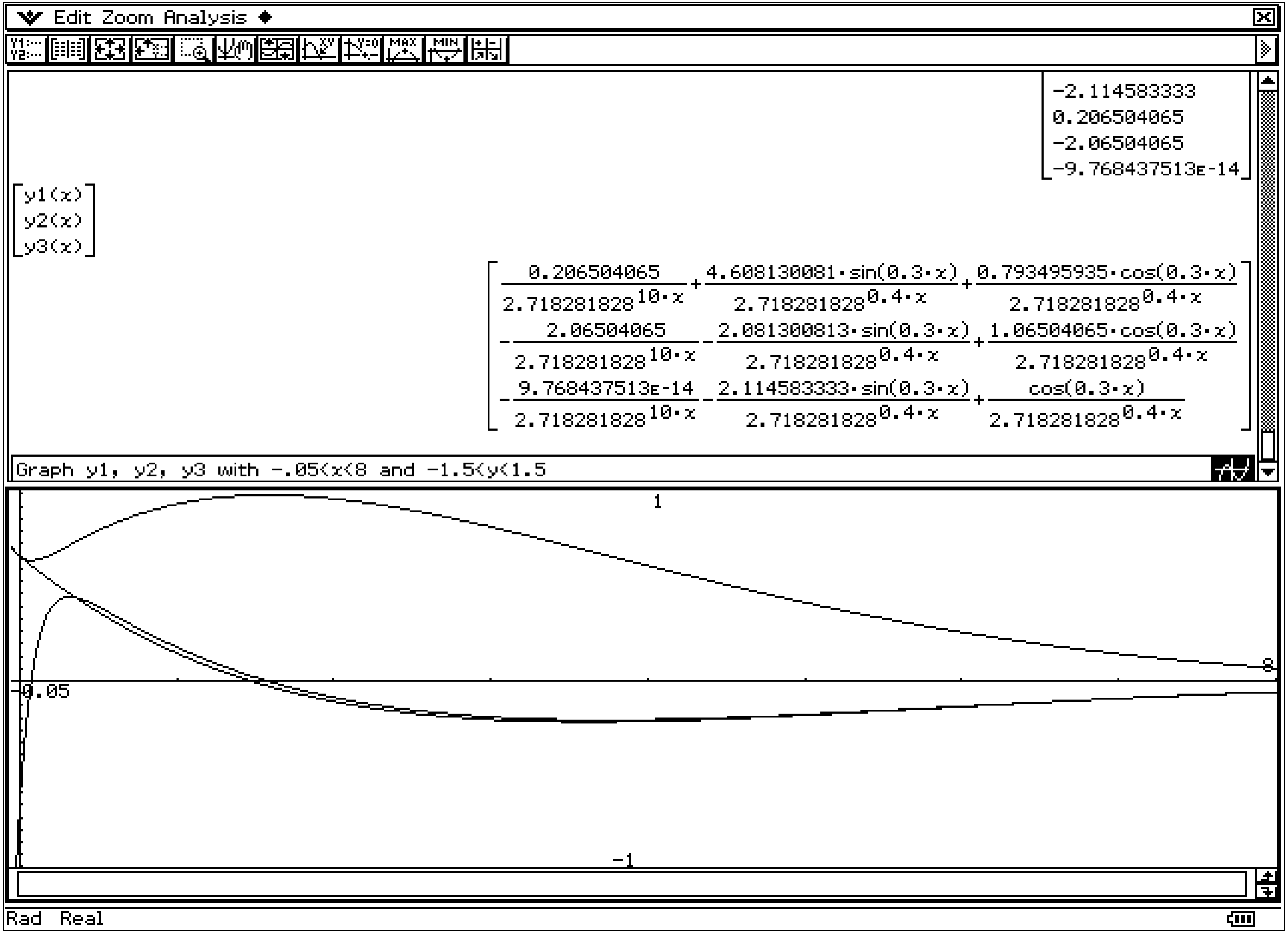
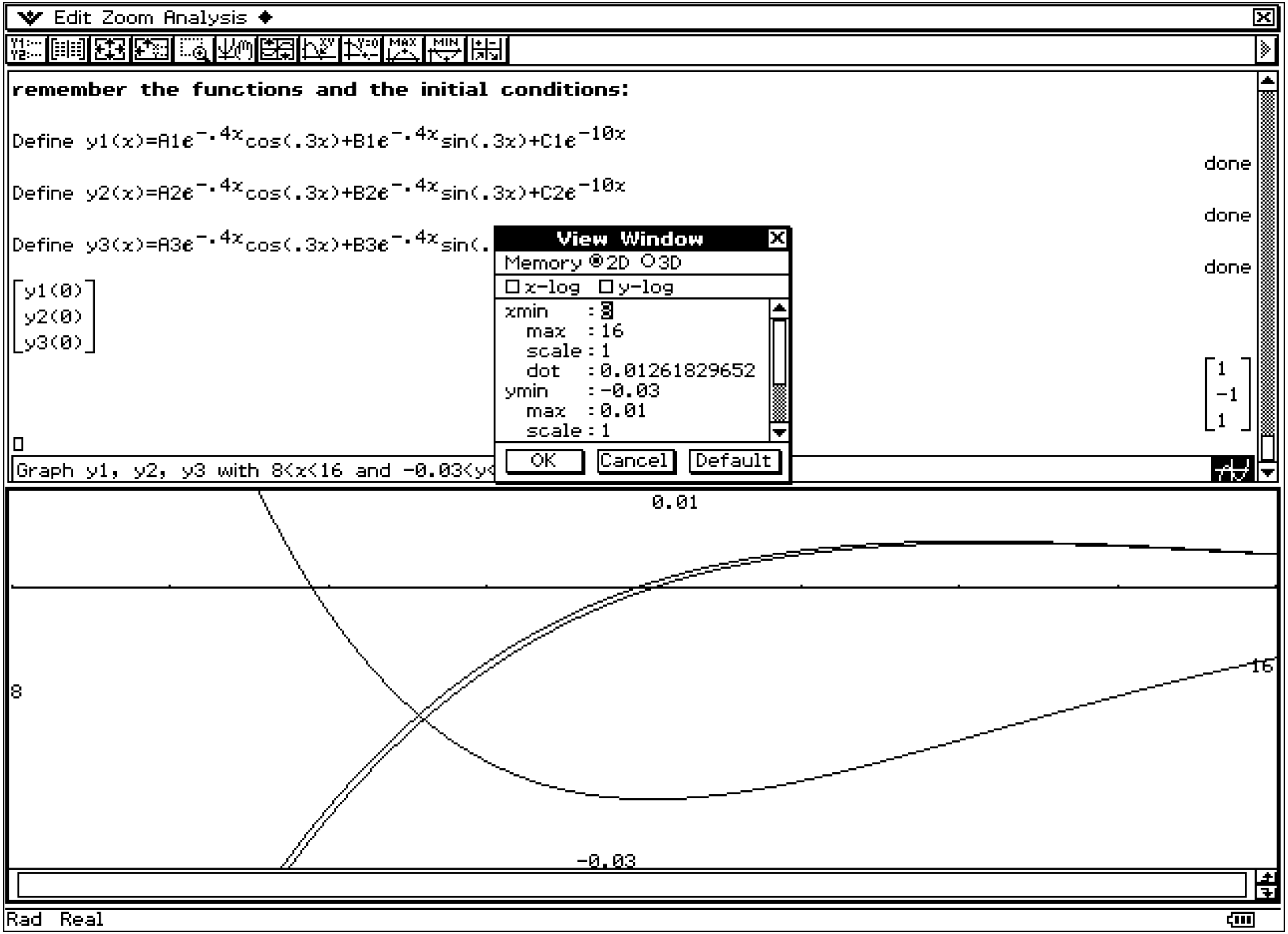


View window: -0.05 < x < 8 and -1.5 < y < 1.5 and graphical representation of y1, y2, y3

View window: 8 < x < 16 and -0.03 < y < 0.01



Now solving the system of order 3 by the help of one equation of 3rd order for y1 (Laplace Transformation):

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Now solving the system of order 3 by the help of one equation of 3rd order for y1 **(Laplace Transformation)**:

$\lambda^3-(matAK[2,2]+matAK[3,3])\lambda^2+det(subMat(matAK,2,2,3,3))\lambda-matAK[2,3]\times matAK[3,1]=0$

$$2.5+8.25\cdot\lambda+10.8\cdot\lambda^2+\lambda^3=0$$

$det(matAK-\lambda\times I)=0$

$$-2.5-8.25\cdot\lambda-10.8\cdot\lambda^2-\lambda^3=0$$

is the characteristic equation for $\frac{d^3}{dt^3}(y1)+10.8\frac{d^2}{dt^2}(y1)+8.25\frac{d}{dt}(y1)+2.5\cdot y1=0$

initial conditions y1(0)=1, y1'(0)=-1, y1''(0)=19.6

laplace(2.5y+8.25y'+10.8y''+y'''=0,t,y,s)

$$2.5\cdot Lp+Lp\cdot s^3-s^2\cdot y(0)-s\cdot y'(0)-y''(0)+8.25\cdot(Lp\cdot s-y(0))+10.8\cdot\left(Lp\cdot s^2-s\cdot y(0)-y'(0)\right)=0$$

ans|y(0)=1 and y'(0)=-1 and y''(0)=19.6

$$-19.6-8.25\cdot(1-Lp\cdot s)+10.8\cdot\left(1-s+Lp\cdot s^2\right)+s+2.5\cdot Lp+Lp\cdot s^3-s^2=0$$

solve(ans,Lp)

$$\left\{Lp=\frac{341+196\cdot s+20\cdot s^2}{50+165\cdot s+216\cdot s^2+20\cdot s^3}\right\}$$

$invlaplace\left(\frac{341+196\cdot s+20\cdot s^2}{50+165\cdot s+216\cdot s^2+20\cdot s^3},s,x\right)$

$$\frac{127\cdot e^{-10\cdot x}}{615}+\frac{2834\cdot\sin\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{615}+\frac{488\cdot\cos\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{615}$$

Define $f(x)=\frac{127\cdot e^{-10\cdot x}}{615}+\frac{2834\cdot\sin\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{615}+\frac{488\cdot\cos\left(\frac{3\cdot x}{10}\right)\cdot e^{\frac{-2\cdot x}{5}}}{615}$

done

check the initial conditions:

$\left[f(x)\quad\frac{d}{dx}(f(x))\quad\frac{d^2}{dx^2}(f(x))\right]|x=0$

$$[1\quad -1\quad 19.6]$$

Alg Decimal Real Rad

Finally another way of solution is the transformation in difference equations:

y'(t) = (y(t+T)-y(t)) / T for small T, say T=0.1.

Now the new system is

x(t+T) = x(t) + T*matAK*x(t) = (I + T*matAK)*x(t).

We use the fixpoint iteration $\mathbf{x_{k+1} = (I + T*matAK)*x_k}$ with $\mathbf{x_o=[1,-1,1]^T}$ and create 3 lists.

Now **matAKI = I + T*matAK**.

The program **DefLis3D** creates the lists for the components of x.

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Finally another way of solution: transformation in difference equations

see program DefLis3D and DefSeq3D.

0.1⇒T

0.1

I+T×matAK⇒matAKI

$$\begin{bmatrix}1 & 0.1 & 0\\ 0 & 0 & 0.96\\ -0.02604166667 & -0.00260416666 & 0.92\end{bmatrix}$$

main\DefLis3D($\begin{bmatrix}1\\ -1\\ 1\end{bmatrix}$,500)

done

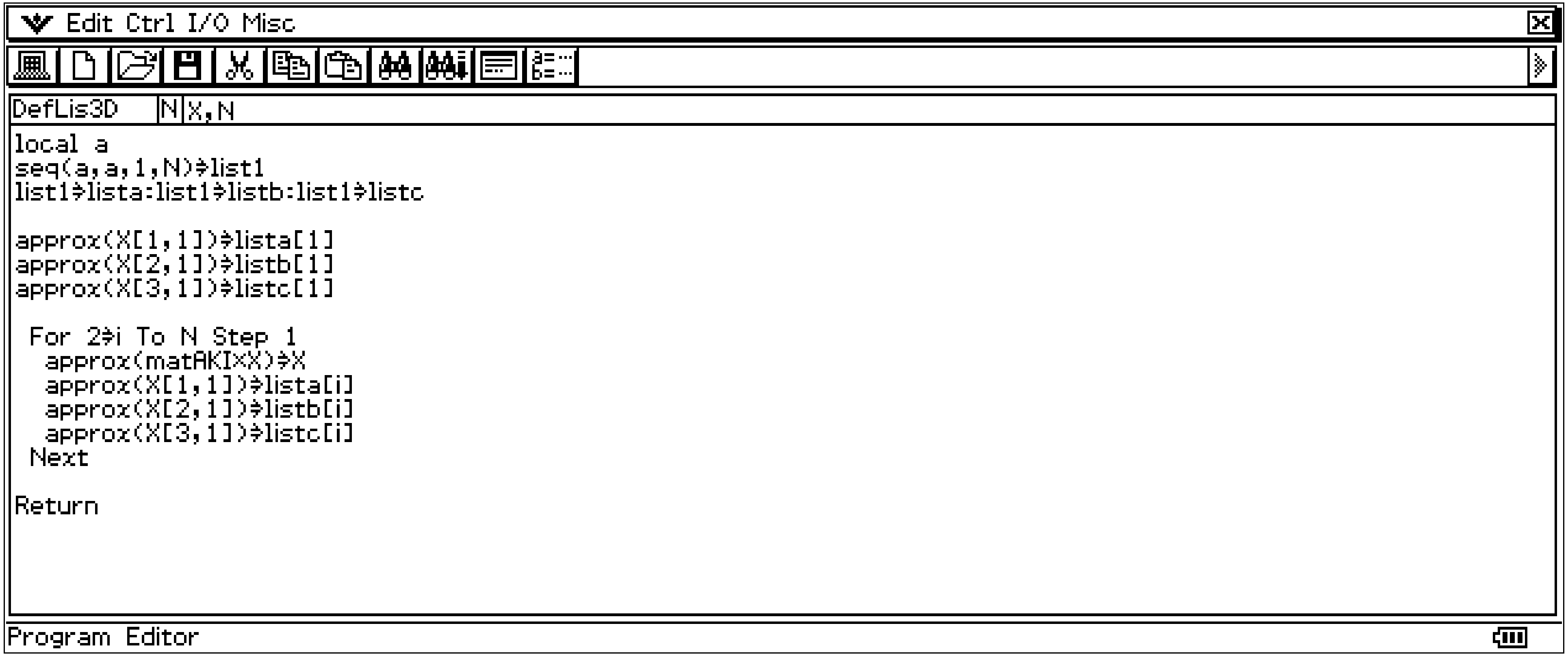
seq(a,a,1,500)⇒list1

{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,▸

	list1	lista	listb	listc	list5	list6
1	1	1	-1	1		
2	2	0.9	0.96	0.8965		
3	3	0.996	0.8607	0.7989		
4	4	1.082	0.7669	0.7068		
5	5	1.1587	0.6785	0.62		
6	6	1.2266	0.5952	0.5385		
7	7	1.2861	0.5169	0.4619		
8	8	1.3378	0.4434	0.3901		
9	9	1.3821	0.3745	0.3229		
10	10	1.4196	0.31	0.2601		
11	11	1.4506	0.2497	0.2015		
12	12	1.4756	0.1934	0.147		
13	13	1.4949	0.1411	0.0963		
14	14	1.509	0.0924	0.0493		
15	15	1.5183	0.0473	5.8E-3		
16	16	1.523	5.5E-3	-0.034		
17	17	1.5236	-0.032	-0.071		
Cal▸						

list= list1

Alg Decimal Real Rad



Finally we use the stat editor menu to create the x-y-lines for the given data in list1and lista, listb, listc respectively.

Edit Calc SetGraph

	list1	lista	listb	listc
1	1	1	-1	1
2	2	0.9	0.96	0.8965625
3	3	0.996	0.8607	0.7989
4	4	1.08207	0.766944	0.706809
5	5	1.1587644	0.6785367	0.6200882
6	6	1.226618	0.5952846	0.5385379
7	7	1.2861465	0.5169964	0.4619615
8	8	1.3378461	0.443483	0.3901648
9	9	1.3821944	0.3745582	0.322957
10	10	1.4196503	0.3100387	0.2601504
11	11	1.4506541	0.2497443	0.2015609
12	12	1.4756286	0.1934984	0.1470082
13	13	1.4949784	0.1411278	0.0963158
14	14	1.5090912	0.0924631	0.0493113
15	15	1.5183375	0.0473388	5.826E-3
16	16	1.5230714	5.593E-3	-0.034303
17	17	1.5236308	-0.03293	-0.071236
18	18	1.5203377	-0.068387	-0.105129
19	19	1.5134989	-0.100924	-0.136133
20	20	1.5034065	-0.130688	-0.164394
21	21	1.4903377	-0.157818	-0.190053
22	22	1.4745558	-0.182451	-0.213249
23	23	1.4563107	-0.204719	-0.234113
24	24	1.4358388	-0.224749	-0.252776
25	25	1.4133639	-0.242665	-0.26936
26	26	1.3890973	-0.258586	-0.283986
27	27	1.3632387	-0.272626	-0.296768
28	28	1.335976	-0.284897	-0.307817
29	29	1.3074863	-0.295505	-0.317241
30	30	1.2779357	-0.304551	-0.325141
31	31	1.2474806	-0.312136	-0.331616
32	32	1.2162669	-0.318352	-0.336761
33	33	1.1844317	-0.32329	-0.340664
34	34	1.1521026	-0.327038	-0.343414
35	35	1.1193988	-0.329677	-0.345092
36	36	1.086431	-0.331288	-0.345777
37	37	1.0533022	-0.331946	-0.345544
38	38	1.0201075	-0.331723	-0.344466
39	39	0.9869352	-0.330687	-0.34261

Cal▸

[1]= 1

Deg Auto Decimal

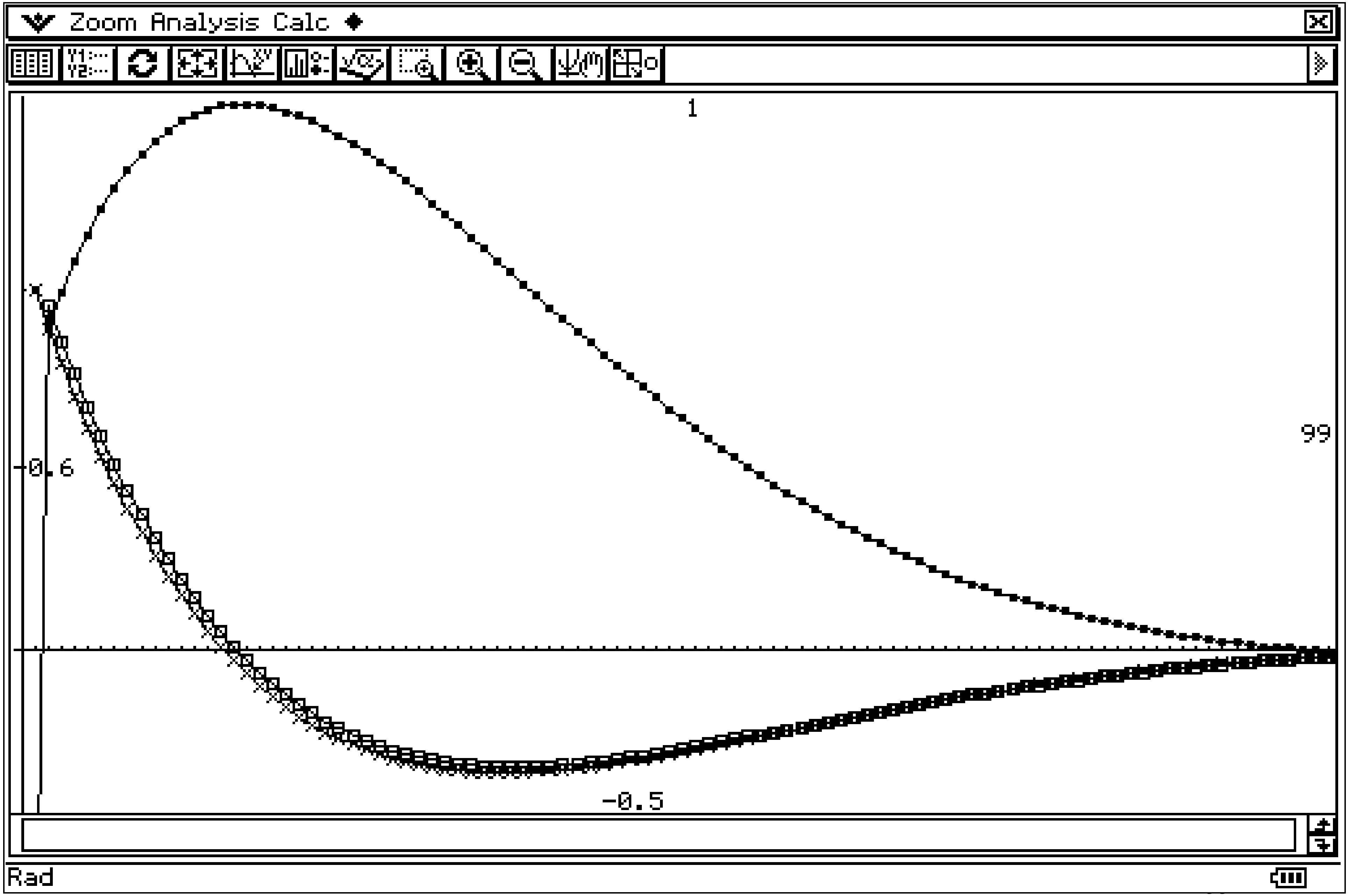
Edit Calc SetGraph

	list1	lista	listb	listc
462	462	3.565E-8	-1.92E-8	-1.93E-8
463	463	3.373E-8	-1.85E-8	-1.86E-8
464	464	3.187E-8	-1.79E-8	-1.8E-8
465	465	3.008E-8	-1.73E-8	-1.73E-8
466	466	2.835E-8	-1.66E-8	-1.67E-8
467	467	2.668E-8	-1.6E-8	-1.6E-8
468	468	2.508E-8	-1.54E-8	-1.54E-8
469	469	2.353E-8	-1.48E-8	-1.48E-8
470	470	2.205E-8	-1.42E-8	-1.42E-8
471	471	2.063E-8	-1.36E-8	-1.36E-8
472	472	1.926E-8	-1.3E-8	-1.3E-8
473	473	1.796E-8	-1.25E-8	-1.24E-8
474	474	1.671E-8	-1.19E-8	-1.18E-8
475	475	1.551E-8	-1.14E-8	-1.13E-8
476	476	1.437E-8	-1.08E-8	-1.08E-8
477	477	1.328E-8	-1.03E-8	-1.02E-8
478	478	1.225E-8	-9.87E-9	-9.78E-9
479	479	1.126E-8	-9.39E-9	-9.29E-9
480	480	1.032E-8	-8.92E-9	-8.82E-9
481	481	9.43E-9	-8.46E-9	-8.36E-9
482	482	8.584E-9	-8.02E-9	-7.91E-9
483	483	7.781E-9	-7.59E-9	-7.48E-9
484	484	7.021E-9	-7.18E-9	-7.06E-9
485	485	6.303E-9	-6.78E-9	-6.66E-9
486	486	5.624E-9	-6.4E-9	-6.28E-9
487	487	4.984E-9	-6.02E-9	-5.9E-9
488	488	4.381E-9	-5.67E-9	-5.54E-9
489	489	3.814E-9	-5.32E-9	-5.2E-9
490	490	3.281E-9	-4.99E-9	-4.87E-9
491	491	2.782E-9	-4.67E-9	-4.55E-9
492	492	2.314E-9	-4.37E-9	-4.25E-9
493	493	1.876E-9	-4.08E-9	-3.96E-9
494	494	1.468E-9	-3.8E-9	-3.68E-9
495	495	1.088E-9	-3.53E-9	-3.41E-9
496	496	7.34E-10	-3.27E-9	-3.16E-9
497	497	4.07E-10	-3.03E-9	-2.91E-9
498	498	1.03E-10	-2.8E-9	-2.68E-9
499	499	-1.7E-10	-2.58E-9	-2.46E-9
500	500	-4.3E-10	-2.37E-9	-2.26E-9

Cal▸

[1]= 1

Deg Auto Decimal

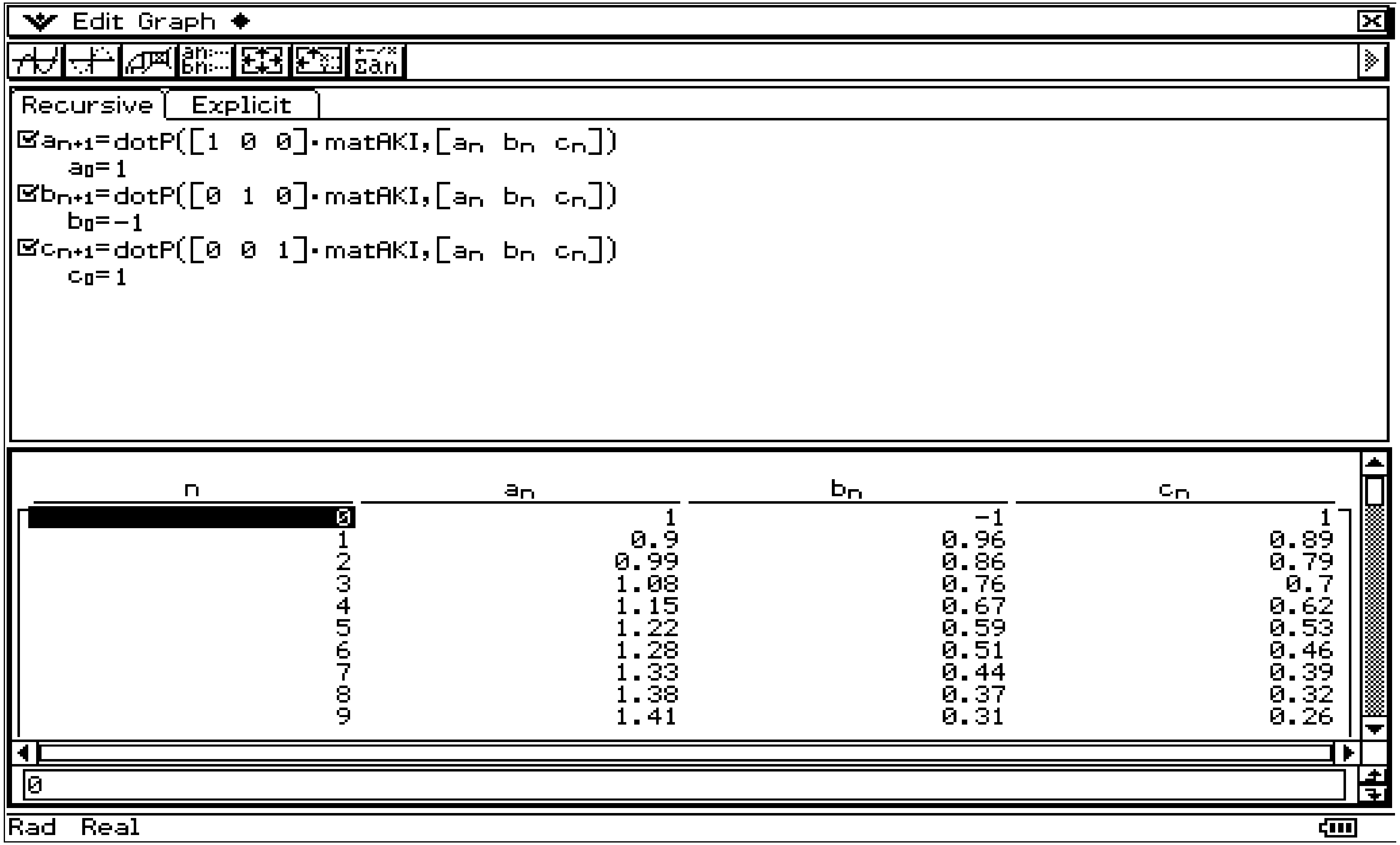


Zoom Analysis Calc
1
99
-0.6
-0.5
Rad

The program DefSeq3D creates the equations for the sequence menu by the help of string commands.

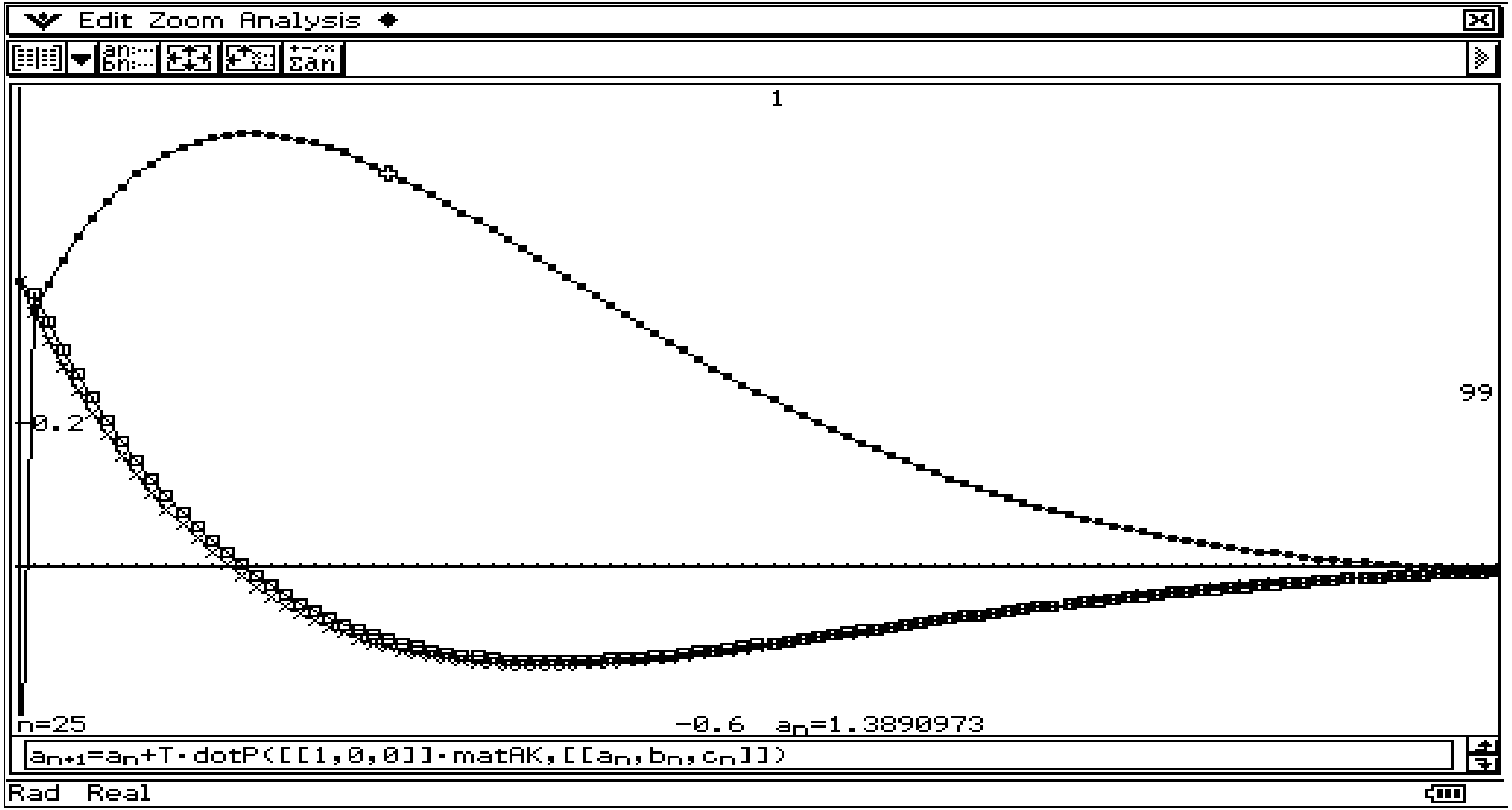
```
Edit Ctrl I/O Misc
DefSeq3D   N  X,N
ViewWindow 0,N,1,-.5,1.5,1
ClrText:local dm,i,j,W
SeqType "an+1a0"
rowDim(A)⇒dm
DelVar a,b,c:[a,b,c]⇒W
For 1⇒i To dm Step 1
  ExpToStr W[1,i],hlp1:StrLeft hlp1,1,hlp:StrJoin hlp,"0",hlp:X[i,1]⇒#hlp
  StrJoin "","dotP([",hlp:StrRotate "1,0,0,",hlp2,2-2i:StrShift hlp2,hlp2,2dm-7
  StrJoin hlp,hlp2,hlp:StrJoin hlp,"]×matAKI,[an,bn,cn])",hlp
  ExpToStr W[1,i],hlp1:StrLeft hlp1,1,hlp1:StrJoin hlp1,"n+1",hlp1
  hlp⇒#hlp1
Next
SeqSelOn an+1:SeqSelOn bn+1:SeqSelOn cn+1:SetΣdisp Off
0⇒SqStart:N⇒SqEnd
DispSeqTbl:Pause:DrawSeqCon
Return
Program Editor
```

The program DefSeq3D creates the equations for the sequence menu.



n	a_n	b_n	c_n
0	1	-1	1
1	0.9	0.96	0.89
2	0.99	0.86	0.79
3	1.08	0.76	0.7
4	1.15	0.67	0.62
5	1.22	0.59	0.53
6	1.28	0.51	0.46
7	1.33	0.44	0.39
8	1.38	0.37	0.32
9	1.41	0.31	0.26

By the help of these sequences we get the same graphical representations of y1(ldot), y2(square), y3(cross).



The file for the classpad manager you can download here:

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