Using the ClassPad300Plus in Statistics to Draw Step Functions and to Compute their Quantiles (Workshop)

Ludwig Paditz, University of Applied Sciences Dresden (FH), Germany

Abstract:

Let us consider the ClassPad300Plus (with the new operating system OS 03.01) and discuss on some new exercises in statistics, e.g. drawing step-functions and computing their quantiles. We consider a $B(n,p)$-distribution function (binomial distribution with the parameters $n = 6$ and $p = 0.3$ say).

At first we try to draw the step function $F(x) = P(X \leq x)$, where X is a $B(n,p)$-distributed random variable. By the help of the **signum**-function we can generate the wished step function, using the possibilities of the algebra. But this step function will not be right continuous. Using the **piecewise**-function, we can generate and draw an exact right continuous step function:

y8(x) =piecewise(x<bx[1],0,
      piecewise(x≥bx[1] and x<bx[2],bf[1,1],
     piecewise(x≥bx[2] and x<bx[3],bf[2,1],
    piecewise(x≥bx[3] and x<bx[4],bf[3,1],
   piecewise(x≥bx[4] and x<bx[5],bf[4,1],
  piecewise(x≥bx[5] and x<bx[6],bf[5,1],
 piecewise(x≥bx[6] and x<bx[7],bf[6,1],1)))))))

Here the list $\mathbf{bx} = \{0,1,2,3,\ldots,n\}$ and the vector $\mathbf{bf} = [p_0, p_1, p_2, \ldots, p_n]^T$ are used. We generate this large term by the help of a program using commands for strings and characters.

Finally we study possibilities to compute the quantiles for a given probability ***prob***, i.e. we try to solve the inequality $F(x\text{-}0) = P(X < x) < prob \leq F(x) = P(X \leq x)$ with an appropriate x. Here we can use the **InvBinomialCD**-function of the ClassPad300.

By the help of several examples the interactive work with the ClassPad300Plus is considered. The student can solve difficult exercises of practical applications step by step using the symbolic calculation and the graphic possibilities of the calculator. Sometimes several fields of mathematics are combined to solve a problem.

Reference: http://www.informatik.htw-dresden.de/~paditz/bquancp3.html
Download ClassPad-file:
http://www.informatik.htw-dresden.de/~paditz/workshop_charlotte_2007.vcp
http://classpad.net/product/Classpad300/cp_manager_03.html
http://www.canyonscorner.com/assets/Reports/CP_StepbyStep_V3_AllApps.pdf

Introduction:

We use the following definition of the quantile of order *gamma*:

The number $\mathbf{b}_{gamma}$, $0 < gamma < 1$, of a **B(n,p)**-distributed random variable **X** is called **quantile** of the order ***gamma***, if the inequality $\mathbf{P(} X < b_{gamma} \mathbf{)} \leq gamma \leq \mathbf{P(} X \leq b_{gamma} \mathbf{)}$ is fulfilled.

The **B(n,p)**-distributed random variable **X** has a discrete distribution function **F(x)** (a right continuous step-function).

If the equation *gamma* = **F(x)** has no solution, than the wished quantile b_{gamma} is the integer number **x**, which fulfils the inequality **F(x-0) < *gamma* < F(x) = F(x+0)**.

In the other case (the equation *gamma* = **F(x)** has (at least) one solution) the number b_{gamma} is not determined in an unique manner and the wished quantile is one of the solutions of the equation *gamma* = **F(x)** with a value **x** of the interval **[k, k+1)**, in which the (cumulative) distribution function **F(x)** has the value **F(x) = *gamma***, or we choose **x=k+1**.

In the statistical theory the quantiles are called critical values and we need these values to construct confidence intervals or to study statistical hypotheses. In such cases the probability *gamma* or **1-*gamma* = *alpha*/2** is called confidence level or significance level.

The binomial distribution has the well-known probability density function, given by **P(X=x) = C(n,x) * p^n * (1-p)$^{n-x}$** for **x = 0, 1, 2, …, n.** Here **C(n,x) = n!/(x!(n - x)!)** is the binomial coefficient. Sometimes the probability density function is called probability mass function. Note that this function is the general term in the binomial expansion of **(p + (1-p))^n**.

Line diagram (BinomPMF):

Let us draw the line diagram for the probability mass function **P(X=x)** by the help of CP300. We use a small program BinomPMF and suppose **p=0.3** and **n=6**.
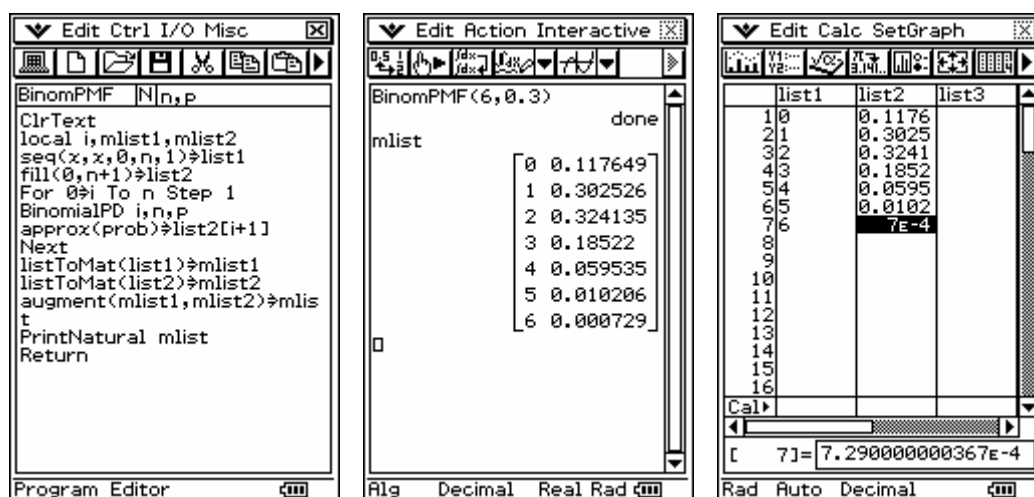
The screenshots are from Program Editor and Main and Stat Editor respectively.
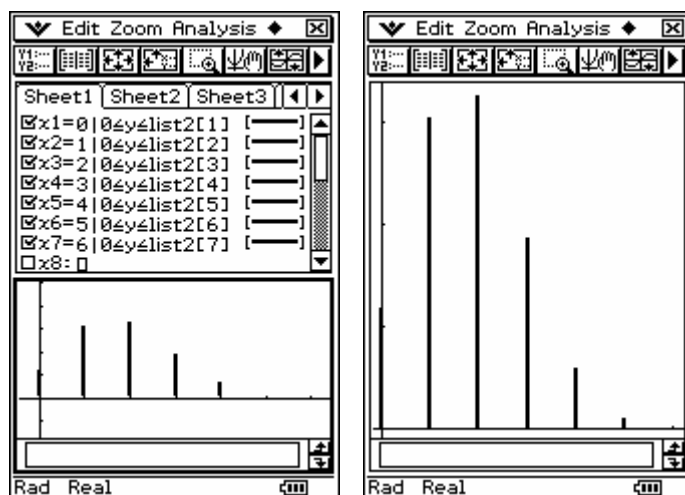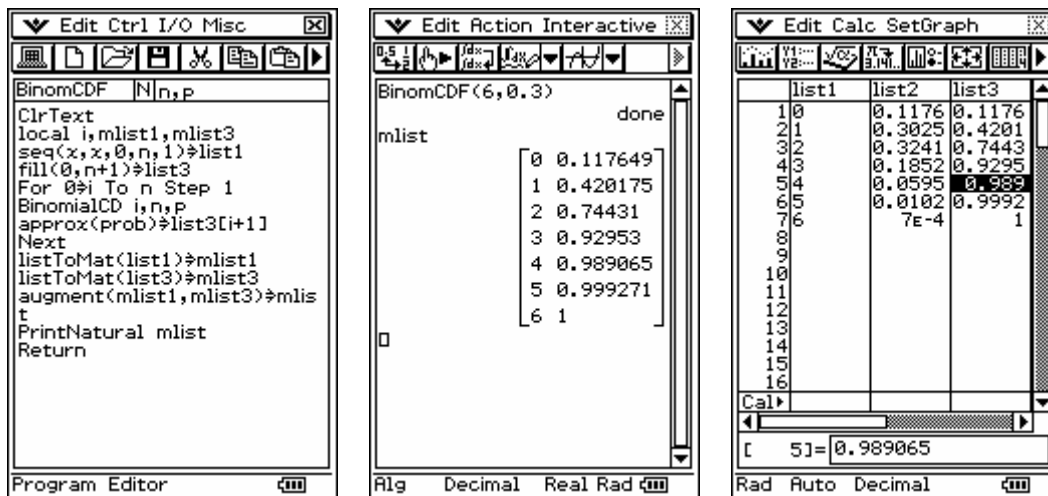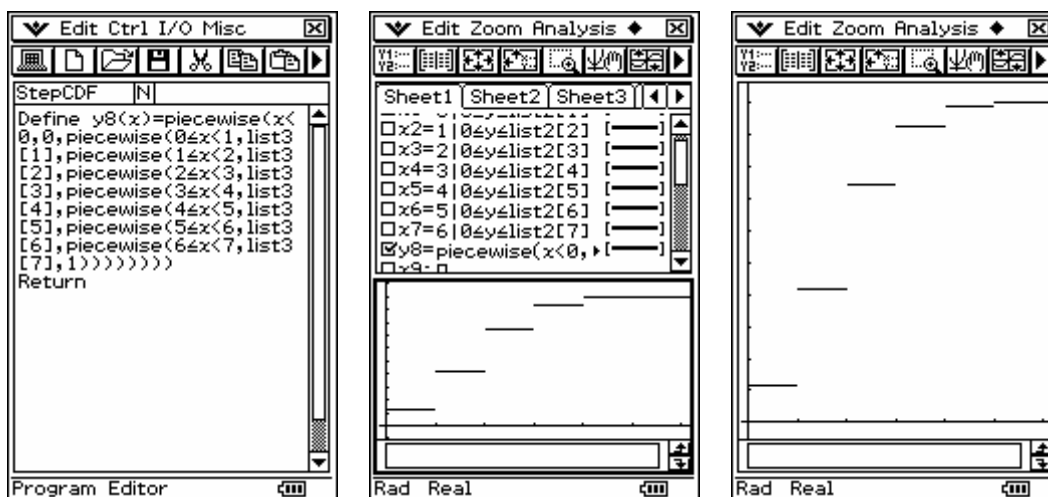
 with Resize and Zoom Box

Step function of the cumulative distribution function (BinomCDF):

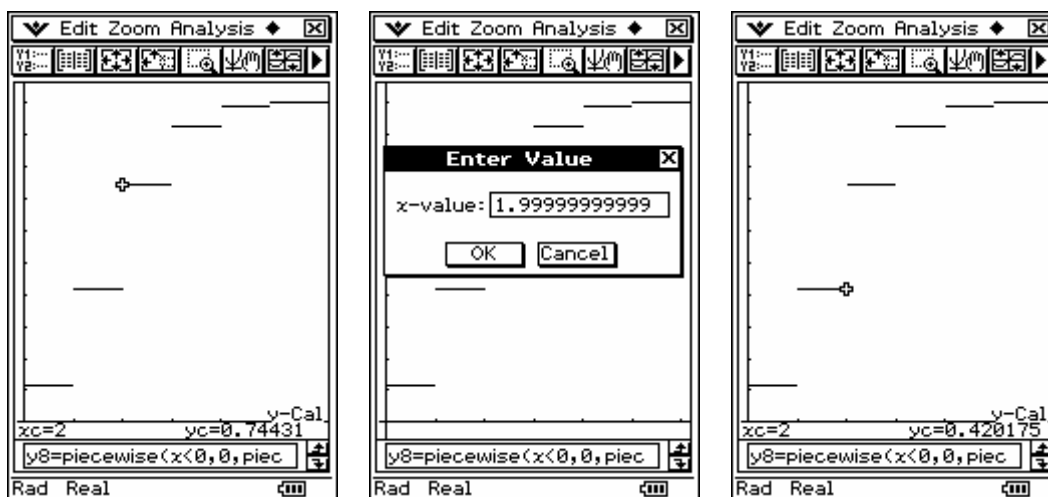Let us draw the step function for the cumulative distribution function by the help of CP300. We use a small program BinomCDF to generate a table of values and suppose **p=0.3** and **n=6**.


```
Edit Ctrl I/O Misc          [X]
BinomCDF    N n,p
ClrText
local i,mlist1,mlist3
seq(x,x,0,n,1)⇒list1
fill(0,n+1)⇒list3
For 0⇒i To n Step 1
BinomialCD i,n,p
approx(prob)⇒list3[i+1]
Next
listToMat(list1)⇒mlist1
listToMat(list3)⇒mlist3
augment(mlist1,mlist3)⇒mlis
t
PrintNatural mlist
Return

Program Editor
```

```
Edit Action Interactive [X]
BinomCDF(6,0.3)
                    done
mlist
          ⌈0  0.117649⌉
          │1  0.420175│
          │2  0.74431 │
          │3  0.92953 │
          │4  0.989065│
          │5  0.999271│
          └6  1       ┘

Alg   Decimal   Real Rad
```

```
Edit Calc SetGraph      [X]
        list1  list2   list3
1  0    0.1176 0.1176
2  1    0.3025 0.4201
3  2    0.3241 0.7443
4  3    0.1852 0.9295
5  4    0.0595 0.989
6  5    0.0102 0.9992
7  6    7ε-4   1
8
9
10
11
12
13
14
15
16
Cal▶
[   5]=0.989065
Rad  Auto  Decimal
```

The screenshots are from Program Editor and Main and Stat Editor respectively. Now we can define the step function by the help of the piecewise function and the program StepCDF:



```
Edit Ctrl I/O Misc          [X]
StepCDF    N
Define y8(x)=piecewise(x<
0,0,piecewise(0≤x<1,list3
[1],piecewise(1≤x<2,list3
[2],piecewise(2≤x<3,list3
[3],piecewise(3≤x<4,list3
[4],piecewise(4≤x<5,list3
[5],piecewise(5≤x<6,list3
[6],piecewise(6≤x<7,list3
[7],1)))))))))
Return

Program Editor
```

```
Edit Zoom Analysis ♦  [X]
Sheet1 Sheet2 Sheet3  ◄ ►
☐x2=1|0≤y≤list2[2] [—]
☐x3=2|0≤y≤list2[3] [—]
☐x4=3|0≤y≤list2[4] [—]
☐x5=4|0≤y≤list2[5] [—]
☐x6=5|0≤y≤list2[6] [—]
☐x7=6|0≤y≤list2[7] [—]
☑y8=piecewise(x<0,▶[—]
☐x9=

Rad  Real
```

```
Edit Zoom Analysis ♦  [X]



Rad  Real
```

The step function is right continuous and we can check it with the y-value function of CP300:


```
Edit Zoom Analysis ♦  [X]



                    y-Cal
xc=2          yc=0.74431
y8=piecewise(x<0,0,piec
Rad  Real
```

```
Edit Zoom Analysis ♦  [X]

┌─── Enter Value ────[X]┐
│                       │
│ x-value:1.99999999999 │
│                       │
│   [OK]   [Cancel]     │
└───────────────────────┘


y8=piecewise(x<0,0,piec
Rad  Real
```

```
Edit Zoom Analysis ♦  [X]



                    y-Cal
xc=2          yc=0.420175
y8=piecewise(x<0,0,piec
Rad  Real
```

## Step function and vector calculation:

Now we define a step function by the help of the vector calculation to see another possibility to get step functions:
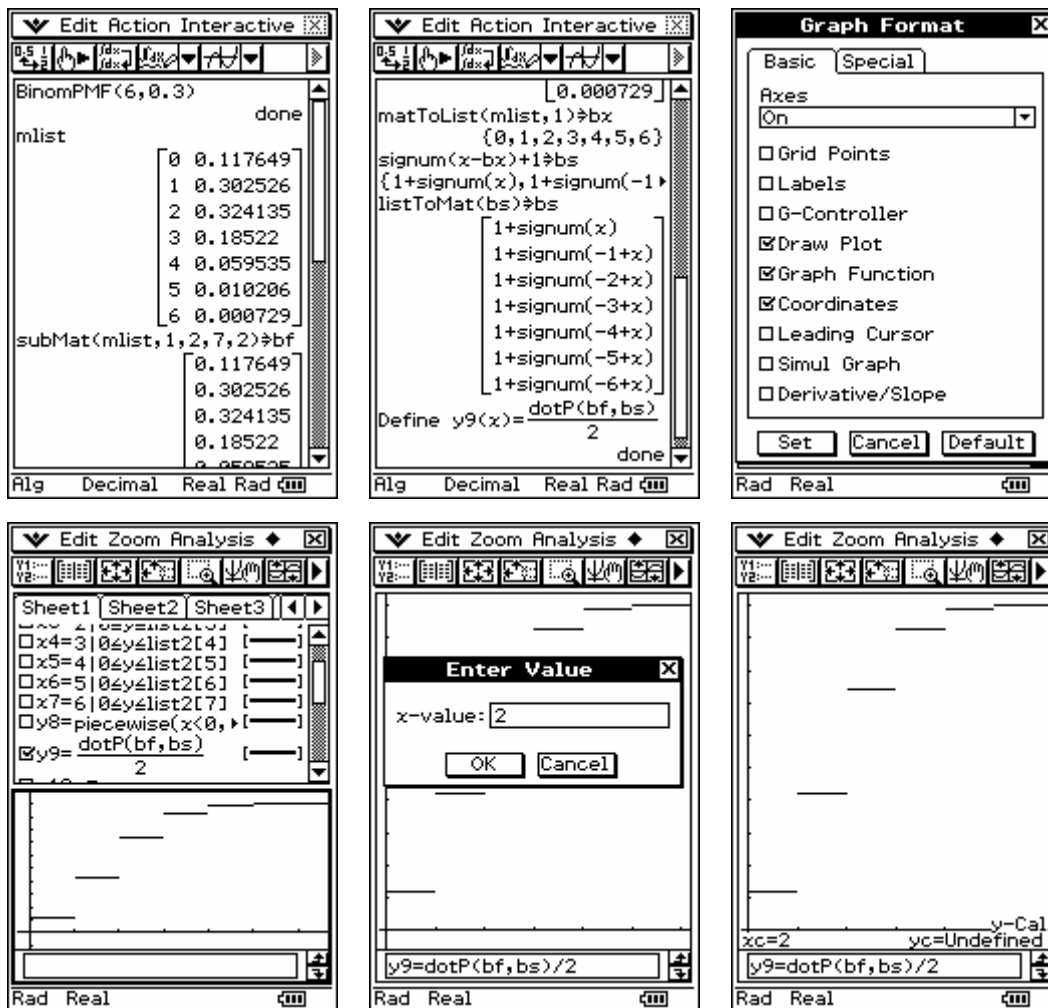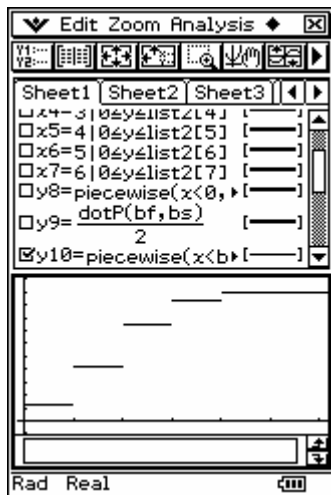

This definition shows that we have no value in the discontinuous points, thus the piecewise definition is a better one.

Step function using string commands:

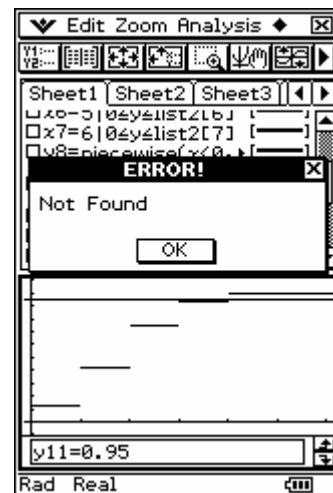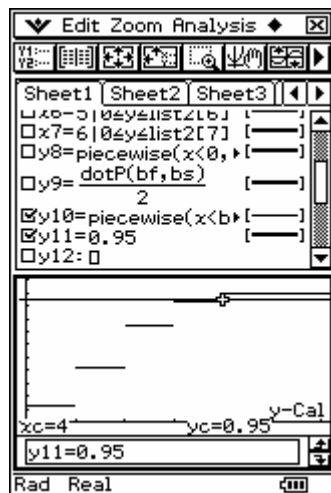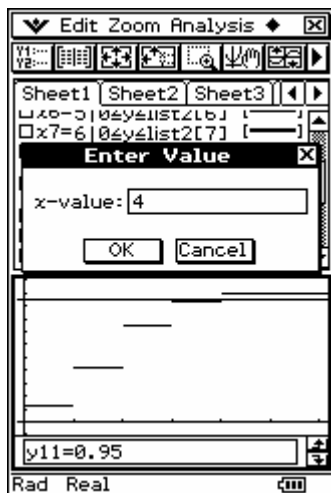It is possible to create a long iterated term of a function by the help of string commands:
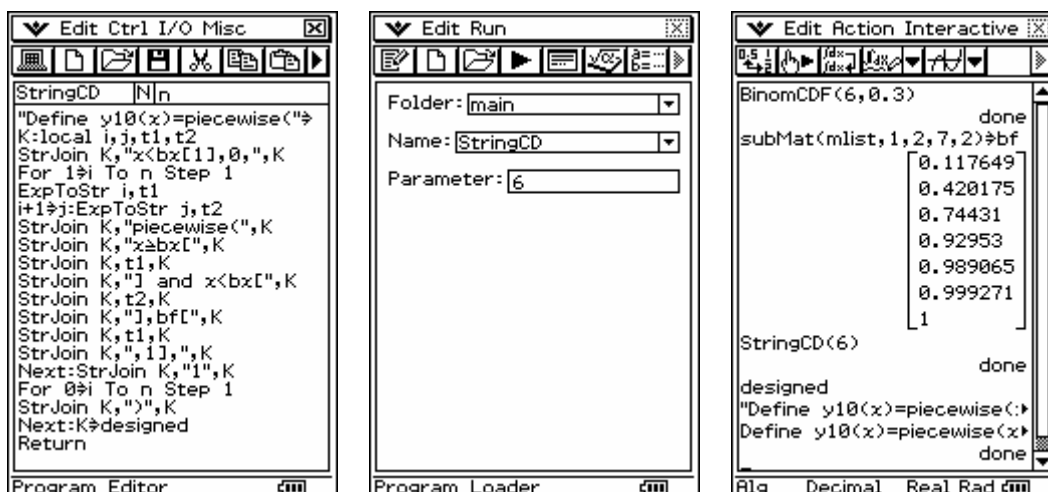
 Thus y8(x) and y10(x) are the same functions.

Computing of the quantiles of the binomial distribution (graphic solution)

Again let be **n=6** and **p=0.3** and *gamma*=**0.95.** We try to solve the equation **F(x)=0.95** by the help of the intersection of **y10(x)=F(x)** and **y11(x)=0.95** and get no solution (cp. ERROR).

In the graphic representation we can observe, the solution is **x = b_{gamma} = 4,** because this value fulfils the definition **F(x-0) = P(X < b_{gamma}) ≤ gamma ≤ P(X ≤ b_{gamma}) = F(x+0) = F(x).** Now let be *gamma* = **0.92953** [= F(3)]. Put **y12(x) = bf[4,1] = 0.92953** and see what happens with the intersection. We get the smallest graphical solution **x = b_{gamma} = 3.**
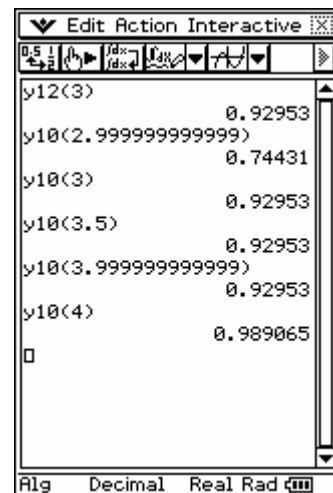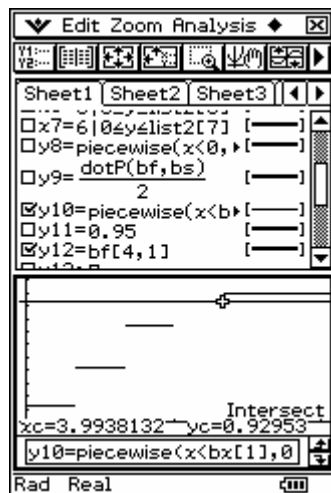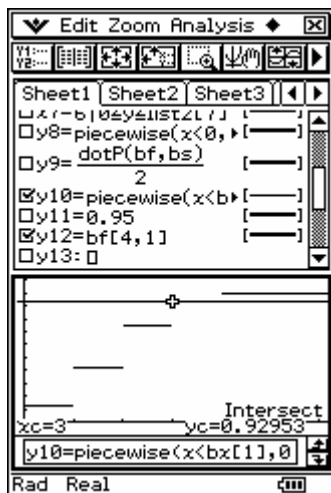
Finally $3 \le b_{gamma} \le 4$ is the set of all solutions of $P(X < b_{gamma}) \le gamma \le P(X \le b_{gamma})$. The same solutions we get considering the table of the values of the function $F(x)$, cp. list1 and list3 in the Stat-List-Editor.

Computing of the quantiles of the binomial distribution (ClassPad's solution)
Open the Stat-List-Editor and choose Calc and Distribution. Open Inverse Binomial CD:
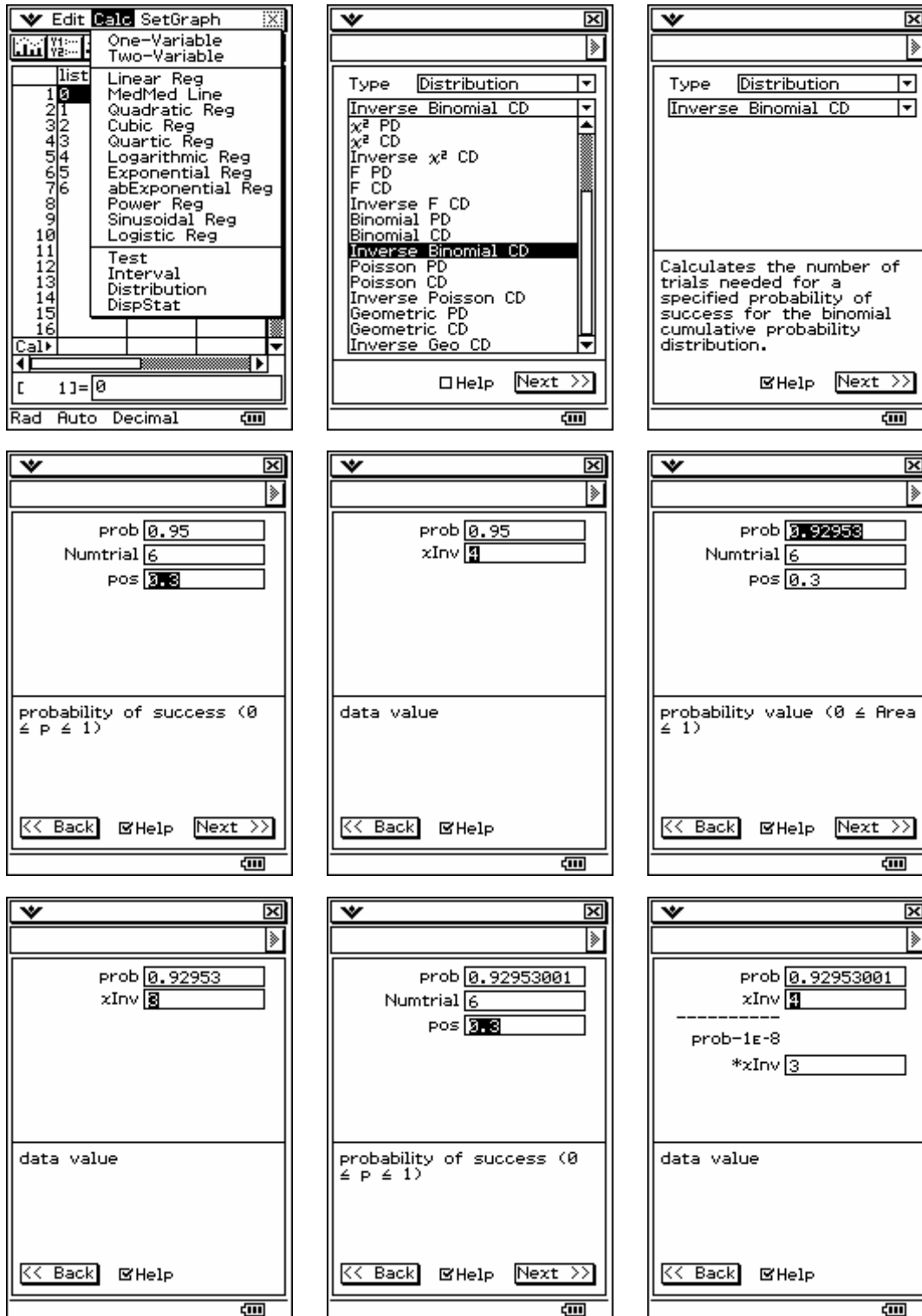
The last screenshot shows, that a change of 1E-8 will change the quantile.
Contact: paditz@informatik.htw-dresden.de