Vorl. Prof. Oestreich - Vertretung Prof. Paditz

10.01.2018 - Repetitorium 2.KW

Aufg. 2.2. 3e, g, j, k, 6, 13b, 14c, 18, 29a

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Aufg. 3e, g, j, k

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$\frac{d}{dx}$(sin(x^2+1)cos(4x))

$$2\cdot x\cdot \cos\left(x^{2}+1\right)\cdot \cos(4\cdot x)-4\cdot \sin\left(x^{2}+1\right)\cdot \sin(4\cdot x)$$

$\frac{d}{dx}$(1/tan(|x|))

$$\frac{-\left((\tan(|x|))^{2}+1\right)\cdot \mathrm{signum}(x)}{(\tan(|x|))^{2}}$$

simplify(ans)

$$\frac{-\mathrm{signum}(x)}{(\sin(|x|))^{2}}$$

$\frac{d}{dx}$((a^2+x^2)^2/(a^3+x^3)^2)

$$\frac{-\left(2\cdot x^{6}-4\cdot a^{5}\cdot x+8\cdot a^{2}\cdot x^{4}+6\cdot a^{4}\cdot x^{2}-4\cdot a^{3}\cdot x^{3}\right)}{\left(x^{3}+a^{3}\right)^{3}}$$

simplify(ans)

$$\frac{-2\cdot x\cdot (x^3-2\cdot a^3+3\cdot a^2\cdot x)\cdot (x^2+a^2)}{(x^2+a^2-a\cdot x)^3\cdot (x+a)^3}$$

$\frac{d}{dx}((\sin(x))\hat{}x)$

$$x\cdot\cos(x)\cdot(\sin(x))^{x-1}+\ln(\sin(x))\cdot(\sin(x))^x$$

stop

Aufg. 2.2.6

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(Handskizze!)

Schnittpunkt bei $x=\pi/4$

Anstieg sin(x):

solve(tan(α)=cos($\pi/4$), α)

$$\left\{\alpha=\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)+\pi\cdot \text{constn}(1)\right\}$$

$\alpha=\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$

Anstieg cos(x):

solve(tan(β)=−sin($\pi/4$), β)

$$\left\{\beta=-\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)+\pi\cdot \text{constn}(1)\right\}$$

$\beta=-\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$

Ergebnis: Schnittwinkel=2α=70,53°

approx($2\tan^{-1}\left(\frac{\sqrt{2}}{2}\right)$)*180/$\pi$

$$70.52877937$$

approx(tan$^{-1}\left(\frac{2\sqrt{2}}{1}\right)$

1.230959417

approx(ans/π)

0.391826552

stop

Aufg. 2.2.13b

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Ellipse (x/5)^2+((y+1)/2)^2=1

Define xt1(t)=5*cos(t)

done

Define yt1(t)=-1+2*sin(t)

done

Ellipse

Po mit $x_0=0$, $y<0$, d.h. $y_0=-3$

solve((x/5)^2+((y+1)/2)^2=1,y)

$$\left\{y=\frac{-2\cdot\sqrt{-x^2+25}}{5}-1,\ y=\frac{2\cdot\sqrt{-x^2+25}}{5}-1\right\}$$

Define $y(x)=\frac{-2\cdot\sqrt{-x^2+25}}{5}-1$

done

$\frac{d}{dx}(y(x))$

$$\frac{2\cdot x}{5\cdot\sqrt{-x^2+25}}$$

ans|x=0

0

Tangente y=-3

stop

Aufg. 2.2.14c

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DelVar x, y, r, φ

done

r(φ)=1/φ

x(r, φ)=r*cos(φ), y((r, φ)=r*sin(φ)

somit

x(φ)=cos(φ)/φ, y(φ)=sin(φ)/φ

nun

$dy/dx=dy/d\varphi*d\varphi/dx=\dfrac{dy/d\varphi}{dx/d\varphi}$

$$\frac{\frac{d}{d\varphi}(\sin(\varphi)/\varphi)}{\frac{d}{d\varphi}(\cos(\varphi)/\varphi)}$$

$$\frac{-(\varphi\cdot\cos(\varphi)-\sin(\varphi))}{\varphi\cdot\sin(\varphi)+\cos(\varphi)}$$

nun φ=π/2

$$\frac{-(\varphi\cdot\cos(\varphi)-\sin(\varphi))}{\varphi\cdot\sin(\varphi)+\cos(\varphi)}\,|\,\varphi=\pi/2$$

$$\frac{2}{\pi}$$

$y'=\dfrac{2}{\pi}$

x(φ)=cos(φ)/φ|φ=π/2

$$x\left(\frac{\pi}{2}\right)=0$$

y(φ)=sin(φ)/φ|φ=π/2

$$y\left(\frac{\pi}{2}\right)=\frac{2}{\pi}$$

$\frac{y-\frac{2}{\pi}}{x-0}=y'=\frac{2}{\pi}$ ergibt: $y=\frac{2}{\pi}x+\frac{2}{\pi}$

Spirale	Y1:··· Y2:···

stop

Aufg. 2.2.18

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Define y(x)=|x/2−3|

done

abgeknickte Gerade:

y(x)

$$\left|\frac{x}{2}-3\right|$$

y' in x=6 nicht definiert (Knickstelle)

$\frac{d}{dx}(y(x))$

$$\frac{\mathrm{signum}\left(\frac{x}{2}-3\right)}{2}$$

y''=0 außerhalb der Knickstelle:

$\frac{d^2}{dx^2}(y(x))$

$$\frac{\frac{d}{dx}\left(\mathrm{signum}\left(\frac{x}{2}-3\right)\right)}{2}$$

Define y1(x)=|x/2−3|

done

Define $y2(x)=\dfrac{\text{signum}\left(\frac{x}{2}-3\right)}{2}$

done

Define $y3(x)=0$

Grafik

stop

Aufg. 2.2.29a

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Define $y(x)=\sqrt{1+x^{\wedge}2}$

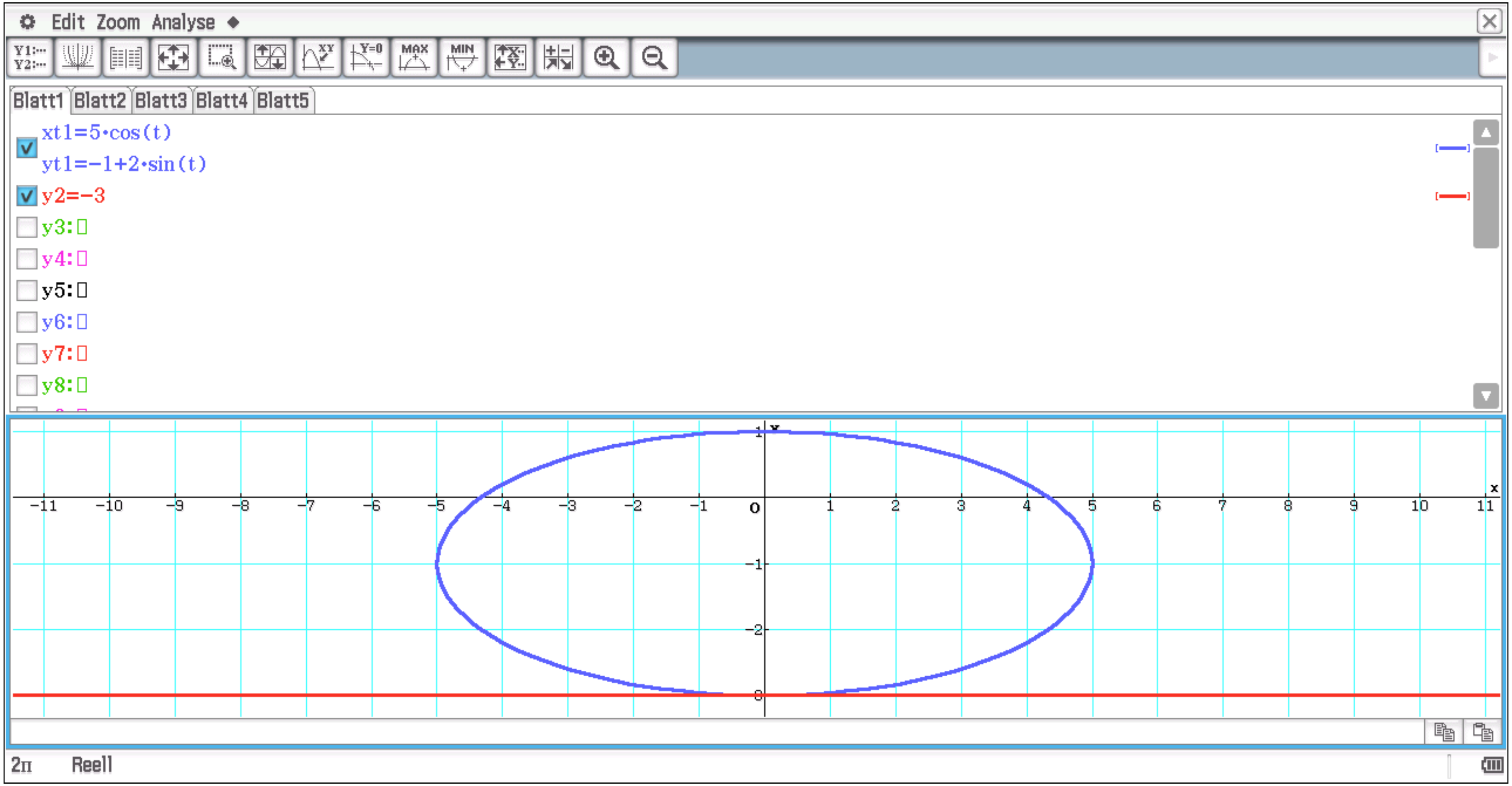
done

$\dfrac{d}{dx}(y(x))$

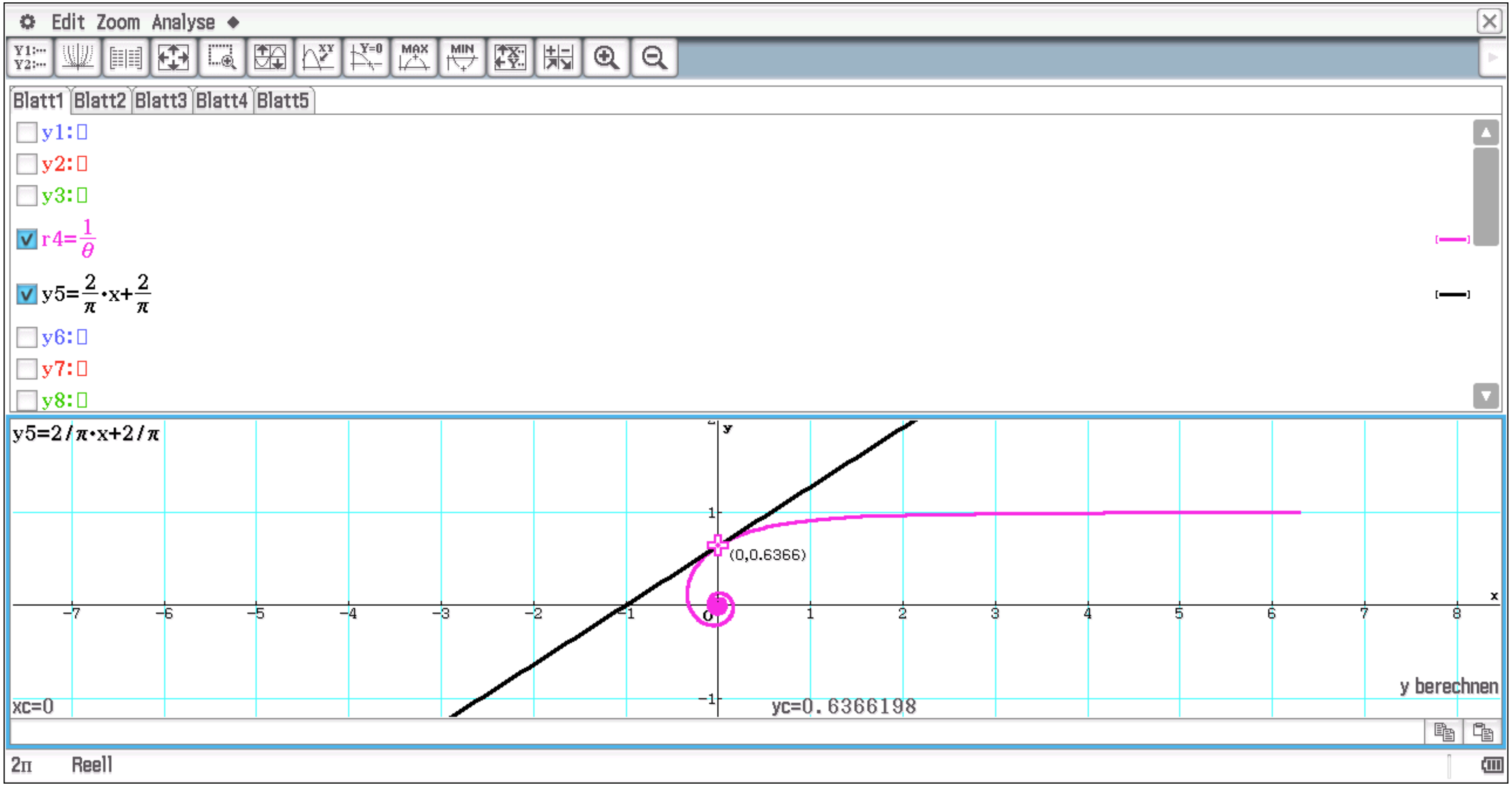
$$\frac{x}{\sqrt{x^2+1}}$$

somit $dy=\dfrac{x}{\sqrt{x^2+1}}dx$

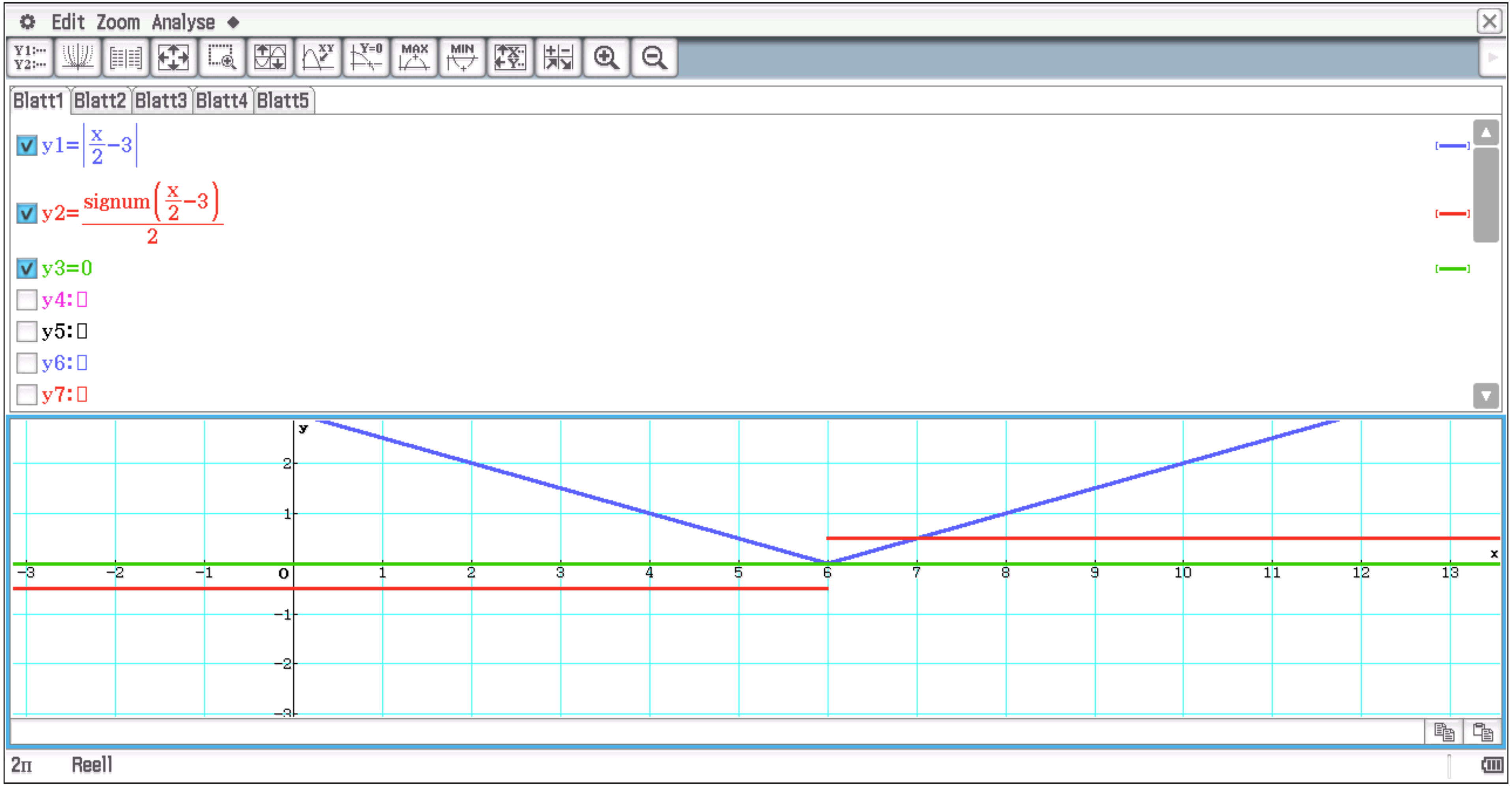
stop



Grafik zu Aufgabe 2.2.13b



Grafik zu Aufgabe 2.2.14c



Grafik zu Aufgabe 2.2.18