

Using the ClassPad300Plus in Analysis to Solve a System of Linear Differential Equations

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Abstract:

In real life situations quantities and their rate of change depend on more than one variable. For example, the rabbit population, though it may be represented by a single number, depends on the size of predator populations and the availability of food. In order to represent and study such complicated problems we need to use more than one dependent variable and more than one equation. Systems of differential equations are the models to use. The nonlinear systems are very hard to solve explicitly, but qualitative and numerical techniques may help us to get some information on the behaviour of the solutions.

Let us consider the ClassPad300Plus (with the new operating system OS 03.01) and discuss on some new exercises in analysis, e.g. solving a linear system of differential equations.

We know several ways to get a solution. The techniques for studying systems fall into the following three categories: *analytic*, *graphic* and *numeric*.

We can transform a system of equations in one equation of higher order and we have for linear systems with initial conditions the possibility to use the Laplace transformation.

On the other hand we can transform a system of differential equations in a system of difference equations, i.e. sequences of numbers given by the help of recursive equations. These sequences are used as a discrete mathematical model for differential equations.

The ClassPad300 has the **dSolve**- and the **rSolve**-function to study systems of differential and difference equations respectively and additionally the Laplace and inverse Laplace transformation. Finally we have the possibility to generate large **dSolve**- or **rSolve**-terms by the help of commands for strings and characters. Thus the calculator can generate the large syntax for the used **dSolve**- and **rSolve**-function. This is a convenient method to input a long command row not manually but by the help of a program.

By the help of several examples the interactive work with the ClassPad300Plus is considered. The student can solve difficult exercises of practical applications step by step using the symbolic calculation and the graphic possibilities of the calculator. Sometimes several fields of mathematics are combined to solve a problem.

References:

<http://www.sosmath.com/diffeq/diffeq.html>

http://www.informatik.htw-dresden.de/~paditz/Pendulum_Program.pdf

<http://www.informatik.htw-dresden.de/~paditz/LaplaceTransf2006.pdf>

Example of finding the mathematical model and several ways of solution:

The following mathematical model due to an inverted pendulum, cp.

[http://www.fh-](http://www.fh-kempten.de/deu/hochschule/fachbereiche/fbe/labore/digital/homepage/swpr/ss98/Staude_Sommer/Pendel/Pendelengl.htm)

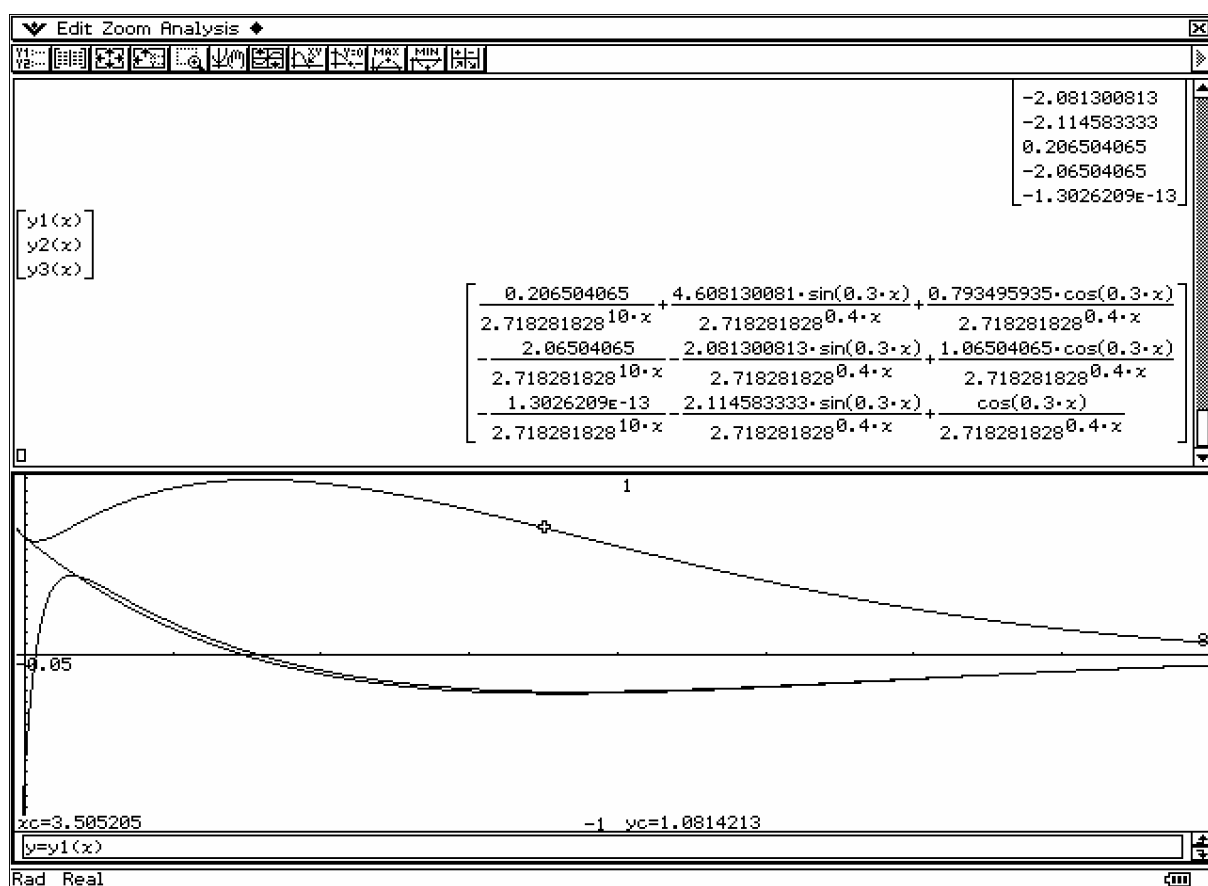
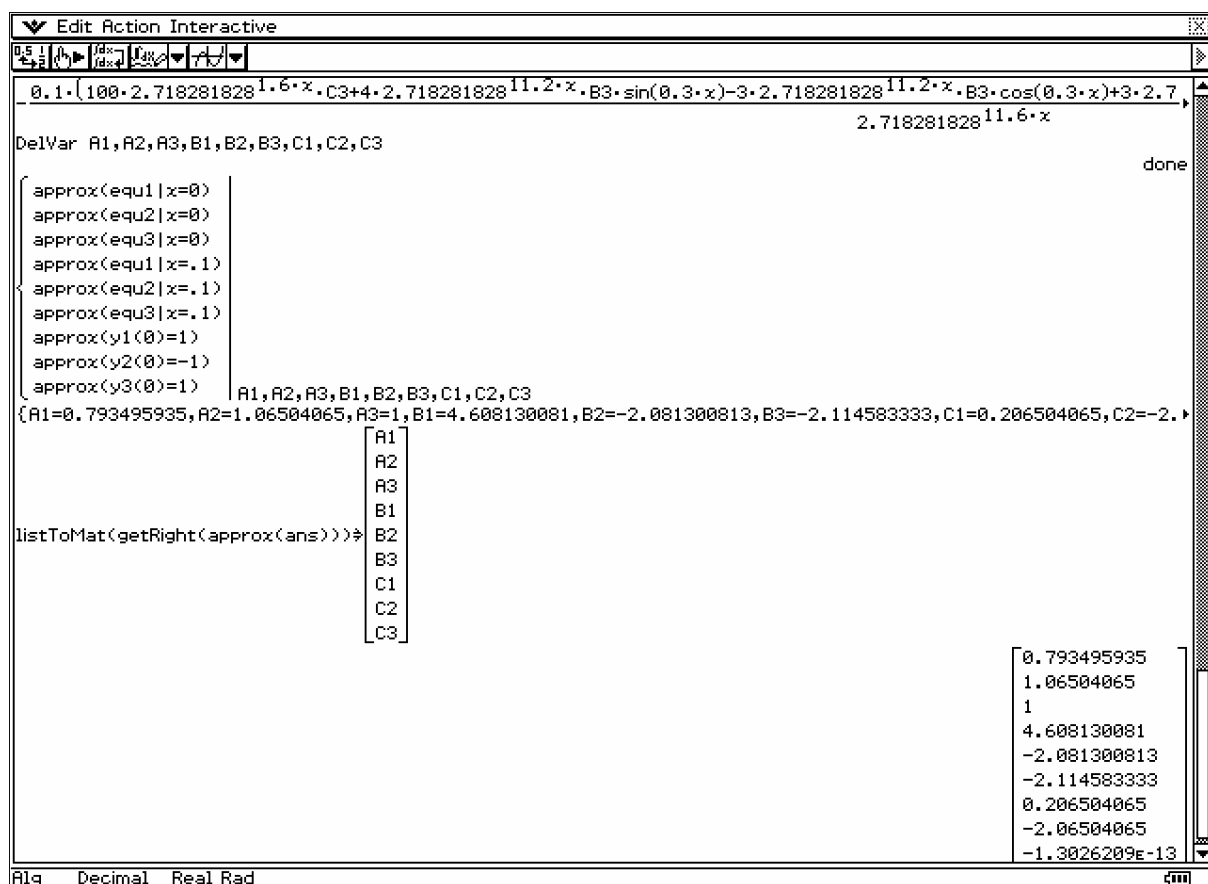
[kempten.de/deu/hochschule/fachbereiche/fbe/labore/digital/homepage/swpr/ss98/Staude_Sommer/Pendel/Pendelengl.htm](http://www.fh-kempten.de/deu/hochschule/fachbereiche/fbe/labore/digital/homepage/swpr/ss98/Staude_Sommer/Pendel/Pendelengl.htm)

http://www.htw-dresden.de/~kaestner/www/pa/pendel/pendel_text.htm

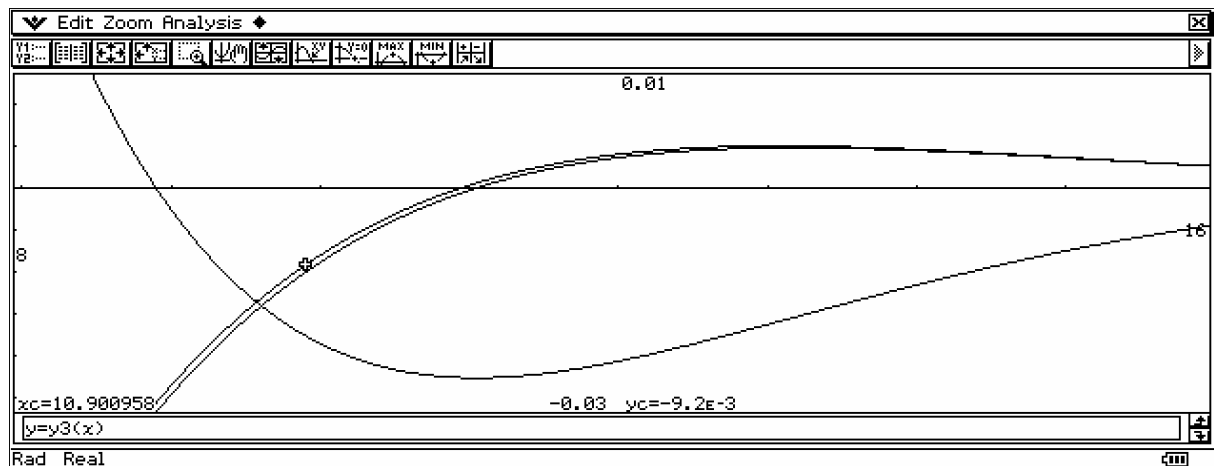
<http://ieeexplore.ieee.org/iel5/41/33886/01614147.pdf>

▼ Edit Action Interactive

0.5 1 1/2 3/4 1/4 1/8 1/16 1/32 1/64 1/128 1/256 1/512 1/1024 1/2048 1/4096 1/8192 1/16384 1/32768 1/65536 1/131072 1/262144 1/524288 1/1048576 1/2097152 1/4194304 1/8388608 1/16777216 1/33554432 1/67108864 1/134217728 1/268435456 1/536870912 1/1073741824 1/2147483648 1/4294967296 1/8589934592 1/17179869184 1/34359738368 1/68719476736 1/137438953472 1/274877906944 1/549755813888 1/1099511627776 1/2199023255552 1/4398046511104 1/8796093022208 1/17592186044416 1/35184372088832 1/70368744177664 1/140737488355328 1/281474976710656 1/562949953421312 1/1125899906842624 1/2251799813685248 1/4503599627370496 1/9007199254740992 1/18014398509481984 1/36028797018963968 1/72057594037927936 1/144115188075855872 1/288230376151711744 1/576460752303423488 1/1152921504606846976 1/2305843009213693952 1/4611686018427387904 1/9223372036854775808 1/18446744073709551616 1/36893488147419103232 1/73786976294838206464 1/147573952589676412928 1/295147905179352825856 1/590295810358705651712 1/1180591620717411303424 1/2361183241434822606848 1/4722366482869645213696 1/9444732965739290427392 1/18889465931478580854784 1/37778931862957161709568 1/75557863725914323419136 1/151115727451828646838272 1/302231454903657293676544 1/604462909807314587353088 1/1208925819614629174706176 1/2417851639229258349412352 1/4835703278458516698824704 1/9671406556917033397649408 1/19342813113834066795298816 1/38685626227668133590597632 1/77371252455336267181195264 1/154742504910672534362390528 1/309485009821345068724781056 1/618970019642690137449562112 1/1237940039285380274899124224 1/2475880078570760549798248448 1/4951760157141521099596496896 1/9903520314283042199192993792 1/19807040628566084398385987584 1/39614081257132168796771975168 1/79228162514264337593543950336 1/158456325028528675187087900672 1/316912650057057350374175801344 1/633825300114114700748351602688 1/1267650600228229401496703205376 1/2535301200456458802993406410752 1/5070602400912917605986812821504 1/10141204801825835211973625643008 1/20282409603651670423947251286016 1/40564819207303340847894502572032 1/81129638414606681695789005144064 1/162259276829213363391578010288128 1/324518553658426726783156020576256 1/649037107316853453566312041152512 1/1298074214633706907132624082305024 1/2596148429267413814265248164610048 1/5192296858534827628530496329220096 1/10384593717069655257060992658440192 1/20769187434139310514121985316880384 1/41538374868278621028243970633760768 1/83076749736557242056487941267521536 1/166153499473114484112975882535043072 1/332306998946228968225951765070086144 1/664613997892457936451903530140172288 1/1329227995784915872903807060280344576 1/2658455991569831745807614120560689152 1/5316911983139663491615228241121378304 1/10633823966279326983230456482242756608 1/21267647932558653966460912964485513216 1/42535295865117307932921825928971026432 1/85070591730234615865843651857942052864 1/170141183460469231731687303715884105728 1/340282366920938463463374607431768211456 1/680564733841876926926749214863536422912 1/1361129467683753853853498429727072845824 1/2722258935367507707706996859454145691648 1/5444517870735015415413993718908291383296 1/10889035741470030830827987437816582766592 1/21778071482940061661655974875633165533184 1/43556142965880123323311949751266331066368 1/87112285931760246646623899502532662132736 1/174224571863520493293247799005065324265472 1/348449143727040986586495598010130648530944 1/696898287454081973172991196020261297061888 1/1393796574908163946345982392040522594123776 1/2787593149816327892691964784081045188247552 1/5575186299632655785383929568162090376495104 1/11150372599265311570767859136324180752990208 1/22300745198530623141535718272648361505980416 1/44601490397061246283071436545296723011960832 1/89202980794122492566142873090593446023921664 1/178405961588244985132285746181186892047843328 1/356811923176489970264571492362373784095686656 1/71362384635297994052914298



View window: $-0.05 < x < 8$ and $-1 < y < 1$ and graphical representation of y_1, y_2, y_3



View window: $8 < x < 16$ and $-0.03 < y < 0.01$

Solving the system of order 3 by the help of one equation of 3rd order for y_1 :

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Edit Action Interactive
λ³-(matAK[2,2]+matAK[3,3])λ²+det(subMat(matAK,2,2,3,3))λ-matAK[2,3]×matAK[3,1]=0
2.5+8.25·λ+10.8·λ²+λ³=0
det(matAK-λ×I)=0
-2.5-8.25·λ-10.8·λ²-λ³=0
"characteristic equation for d³(y1)/dt³+10.8 d²(y1)/dt²+8.25 d(y1)/dt+2.5·y1=0"
"characteristic equation for diff(y1,t,3)+10.8diff(y1,t,2)+8.25 diff(y1,t)+2.5·y1=0"
"Laplace transformation"
"initial conditions y1(0)=1, y1'(0)=-1, y1''(0)=19.6"
"initial conditions y1(0)=1, y1'(0)=-1, y1''(0)=19.6"
laplace(2.5y+8.25·y'+10.8·y''+y'''=0,t,y,s)
2.5·Lp+Lp·s³-s²·y(0)-s·y'(0)-y''(0)+8.25·(Lp·s-y(0))+10.8·(Lp·s²-s·y(0)-y'(0))=0
ans|y(0)=1 and y'(0)=-1 and y''(0)=19.6
-19.6-8.25·(1-Lp·s)+10.8·(1-s+Lp·s²)+s+2.5·Lp+Lp·s³-s²=0
solve(ans,Lp)
{Lp= (341+196·s+20·s²)/(50+165·s+216·s²+20·s³)}
invlaplace((341+196·s+20·s²)/(50+165·s+216·s²+20·s³),s,x)
20·( (0.05420054201/(2.718281828¹⁰·x) + 1.08401084E-3·sin(0.3·x)/(2.718281828⁰·⁴·x) - 4.200542005E-3·cos(0.3·x)/(2.718281828⁰·⁴·x) ) - 196·( (5.420054201E-3/(2.718281828¹⁰·x) + 6.7750677/(2.718281828¹⁰·x) ) )
Define f(x)=20·( (0.05420054201/(2.718281828¹⁰·x) + 1.08401084E-3·sin(0.3·x)/(2.718281828⁰·⁴·x) - 4.200542005E-3·cos(0.3·x)/(2.718281828⁰·⁴·x) ) - 196·( (5.420054201E-3/(2.718281828¹⁰·x) + 6.7750677/(2.718281828¹⁰·x) ) )
done
[ f(0) d(f(x))/dx d²(f(x))/dx² ]|x=0
[1 -1 19.6]
simplify((f(x)))
(0.2065040651/(2.718281828¹⁰·x) + 4.608130081·sin(0.3·x)/(2.718281828⁰·⁴·x) + 0.793495935·cos(0.3·x)/(2.718281828⁰·⁴·x))
Alg Decimal Cplx Rad

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For the Laplace transformation again used the initial conditions $y_1=1, y_2=-1, y_3=1$ for $x=0$.

Finally another way of solution is the transformation in difference equations:

$y'(t) = (y(t+T)-y(t)) / T$ for small T , say $T=0.1$.

Now the new system is $x(t+T) = x(t) + T \cdot \text{matAK} \cdot x(t) = (I + T \cdot \text{matAK}) \cdot x(t)$.

We use the fixpoint iteration $x_{k+1} = (I + T \cdot \text{matAK}) \cdot x_k$ with $x_0=[1, -1, 1]^T$ and create 3 lists.

Here $\text{MatAKI} = I + T \cdot \text{matAK}$.

The program DefLis3D creates the lists.

```

Edit Ctrl I/O Misc
DefLis3D N|X,N
local a:=seq(a,a,1,N)→list1
list1→lista:list1→listb:list1→listc
approx(X[1,1])→lista[1]:approx(X[2,1])→listb[1]:approx(X[3,1])→listc[1]
For 2 To N Step 1
approx(matA[K]X)→X:approx(X[1,1])→lista[i]:approx(X[2,1])→listb[i]:approx(X[3,1])→listc[i]
Next
Return
Program Editor

```

Edit Calc SetGraph

list1	lista	listb	listc
1	1	-1	1
2	0.9	0.96	0.8965625
3	0.996	0.8607	0.7989
4	1.08207	0.766944	0.706809
5	1.1587644	0.6785367	0.6200882
6	1.226618	0.5952846	0.5385379
7	1.2861465	0.5169964	0.4619615
8	1.3378461	0.443483	0.3901648
9	1.3821944	0.3745582	0.322957
10	1.4196503	0.3100387	0.2601504
11	1.4506541	0.2497443	0.2015609
12	1.4756286	0.1934984	0.1470082
13	1.4949784	0.1411278	0.0963158
14	1.5090912	0.0924631	0.0493113
15	1.5183375	0.0473388	5.826E-3
16	1.5230714	5.593E-3	-0.034303
17	1.5236308	-0.03293	-0.071236
18	1.5203377	-0.068387	-0.105129
19	1.5134989	-0.100924	-0.136133
20	1.5034065	-0.130688	-0.164394
21	1.4903377	-0.157818	-0.190053
22	1.4745558	-0.182451	-0.213249
23	1.4563107	-0.204719	-0.234113
24	1.4358388	-0.224749	-0.252776
25	1.4133639	-0.242665	-0.26936
26	1.3890973	-0.258586	-0.283986
27	1.3632387	-0.272626	-0.296768
28	1.335976	-0.284897	-0.307817
29	1.3074863	-0.295505	-0.317241
30	1.2779357	-0.304551	-0.325141
31	1.2474806	-0.312136	-0.331616
32	1.2162669	-0.318352	-0.336761
33	1.1844317	-0.32329	-0.340664
34	1.1521026	-0.327038	-0.343414
35	1.1193988	-0.329677	-0.345092
36	1.086431	-0.331288	-0.345777
37	1.0533022	-0.331946	-0.345544
38	1.0201075	-0.331723	-0.344466
39	0.9869352	-0.330687	-0.34261

Cal 11=1

Deg Auto Decimal

Edit Calc SetGraph

list1	lista	listb	listc
462	462	3.565E-8	-1.92E-8
463	463	3.373E-8	-1.85E-8
464	464	3.187E-8	-1.79E-8
465	465	3.008E-8	-1.73E-8
466	466	2.835E-8	-1.66E-8
467	467	2.668E-8	-1.6E-8
468	468	2.508E-8	-1.54E-8
469	469	2.353E-8	-1.48E-8
470	470	2.205E-8	-1.42E-8
471	471	2.063E-8	-1.36E-8
472	472	1.926E-8	-1.3E-8
473	473	1.796E-8	-1.25E-8
474	474	1.671E-8	-1.19E-8
475	475	1.551E-8	-1.14E-8
476	476	1.437E-8	-1.08E-8
477	477	1.328E-8	-1.03E-8
478	478	1.225E-8	-9.87E-9
479	479	1.126E-8	-9.39E-9
480	480	1.032E-8	-8.92E-9
481	481	9.43E-9	-8.46E-9
482	482	8.584E-9	-8.02E-9
483	483	7.781E-9	-7.59E-9
484	484	7.021E-9	-7.18E-9
485	485	6.303E-9	-6.78E-9
486	486	5.624E-9	-6.4E-9
487	487	4.984E-9	-6.02E-9
488	488	4.381E-9	-5.67E-9
489	489	3.814E-9	-5.32E-9
490	490	3.281E-9	-4.99E-9
491	491	2.782E-9	-4.67E-9
492	492	2.314E-9	-4.37E-9
493	493	1.876E-9	-4.08E-9
494	494	1.468E-9	-3.8E-9
495	495	1.088E-9	-3.53E-9
496	496	7.34E-10	-3.27E-9
497	497	4.07E-10	-3.03E-9
498	498	1.03E-10	-2.8E-9
499	499	-1.7E-10	-2.58E-9
500	500	-4.3E-10	-2.37E-9

Cal 11=1

Deg Auto Decimal

By the help of these lists we get the same graphical representations of y_1 , y_2 , y_3 .
 Finally we use the sequence menu to create the sequences given in lista, listb, listc.

The file for the classpad manager you can download here:

http://www.informatik.htw-dresden.de/~paditz/paper_charlotte_2007.vcp

The program DefSeq3D creates the equations for the sequence menu.

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