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# Using the ClassPad300Plus in Analysis to Solve a System of Linear Differential Equations

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## Abstract:

In real life situations quantities and their rate of change depend on more than one variable. For example, the rabbit population, though it may be represented by a single number, depends on the size of predator populations and the availability of food. In order to represent and study such complicated problems we need to use more than one dependent variable and more than one equation. Systems of differential equations are the models to use. The nonlinear systems are very hard to solve explicitly, but qualitative and numerical techniques may help us to get some information on the behaviour of the solutions.
Let us consider the ClassPad300Plus (with the new operating system OS 03.01) and discuss on some new exercises in analysis, e.g. solving a linear system of differential equations.
We know several ways to get a solution. The techniques for studying systems fall into the following three categories: ***analytic***, ***graphic*** and ***numeric***.
We can transform a system of equations in one equation of higher order and we have for linear systems with initial conditions the possibility to use the Laplace transformation.
On the other hand we can transform a system of differential equations in a system of difference equations, i.e. sequences of numbers given by the help of recursive equations. These sequences are used as a discrete mathematical model for differential equations.
The ClassPad300 has the **dSolve**- and the **rSolve**-function to study systems of differential and difference equations respectively and additionally the Laplace and inverse Laplace transformation. Finally we have the possibility to generate large **dSolve-** or **rSolve-**terms by the help of commands for strings and characters. Thus the calculator can generate the large syntax for the used **dSolve**- and **rSolve**-function. This is a convenient method to input a long command row not manually but by the help of a program.

By the help of several examples the interactive work with the ClassPad300Plus is considered. The student can solve difficult exercises of practical applications step by step using the symbolic calculation and the graphic possibilities of the calculator. Sometimes several fields of mathematics are combined to solve a problem.

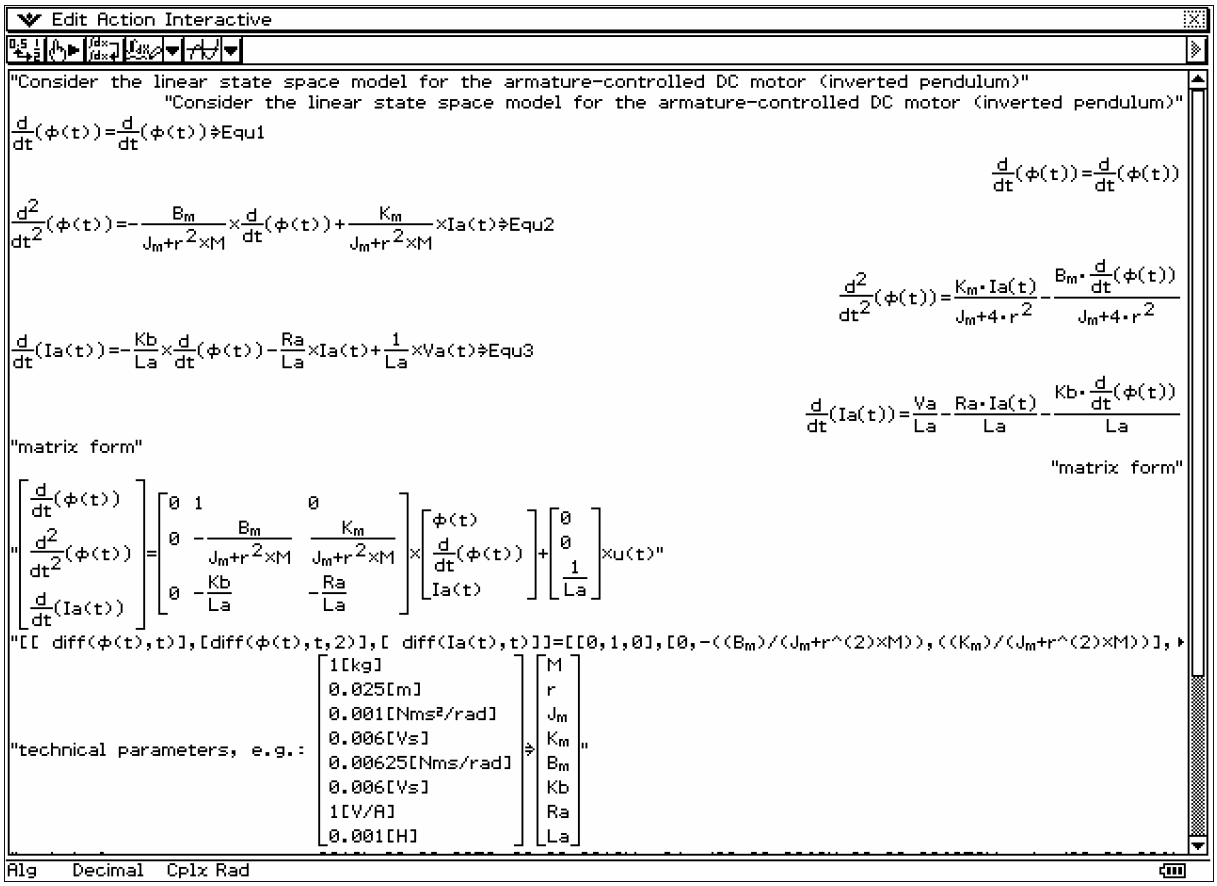
References:
http://www.sosmath.com/diffeq/diffeq.html
http://www.informatik.htw-dresden.de/~paditz/Pendulum_Program.pdf
http://www.informatik.htw-dresden.de/~paditz/LaplaceTransf2006.pdf

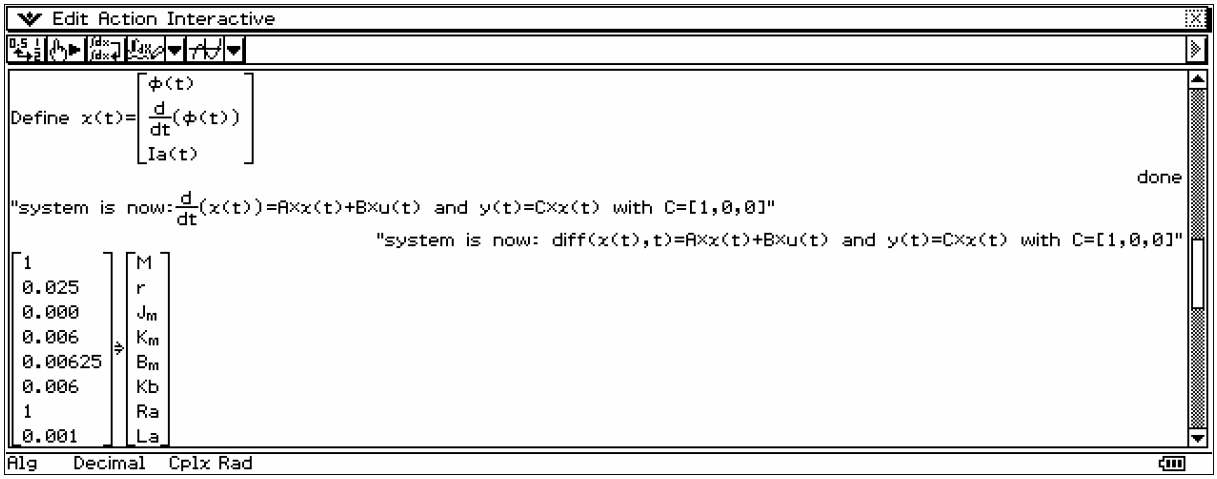
## Example of finding the mathematical model and several ways of solution:

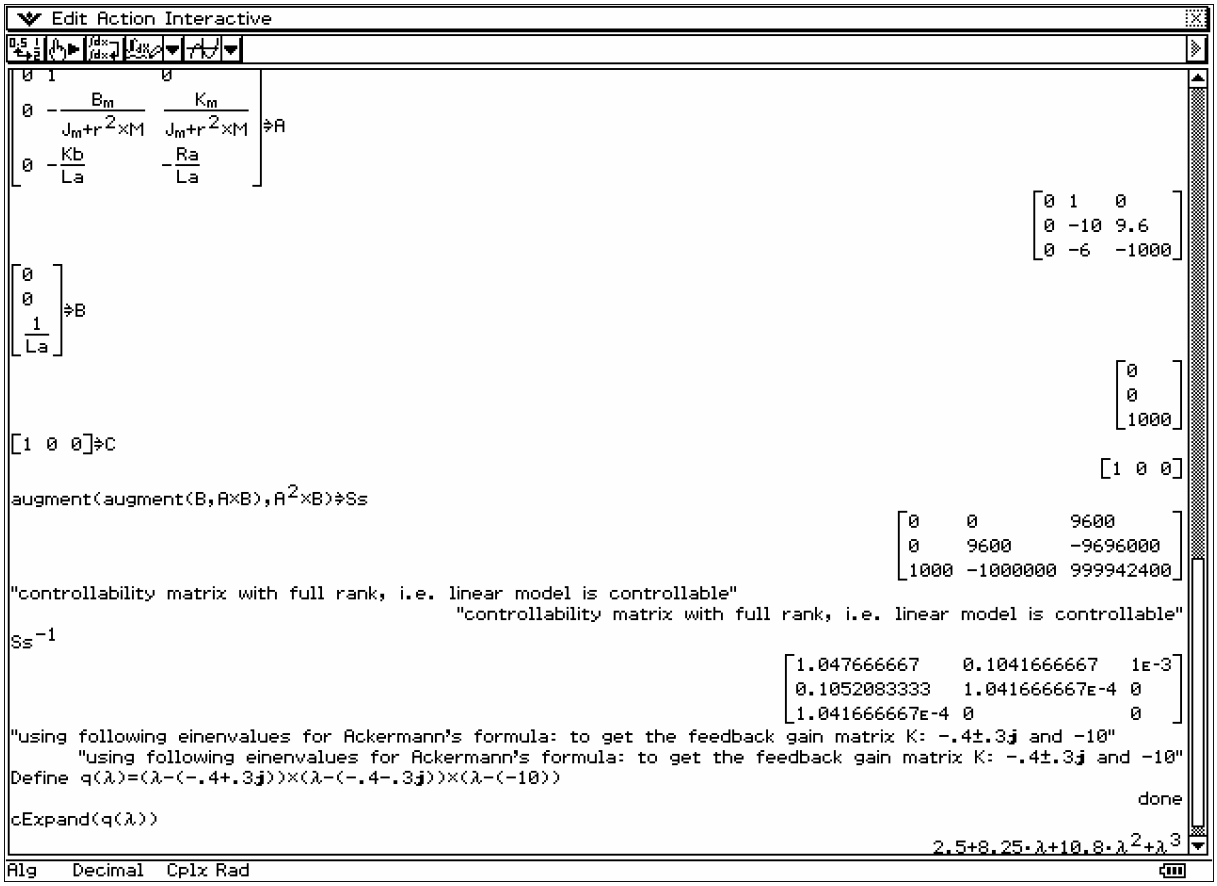
The following mathematical model due to an inverted pendulum, cp.
http://www.fh-kempten.de/deu/hochschule/fachbereiche/fbe/labore/digital/homepage/swpr/ss98/Staude_Sommer/Pendel/Pendel engl.htm

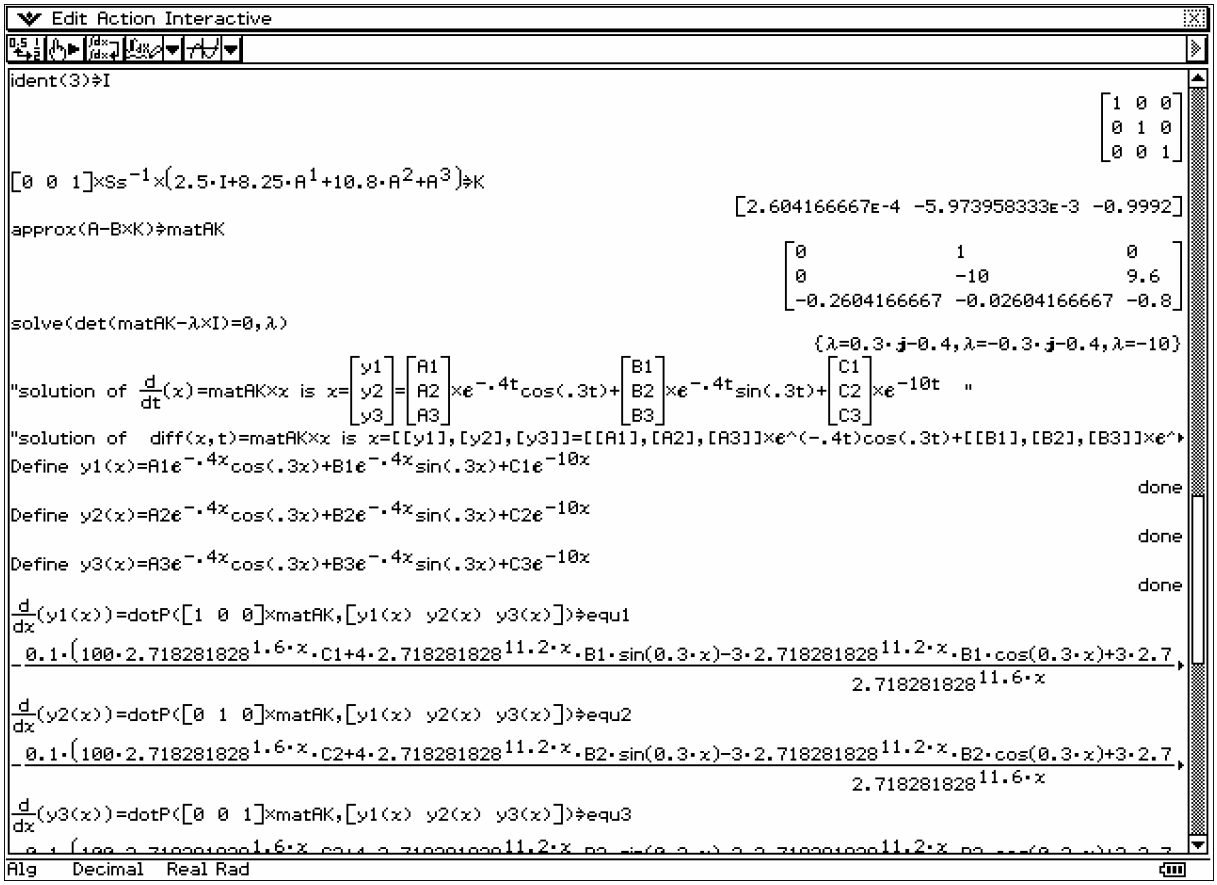
http://instruct1.cit.cornell.edu/courses/ee476/FinalProjects/s2003/es89kh98/es89kh98/Inverted_Pendulum_Balancer.mov
http://www.htw-dresden.de/~kaestner/www/pa/pendel/simulation14/InversesPendel.html
http://www.htw-dresden.de/~kaestner/www/pa/pendel/pendel_modell.htm
http://www.htw-dresden.de/~kaestner/www/pa/pendel/pendel_text.htm

A complete analytic model of the inverted pendulum controlled by a DC motor is derived in three parts, the pendulum-cart dynamics, the friction model, and the motor dynamics. Here we will study the dynamics of the DC motor by the following equations, cp.
http://www.sei.cmu.edu/pub/documents/99.reports/pdf/99tr023.pdf
http://ieeexplore.ieee.org/iel5/41/33886/01614147.pdf

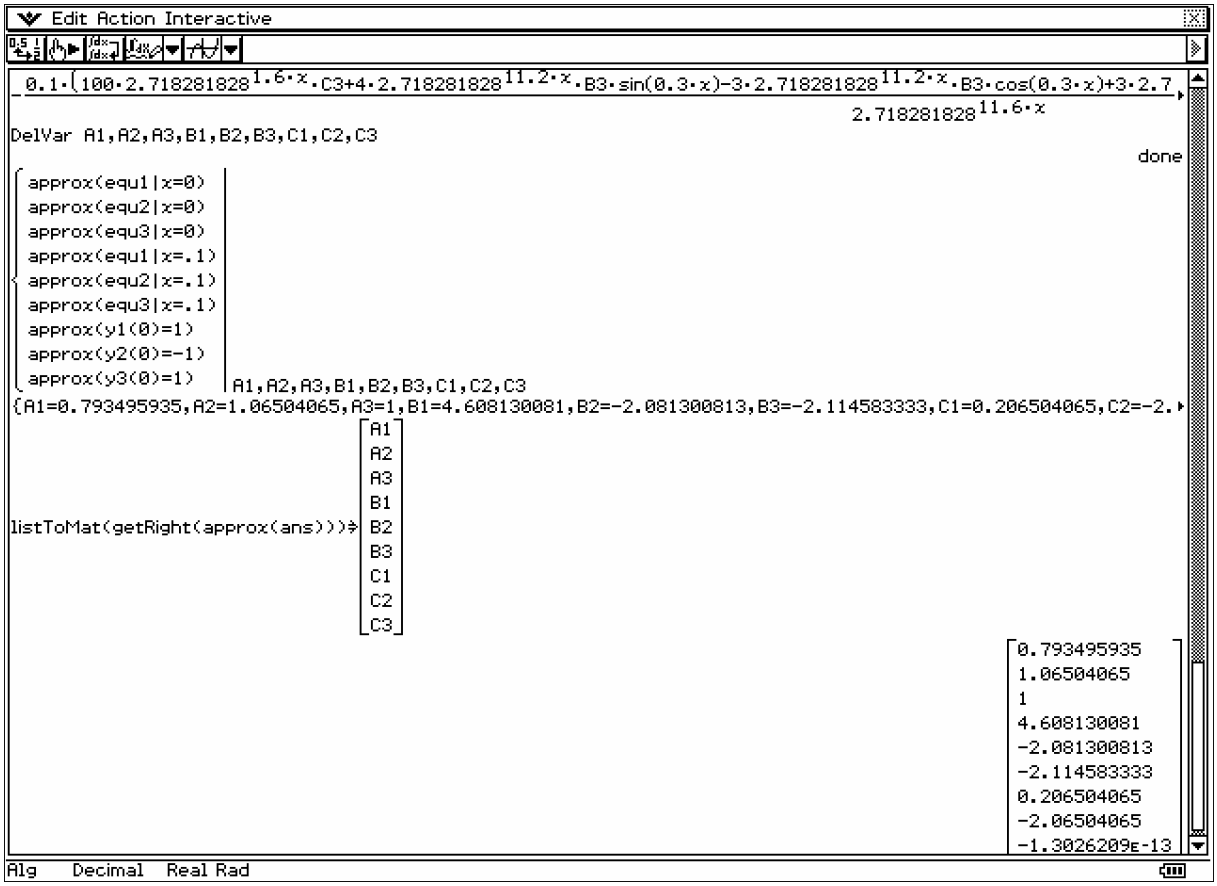


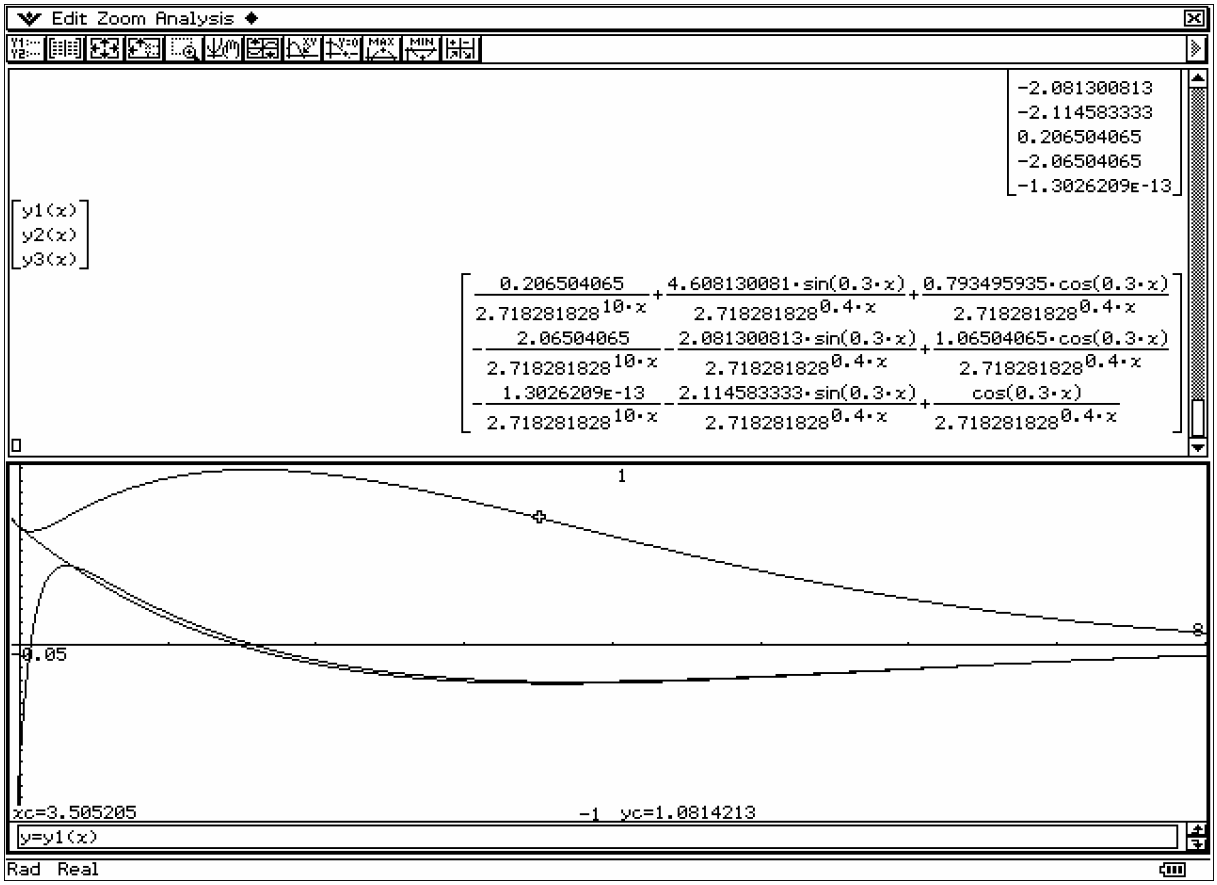





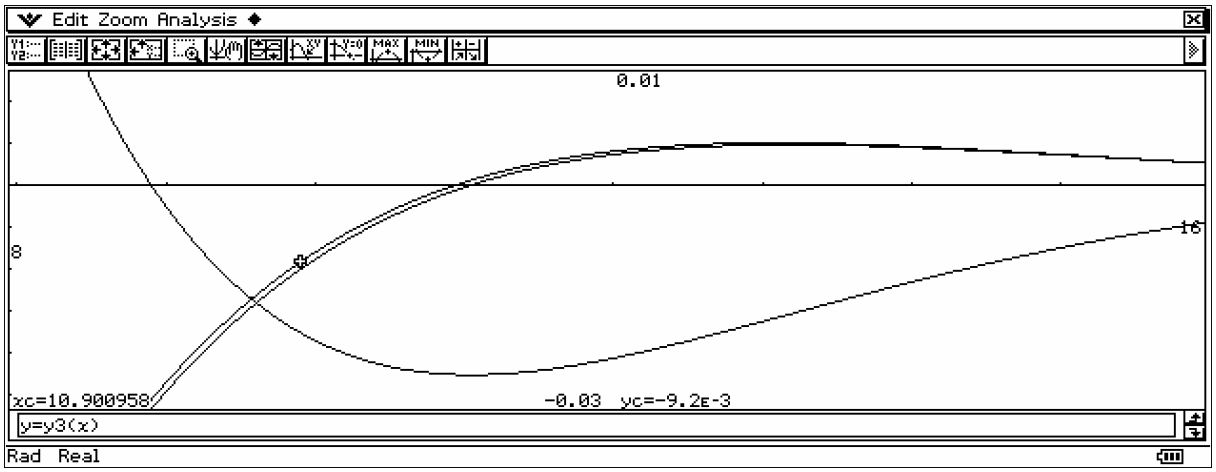




Solution of the linear system with unknown coefficients A1, A2, …, C2, C3.
Determination of the coefficients with the initial conditions y1=1, y2=-1, y3=1 for x=0:



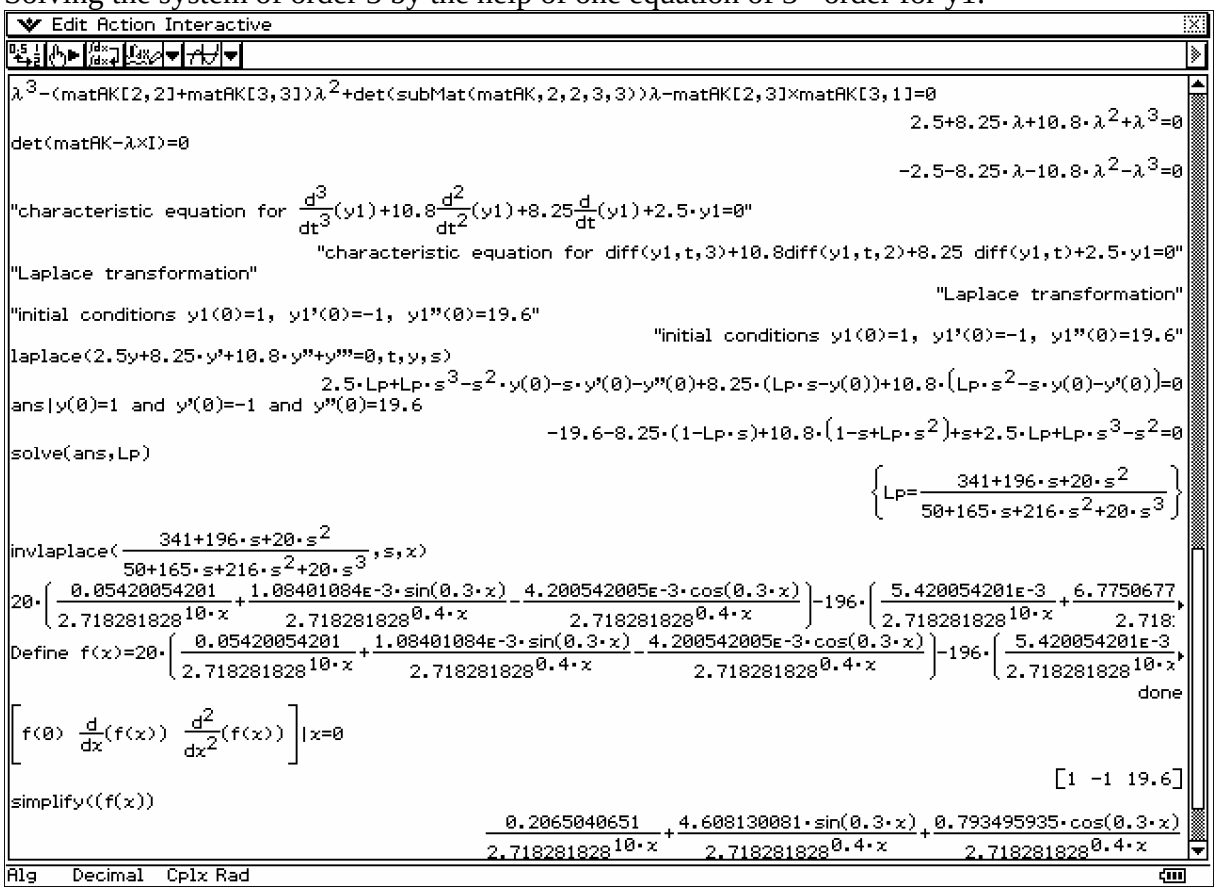




View window: -0.05 < x < 8 and -1 < y < 1 and graphical representation of y1, y2, y3




View window: 8 < x < 16 and -0.03 < y < 0.01

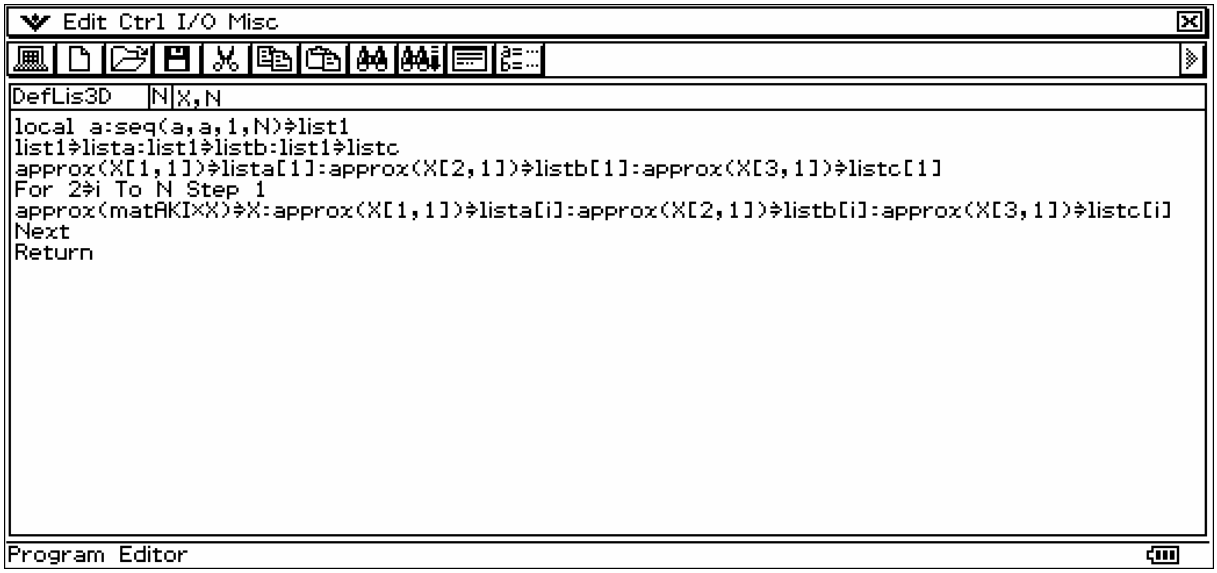
Solving the system of order 3 by the help of one equation of 3rd order for y1:

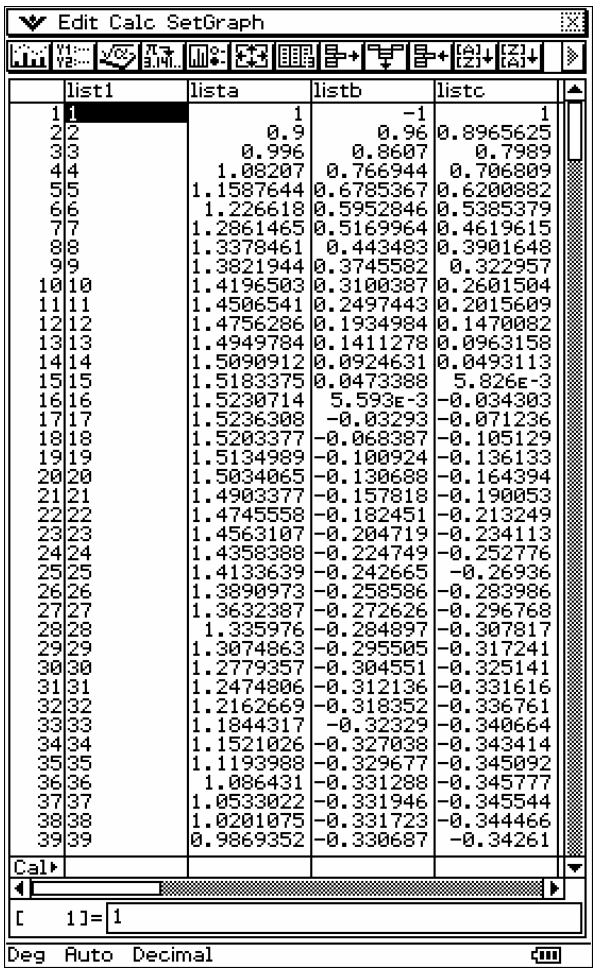


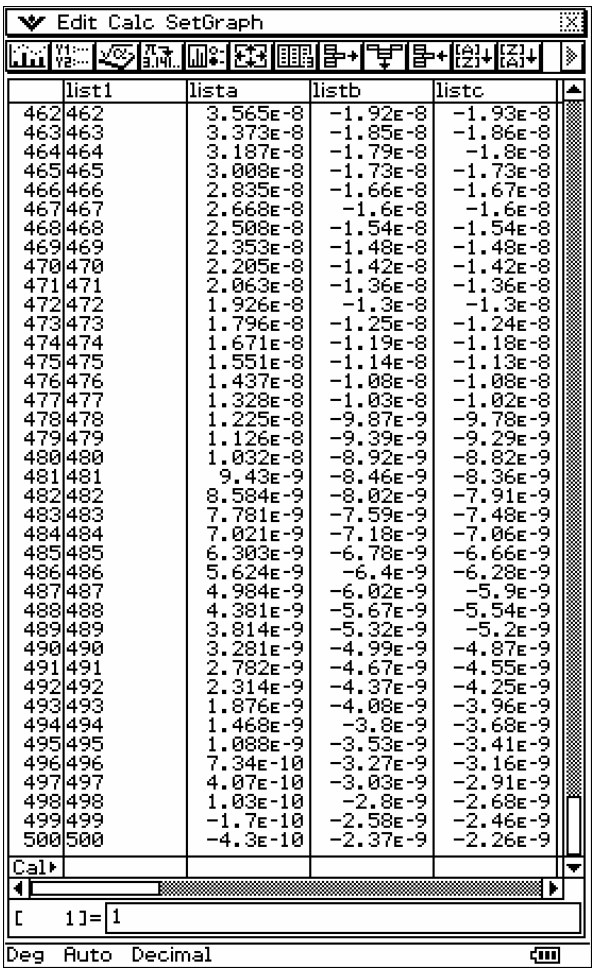

For the Laplace transformation again used the initial conditions y1=1, y2=-1, y3=1 for x=0.

Finally another way of solution is the transformation in difference equations:
y'(t) = (y(t+T)-y(t)) / T for small T, say T=0.1.
Now the new system is x(t+T) = x(t) + T*matAK*x(t) = (I + T*matAK)*x(t).
We use the fixpoint iteration $x_{k+1}$ = (I + T*matAK)*$x_k$ with $x_o$=$[1,-1,1]^T$ and create 3 lists.

Here MatAKI = I + T*matAK.
The program DefLis3D creates the lists.




By the help of these lists we get the same graphical representations of y1, y2, y3.
Finally we use the sequence menu to create the sequences given in lista, listb, listc.

The file for the classpad manager you can download here:
http://www.informatik.htw-dresden.de/~paditz/paper_charlotte_2007.vcp
The program DefSeq3D creates the equations for the sequence menu.

Contact: paditz@informatik.htw-dresden.de

Edit Ctrl I/O Misc

DefSeq3D N,X,N

```
ViewWindow 0,N,1,-.5,1.5,1
ClrText:local dm,i,j,W
SeqType "an+1an"
rowDim(A)⇒dm
DelVar a,b,c:[a,b,c]⇒W
For 1⇒i To dm Step 1
  ExpToStr W[1,i],hlp1:StrLeft hlp1,1,hlp:StrJoin hlp,"n",hlp:X[i,1]⇒#hlp
  StrJoin "","dotP([",hlp:StrRotate "1,0,0,",hlp2,2-2i:StrShift hlp2,hlp2,2dm-7
  StrJoin hlp,hlp2,hlp:StrJoin hlp,"]×matAKI,[an,bn,cn])",hlp
  ExpToStr W[1,i],hlp1:StrLeft hlp1,1,hlp1:StrJoin hlp1,"n+1",hlp1
  hlp⇒#hlp1
Next
SeqSelOn an+1:SeqSelOn bn+1:SeqSelOn cn+1:SetΣdisp Off
0⇒SqStart:N⇒SqEnd
DispSeqTbl:Pause:DrawSeqCon
Return
```

Program Editor

Edit Graph ◆

Recursive | Explicit

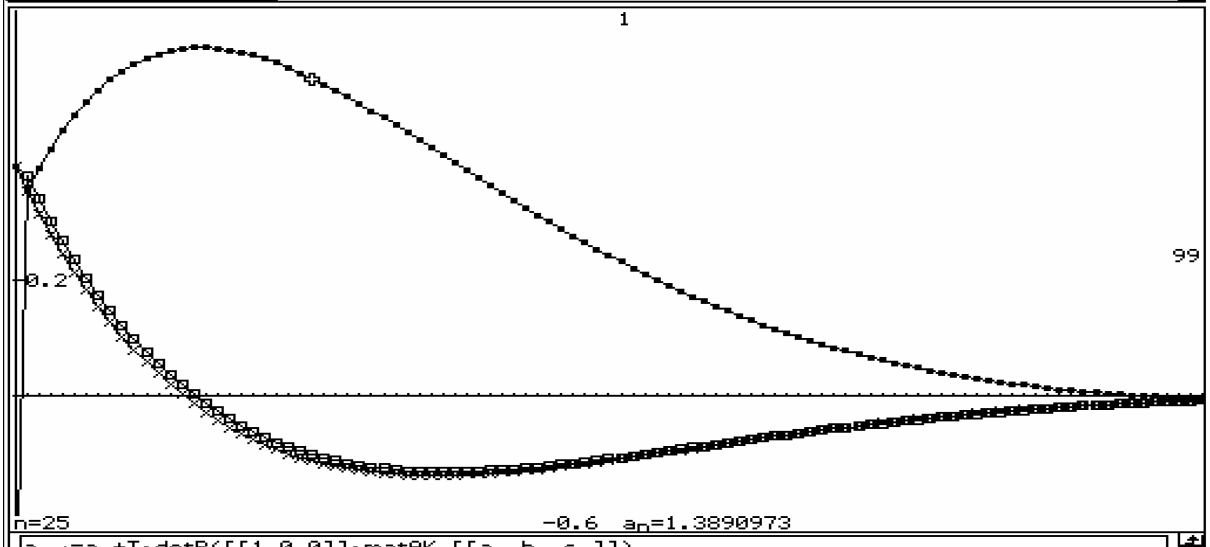
$a_{n+1}=\text{dotP}([1\ 0\ 0]\cdot\text{matAKI},[a_n\ b_n\ c_n])$
$a_0=1$
$b_{n+1}=\text{dotP}([0\ 1\ 0]\cdot\text{matAKI},[a_n\ b_n\ c_n])$
$b_0=-1$
$c_{n+1}=\text{dotP}([0\ 0\ 1]\cdot\text{matAKI},[a_n\ b_n\ c_n])$
$c_0=1$

| n | $a_n$ | $b_n$ | $c_n$ |
|---|---|---|---|
| 0 | 1 | −1 | 1 |
| 1 | 0.9 | 0.96 | 0.89 |
| 2 | 0.99 | 0.86 | 0.79 |
| 3 | 1.08 | 0.76 | 0.7 |
| 4 | 1.15 | 0.67 | 0.62 |
| 5 | 1.22 | 0.59 | 0.53 |
| 6 | 1.28 | 0.51 | 0.46 |
| 7 | 1.33 | 0.44 | 0.39 |
| 8 | 1.38 | 0.37 | 0.32 |
| 9 | 1.41 | 0.31 | 0.26 |

0

Rad Real

Edit Zoom Analysis ◆




$a_{n+1}=a_n+T\cdot\text{dotP}([[1,0,0]]\cdot\text{matAK},[[a_n,b_n,c_n]])$

Rad Real