



Something about myself:

Paul Ernst **Ludwig Paditz**

Education:

Diploma mathematics, TU Dresden, Germany, 1974.

Doctor Natural Science (Dr.rer.nat.), TU, Dresden, Germany, 1977.

Postgrad., State University, St. Petersburg, Russia, 1984-1985.

Habilitation (Dr.rer.nat.habil.), HfV Dresden, Germany, 1989.

Working:

Assistant, department mathematics, TU Dresden, 1977-1979.

Head assistant, HfV Dresden, Institute of Mathematics, 1979-1991,
applied professor, HfV Dresden, 1992.

Professor, Faculty informatics/mathematics, Univ. of Applied Sciences,
Dresden, since 1992.

Opportunities and methods of using ClassPad



Using CAS in Math Education – first experience in Germany with the newest ClassPad technology CP400

Preface:

Ministry of Education in Saxony/Germany introduced 2004:
Modern Math Education with CAS, DGS and TC
beginning in the 8th class upto 12th class,
using graphic calculators (GTR)

CAS – Computer Algebra Systems

DGS – Dynamic Geometry Software

TC – Spreadsheet (Table Calculation)

8th class:

- Knowledge of the use of CAS when forming more complex terms and equations
- Investigating the influence of parameters in the function equation to trace the graph with DGS, TC, GTR or CAS
- Finding equations for measurement series with the help of linear regression with GTR, CAS or TC
- Solving linear systems of equations with more complex coefficients with GTR or CAS (two equations with two unknown variables)

9th class:

Functions and Powers

Mastered of determining zero quadratic functions, graphical solving quadratic equations and solving with GTR or CAS

10th class:

Obtaining the inverse function with CAS, graphical interpretation

Use of CAS to demonstrate the properties of functions
Obtaining illustrative of the limit concept

Know of parametric representation and polar coordinates to describe curves with GTR and CAS

11/12th class:

Differential calculus

The use of CAS in particular, should promote discovery learning, and support for substantive tasks, the reflection on the facts and the interpretation of the result.

Integral Calculus

The use of CAS in particular, should promote discovery learning, and support for substantive tasks, the reflection on the facts and the interpretation of the result.

worksheets:

- ☞ The use of worksheets in mathematics instruction has a long tradition.
- ☞ The use of a worksheet should guide the students to a structured work.
- ☞ Instead of an oral instruction, which requires a synchronous work of all students, the worksheet individually and with their own timing can be processed next.
- ☞ The sequence of the work orders in the worksheet helps to recognize the logical structure of a problem; the work procedures help to penetrate the question.

worksheets:

The disadvantage of a sheet of paper with work orders is seen, that the tools which can be used must be made available about. The students do not know always, how to carry out its solution steps in detail. Moreover, in a classic worksheet are missing the self check of the results, a feedback of the partial steps and also the visualization of the results.

The new developed **eActivity in the ClassPad** represents an extremely rich extension of the worksheet. The eActivity combines the written representation of the setting of tasks of a worksheet with the tool level of the ClassPad.

These tools are the individual menus or modules, which the ClassPad offers: Computer algebra system, dynamic geometry software, computer statistics, curve plotter, and much more.

eActivity:

Thus, all tasks can be worked on with the possibilities of the computer.

At the same time, the documentation of the work can be entered directly.

The found results can be visualized immediately or the results may be in a hidden file, can be viewed.

It is an interactive work of the students, between setting of tasks and the results and control of the results themselves.

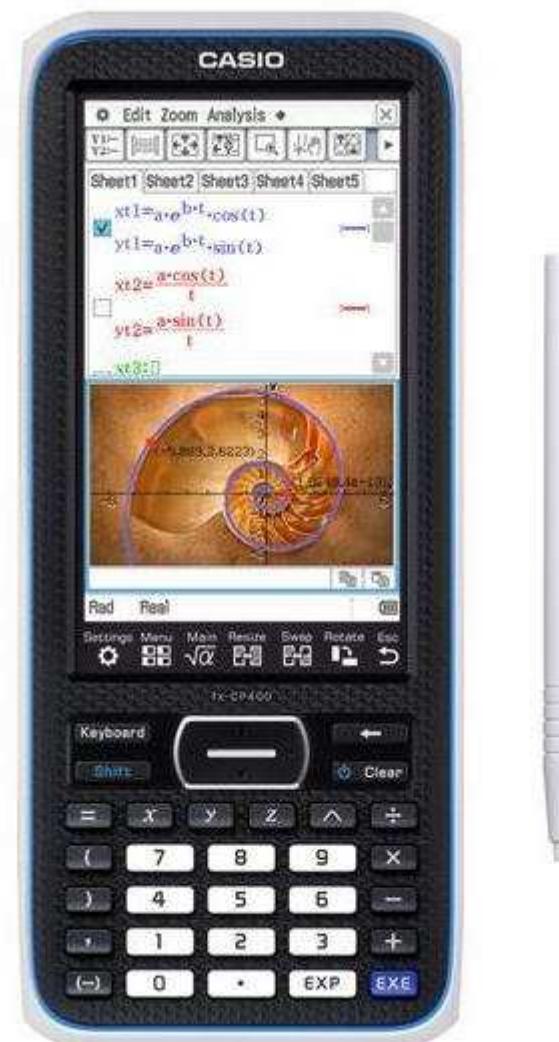
The tool – ClassPad 400:

Графический калькулятор
с сенсорным дисплеем

Fx-CP400 обладает рядом функций,
помогающих школьникам лучше
усваивать материал:

USB поддержка для быстрой и
легкой передачи данных и сов-
местимость с **проектором CASIO**
для отображения информации на
доске.

<http://edu.casio.ru/fx-cp400/>



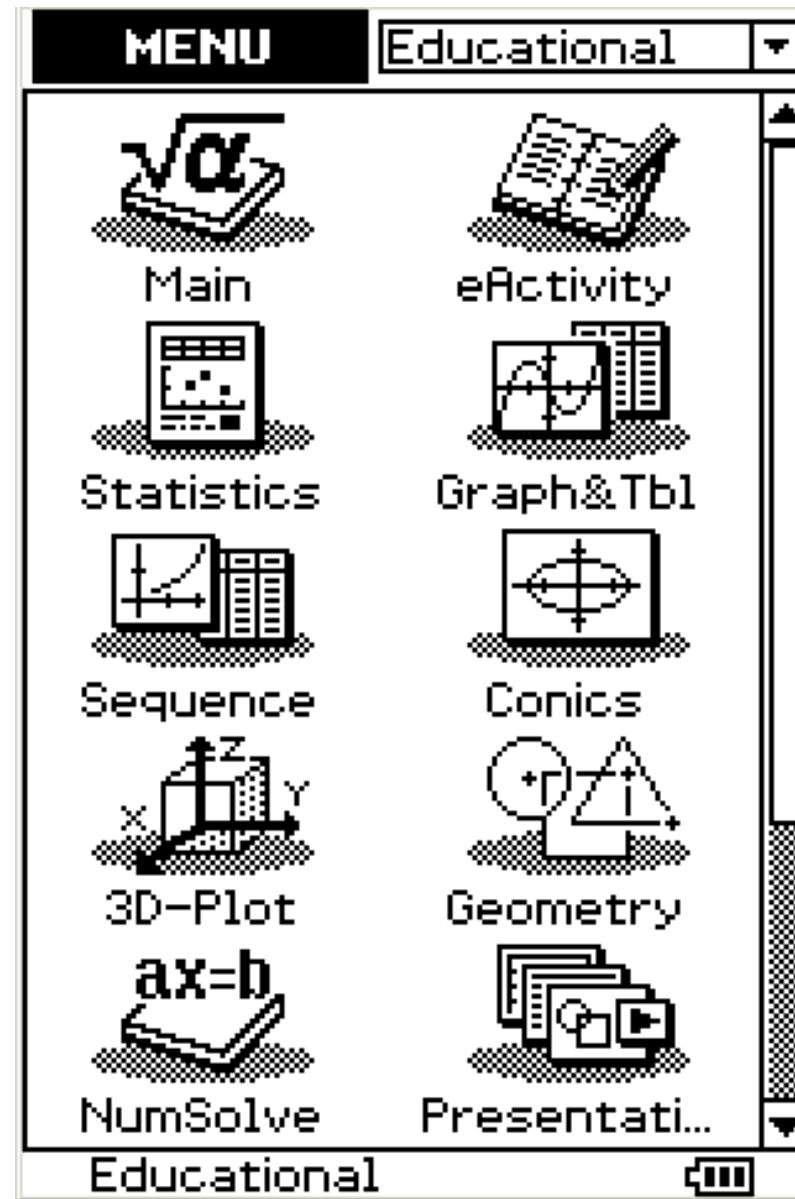
The tool – Beamer XJ-A146:

<http://casio-projectors.ru/products/xja146/>



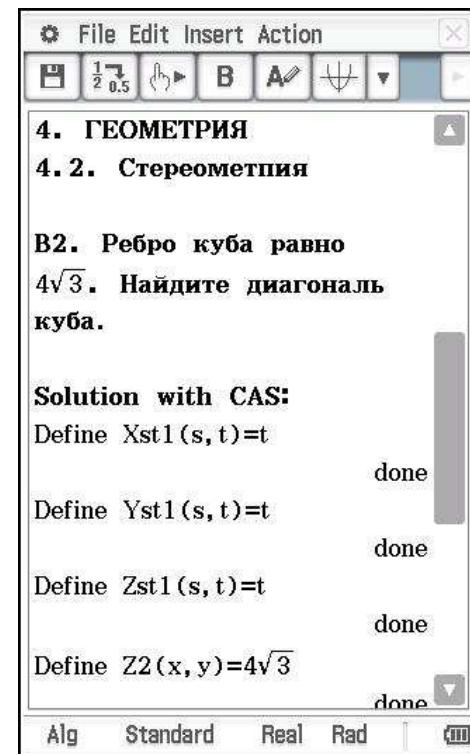
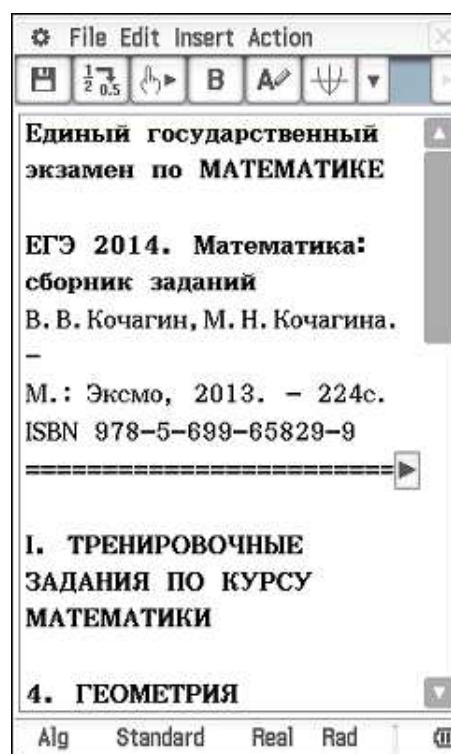
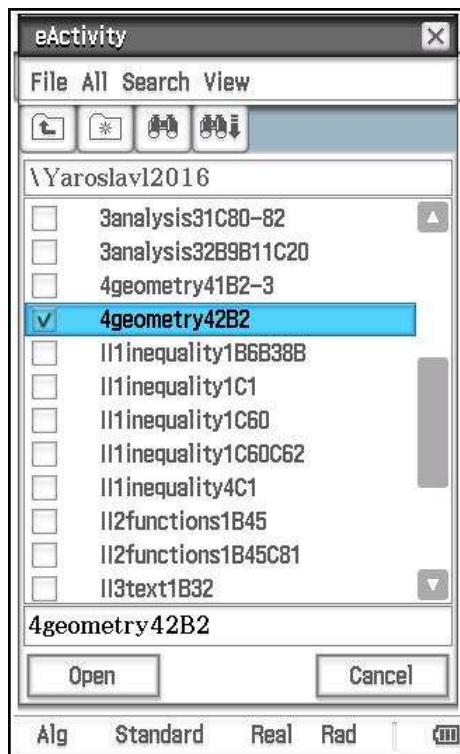
eActivity – an example

According to the
instructions from the
formula construct a
rhombus:



eActivity: 4. ГЕОМЕТРИЯ 4.2. Стереометрия

Ребро куба равно $4\sqrt{3}$. Найдите диагональ куба.



eActivity: 4. ГЕОМЕТРИЯ 4.2. Стереометрия

Ответ: диагональ куба равно 12.

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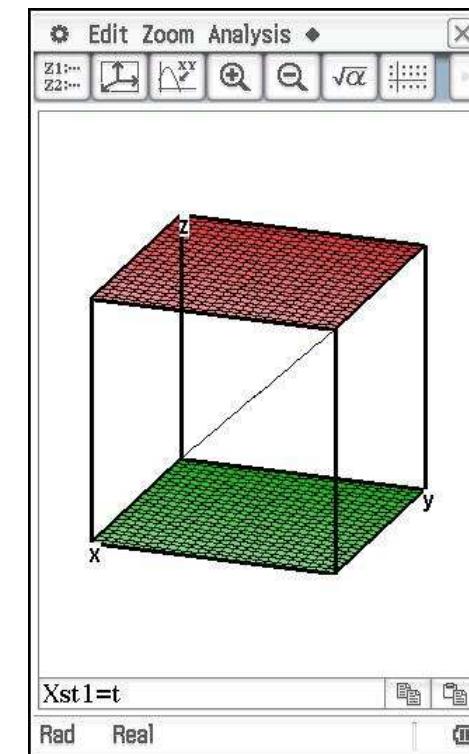
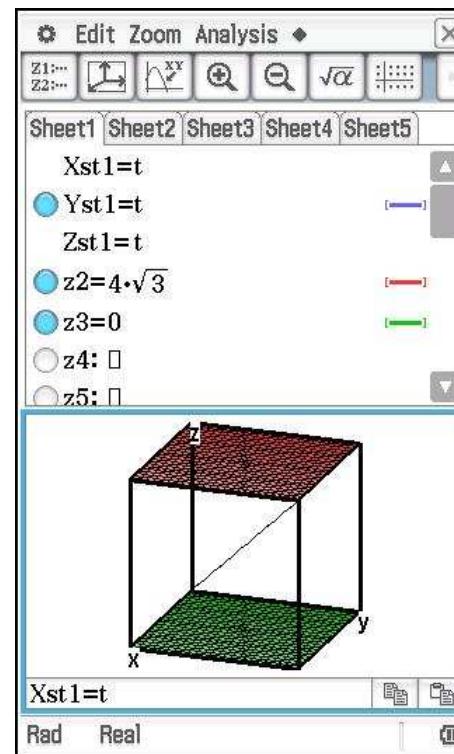
```

Define Zst1(s,t)=t
done
Define Z2(x,y)=4√3
done
Define Z3(x,y)=0
done
solve(Z2(x,y)=Zst1(s,t),t)
{t=4·√3}
norm([Xst1(s,t)]
      [Yst1(s,t)]
      [Zst1(s,t)])
| t=4·√3
12

```

3D - graphic solution

Alg Standard Real Rad



eActivity: 4. ГЕОМЕТРИЯ 4.1. Планиметрия

Найдите диаметр окружности, описанной около квадрата с стороной $8\sqrt{2}$.

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1/2 0.5 B A $\sqrt{ }$ v

Единый государственный экзамен по МАТЕМАТИКЕ

ЕГЭ 2014. Математика: сборник заданий

В. В. Кочагин, М. Н. Кочагина.

М.: Эксмо, 2013. – 224с.
ISBN 978-5-699-65829-9

I. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ПО КУРСУ МАТЕМАТИКИ

4. ГЕОМЕТРИЯ

Alg Standard Real Rad

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1/2 0.5 B A $\sqrt{ }$ v

4.1. Планиметрия

B2. Найдите диаметр окружности, описанной около квадрата с стороной $8\sqrt{2}$.

Solution with CAS:

$$(8\sqrt{2})^2 + (8\sqrt{2})^2 = d^2$$

$$256 = d^2$$

$$\text{solve}(ans, d)$$

$$\{d=-16, d=16\}$$

ответ: $d=16$

graphic construction

Alg Standard Real Rad

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16

$(8\sqrt{2})^2 + (8\sqrt{2})^2 = d^2$

$$256 = d^2$$

$\text{solve}(ans, d)$

$$\{d=-16, d=16\}$$

ответ: $d=16$

graphic construction

eActivity: 4. ГЕОМЕТРИЯ 4.1. Планиметрия

Найдите площадь круга (S), вписанного в прямоугольный треугольник с катетами, равными 24 и 10. В ответе укажите S/π .

B3. Найдите площадь круга (S), вписанного в прямоугольный треугольник с катетами, равными 24 и 10. В ответе укажите $\frac{S}{\pi}$.

Solution with CAS:

A(0, 0), B(24, 0), C(0, 10)
 Center M(r, r)
 $D(r, 0)$, $E(0, r)$, $F(x, y)$ with
 $y = -\frac{10}{24}x + 10$
 $|MD|^2 = r^2$, $|ME|^2 = r^2$, $|MF|^2 =$
 $(x-r)^2 + ((-\frac{10}{24}x+10)-r)^2 = r^2$

$|MD|^2 = r^2$, $|ME|^2 = r^2$, $|MF|^2 =$
 $(x-r)^2 + ((-\frac{10}{24}x+10)-r)^2 = r^2$
 $\text{solve}((r-x)^2 + (r - (-\frac{10}{24}x+10))^2 = r^2)$
 $\left\{ x = \frac{12 \cdot (7 \cdot r - 2 \cdot \sqrt{30} \cdot \sqrt{-r^2 + 34 \cdot r})}{169} \right.$

only one solution for x :

$\text{solve}(-r^2 + 34 \cdot r - 120 = 0, r)$
 $\{r=4, r=30\}$

thus $r=4$:
 $S=r^2\pi=16\pi \Rightarrow \frac{S}{\pi}=16$.

geometry window:

graphic construction

$\text{solve}(-r^2 + 34 \cdot r - 120 = 0, r)$
 $\{r=4, r=30\}$

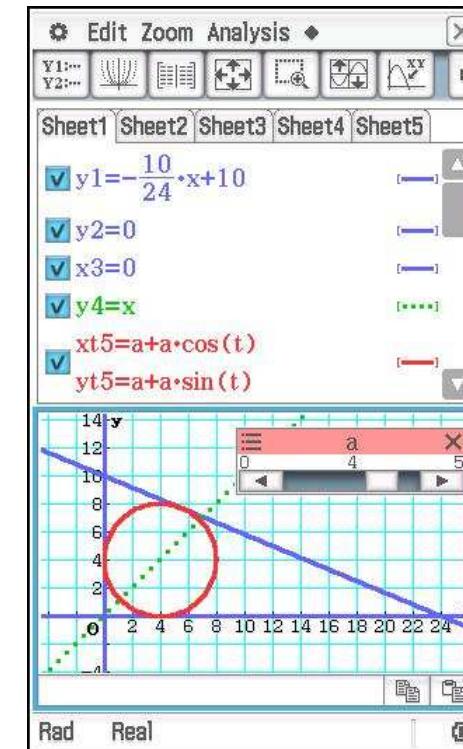
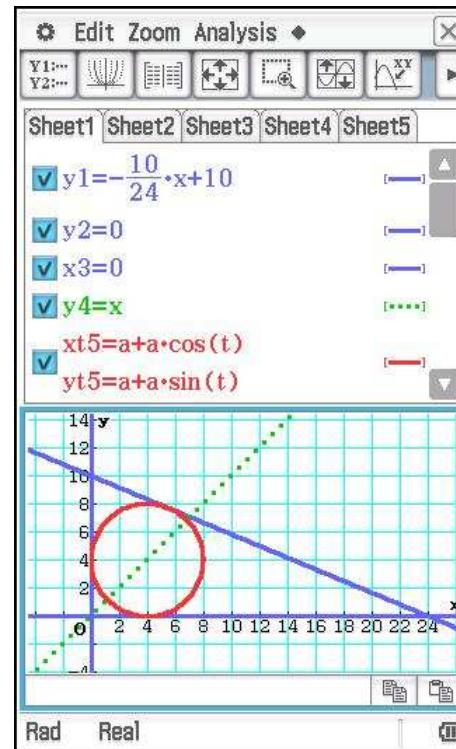
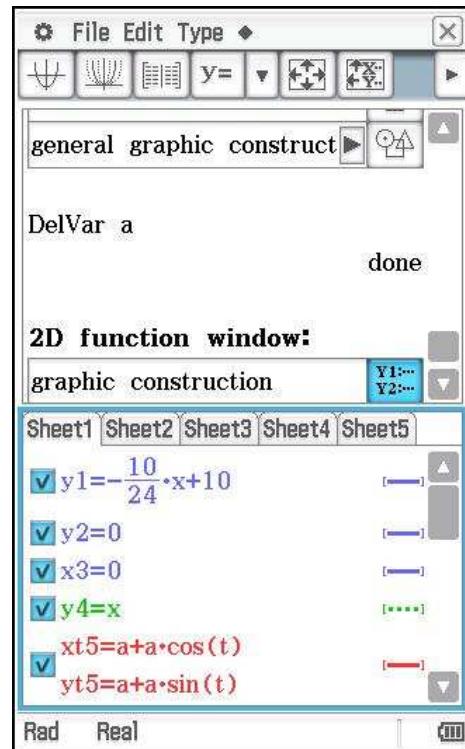
thus $r=4$:
 $S=r^2\pi=16\pi \Rightarrow \frac{S}{\pi}=16$.

geometry window:

graphic construction

eActivity: 4. ГЕОМЕТРИЯ 4.1. Планиметрия

Найдите площадь круга (S), вписанного в прямоугольный треугольник с катетами, равными 24 и 10.
В ответе укажите S/π .



eActivity: 1. ТРИГОНОМЕТРИЯ

1.1. Преобразования тригонометрических выражений



1.1. Преобразования тригонометрических выражений

B19. Найдите значение выражения

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\tan(5\pi+\alpha)}{\sin(\pi-\alpha)-1}, \text{ если } \operatorname{ctg}(\alpha)=0.25$$

Solution with CAS without condition:
(remark: $\operatorname{ctg}(\alpha)$ must be written as $1/\tan(\alpha)$)

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\tan(5\pi+\alpha)}{\sin(\pi-\alpha)-1}$$

Alg Standard Real Rad

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\tan(5\pi+\alpha)}{\sin(\pi-\alpha)-1}$$

$$\frac{\cos(\alpha)-\frac{1}{\tan(\alpha)}}{\sin(\alpha)-1}$$

simplify(ans)

$$\frac{1}{\tan(\alpha)}$$

now with condition
 $\tan(\alpha)=0.25$

$$\frac{1}{\tan(\alpha)} | \tan(\alpha)=0.25$$

4

ответ: $\operatorname{ctg}(\alpha)=4$

=====

Alg Standard Real Rad

eActivity: 1. ТРИГОНОМЕТРИЯ

1.1. Преобразования тригонометрических выражений

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direct computation with condition $\tan(\alpha)=0.25$:

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\frac{1}{\tan(5\pi+\alpha)}}{\sin(\pi-\alpha)-1} \quad | \tan(\alpha)=0.25$$

$$\frac{\cos(\alpha)-4}{\sin(\alpha)-1}$$

direct computation with condition $\alpha=\arctan(0.25)$:

(here in the CAS $\tan^{-1}(0.25)$ means $\arctan(0.25)$)

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\frac{1}{\tan(5\pi+\alpha)}}{\sin(\pi-\alpha)-1} \quad | \alpha=\tan^{-1}(0.25)$$

$$\frac{-4\sqrt{17}+4}{17}$$

$$\frac{-\sqrt{17}+1}{17}$$

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$\frac{\tan(\pi-\alpha)-1}{\sin(\pi-\alpha)-1} \quad | \alpha=\tan^{-1}(0.25)$

$$\frac{-4\sqrt{17}+4}{17}$$

$$\frac{-\sqrt{17}+1}{17}$$

simplify(ans)

$$\frac{4}{17}$$

remark:

$\tan^{-1}(0.25)$

$$\tan^{-1}\left(\frac{1}{4}\right)$$

approx(ans)

0.2449786631

$$\sin\left(\frac{9}{2}\pi-\alpha\right)-\frac{1}{\tan(5\pi+\alpha)} \quad | \alpha=\text{ans}$$

Alg Standard Real Rad

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$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\frac{1}{\tan(5\pi+\alpha)}}{\sin(\pi-\alpha)-1}$

$$= \frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\frac{1}{\tan(\alpha)}}{\sin(\pi-\alpha)-1}$$

$$= \frac{\sin\left(\frac{1}{2}\pi-\alpha\right)-\frac{1}{\tan(\alpha)}}{\sin(\pi-\alpha)-1}$$

$$= \frac{\cos(-\alpha)-\frac{1}{\tan(\alpha)}}{\sin(\pi-\alpha)-1}$$

$$= \frac{\cos(\alpha)-\frac{1}{\tan(\alpha)}}{\sin(\pi-\alpha)-1}$$

$$= \frac{\cos(\alpha)-\frac{1}{\tan(\alpha)}}{-\sin(-\alpha)-1}$$

$$= \frac{\cos(\alpha)-\frac{1}{\tan(\alpha)}}{\cos(\alpha)-\frac{1}{\tan(\alpha)}}$$

ERROR!

Sorry, not equivalent

OK

$\sin(\alpha)-1$

$$= \frac{\cos(\alpha)\cdot\sin(\alpha)-1}{\sin(\alpha)-1}$$

$$= \frac{\cos(\alpha)}{\sin(\alpha)}$$

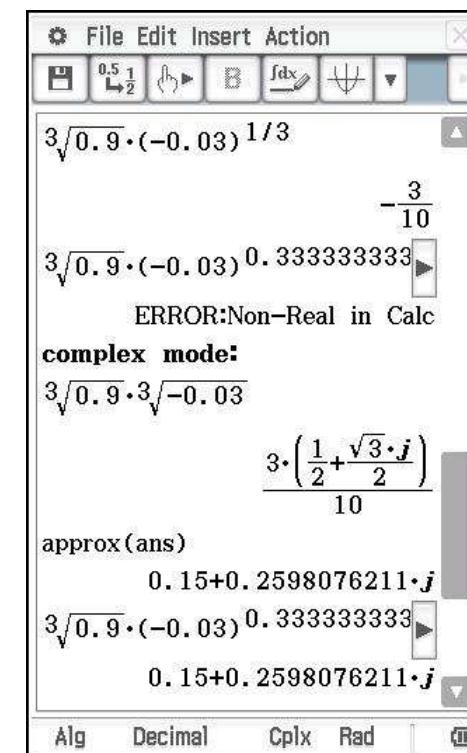
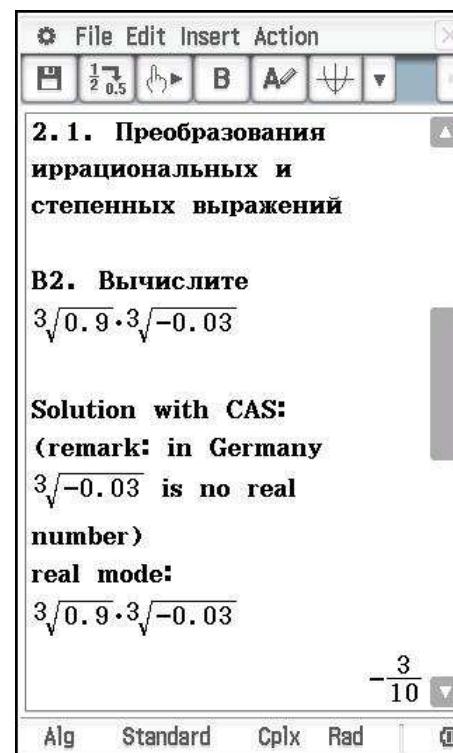
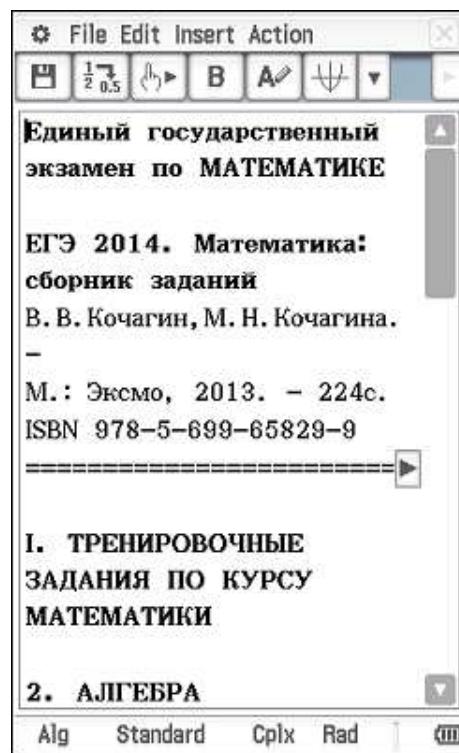
$$= \frac{1}{\tan(\alpha)}$$

$$= 4$$

Exp:((sin(((9)/(2))\cdot\pi-\alpha)-((1)/(\tan(\alpha))))\cdot\sin(\alpha)-1)

eActivity: 2. АЛГЕБРА

2.1. Преобразования иррациональных и степенных выражений



eActivity: 2. АЛГЕБРА

2.1. Преобразования иррациональных и степенных выражений

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2.1. Преобразования иррациональных и степенных выражений

B47. Упростите выражение $\frac{3(\sqrt{a}-3\cdot 4\sqrt{a\cdot b})}{4\sqrt{a\cdot b}-3\cdot \sqrt{b}}$, если $\frac{a}{b}=7\frac{58}{81}$

Solution with CAS:
(remark: in CAS write $7+\frac{58}{81}$)
suppose $a \geq 0$ and $b > 0$

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$\frac{3(\sqrt{a}-3\cdot 4\sqrt{a\cdot b})}{4\sqrt{a\cdot b}-3\cdot \sqrt{b}}$

$\frac{-3\cdot (\sqrt{a}-3\cdot (a\cdot b)^{\frac{1}{4}})}{3\cdot \sqrt{b}-(a\cdot b)^{\frac{1}{4}}}$

$\frac{3(\sqrt{a}-3\cdot 4\sqrt{a\cdot b})}{4\sqrt{a\cdot b}-3\cdot \sqrt{b}} | a=(7+\frac{58}{81})\cdot b$

$\frac{-3\cdot (\frac{25\cdot \sqrt{b}}{9}-5\cdot (b^2)^{\frac{1}{4}})}{3\cdot \sqrt{b}-5\cdot (b^2)^{\frac{1}{4}}}$

simplify(ans|b>0)

5

Alg Standard Real Rad

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$\frac{-3\cdot (\frac{25\cdot \sqrt{b}}{9}-5\cdot (b^2)^{\frac{1}{4}})}{3\cdot \sqrt{b}-5\cdot (b^2)^{\frac{1}{4}}}$

simplify(ans|b>0)

5

ответ: 5

remark:

$7+\frac{58}{81}$

$\frac{625}{81}$

$7\frac{58}{81}=7\cdot\frac{58}{81}$

$\frac{406}{81}=\frac{406}{81}$

Alg Standard Real Rad

eActivity: 2. АЛГЕБРА

2.2. Иррациональные уравнения

2.3. Преобразования логарифмических выражений

2.2. Иррациональные уравнения

B19. Решите уравнение

$$\sqrt{x^2+3x-4} + \sqrt{x^3+12x^2-11x-2} = 0.$$

Solution with CAS:

```
solve(x^2+3x-4=0, x)
{x=-4, x=1}

solve(x^3+12x^2-11x-2=0, x)
{x=1, x=-\frac{\sqrt{161}}{2} - \frac{13}{2}, x=\frac{\sqrt{161}}{2}}
```

ответ: x=1

Alg Standard Real Rad

2.3. Преобразования логарифмических выражений

B17. Найдите значение выражения

$$(\log_5(36) + \log_5(2) - \log_5(8)) \cdot \log_9\left(\frac{1}{25}\right)$$

Solution with CAS:

$$\begin{aligned} & (\log_5(36) + \log_5(2) - \log_5(8)) \\ & - \left(\frac{2 \cdot \ln(3) + 2 \cdot \ln(2)}{\ln(5)} - \frac{2 \cdot \ln(2)}{\ln(5)} \right) \end{aligned}$$

simplify(ans)

Alg Standard Real Rad

$$\begin{aligned} & (\log_5(36) + \log_5(2) - \log_5(8)) \\ & = \log_5\left(36 \cdot \frac{2}{8}\right) \cdot \log_9\left(\frac{1}{25}\right) \\ & = \log_5(9) \cdot \log_9\left(\frac{1}{25}\right) \\ & = \log_5(9) \cdot (-\ln(5)^2) \\ & = \log_5(9) \cdot (-2) \cdot \log_9(5) \\ & = \frac{\ln(9)}{\ln(5)} \cdot (-2) \cdot \log_9(5) \\ & = \frac{\ln(9)}{\ln(5)} \cdot (-2) \cdot \frac{\ln(5)}{\ln(9)} \\ & = -2 \end{aligned}$$

Exp:(log(5,36)+ log(5,2)- log(5,8))

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eActivity: 2. АЛГЕБРА

2.4. Логарифмические уравнения и неравенства

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2.4. Логарифмические уравнения и неравенства

C87. Решение неравенство

$$\frac{\log_{0.1x}(10) \cdot \log(10x)}{\log_{0.01x}(10)} \leq 2.$$

Solution with CAS:

(Remark: $\lg(\dots) = \log(\dots)$)

$$\frac{\log_{0.1x}(10) \cdot \log(10x)}{\log_{0.01x}(10)} \leq 2$$

$$\frac{(\ln(x)-2\ln(5)-2\ln(2))}{(\ln(x)-\ln(5)-\ln(2))} \leq 2$$

simplify(ans)

$$\left(\frac{\ln(x)}{\ln(10)} + 1\right) \cdot \ln\left(\frac{x}{100}\right) \leq 2$$

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File Edit Insert Action

сimplify(ans)

$$\left(\frac{\ln(x)}{\ln(10)} + 1\right) \cdot \ln\left(\frac{x}{100}\right) \leq 2$$

$$\ln\left(\frac{x}{10}\right)$$

solve(ans, x)

$$\{0 < x \leq 1, 10 < x \leq 1000\}$$

Remark: $x \neq 100$,

$\log_{0.01x}(10)$ for $x=100$ is not defined!

$\log_{0.01x}(10) | x=100$

Undefined

$\log_{0.1x}(10) \cdot \log(10x)$

$\log_{0.01x}(10) | x=100$

0

File Edit Insert Action

$\log_{0.01x}(10)$ for $x=100$ is not defined!

$\log_{0.01x}(10) | x=100$

Undefined

$\log_1(10)$

Undefined

$\lim_{x \rightarrow 100^+} (\log_{0.01x}(10))$

0

$\lim_{x \rightarrow 100^-} (\log_{0.01x}(10))$

$-\infty$

Alg Standard Real Rad

eActivity: 2. АЛГЕБРА

2.5. Показательные уравнения и неравенства

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2.5. Показательные уравнения и неравенства

B22. Сколько целых чисел входит в область определения функции

$$y = \sqrt{\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} - 3}$$

Solution with CAS:

Define $y1(x) = \sqrt{\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} - 3}$

done

2D-graphic

Alg Standard Real Rad

Edit Zoom Analysis

Sheet1 Sheet2 Sheet3 Sheet4 Sheet5

y1: $\sqrt{\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} - 3}$

y2: 0

y3: 0

y4: 0

y5: 0

Rad Real

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$\lim_{x \rightarrow 2^-} \sqrt{\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} - 3} = \infty$

$y1(0) = 0$

$y1(1) = 2\sqrt{6}$

$\text{solve}\left(\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} \geq 3, x\right) = \{x \geq 0\}$

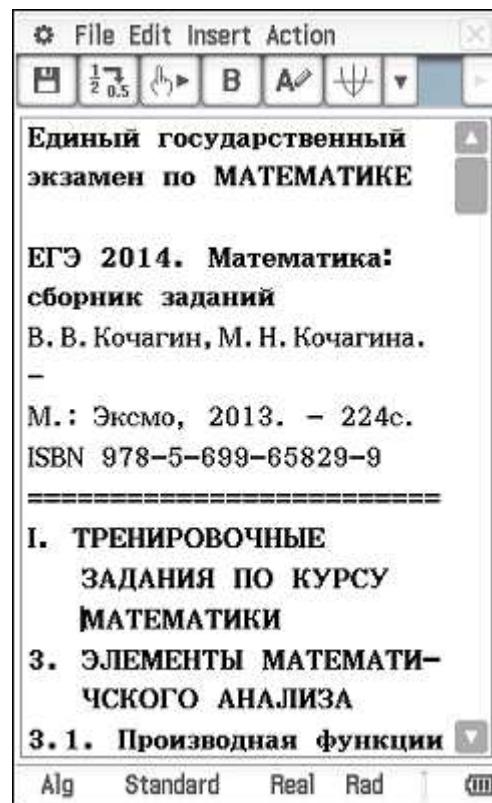
$\text{solve}\left(\frac{x+2}{2-x} \geq 1, x\right) = \{0 \leq x < 2\}$

Alg Standard Real Rad

ответ: 2 ($x=0, x=1$)

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

3.1. Производная функции



C80. Найдите множество значений функции

$$h(x)=2\sqrt{x+14}+\sqrt{6-x}.$$

Solution with CAS:

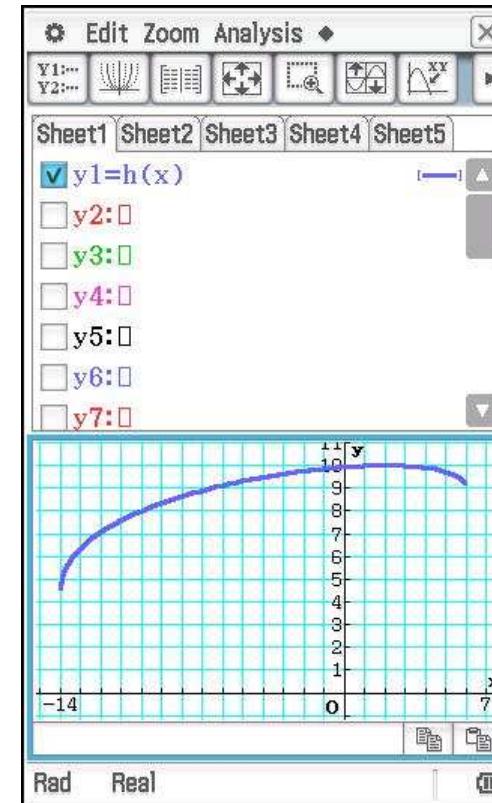
```
Define h(x)=2*sqrt(x+14)+sqrt(6-x)
done
```

```
fMin(h(x),x)
{.MinValue=2*sqrt(5), x=-14}
```

```
fMax(h(x),x)
{.MaxValue=10, x=2}
```

graphical solution

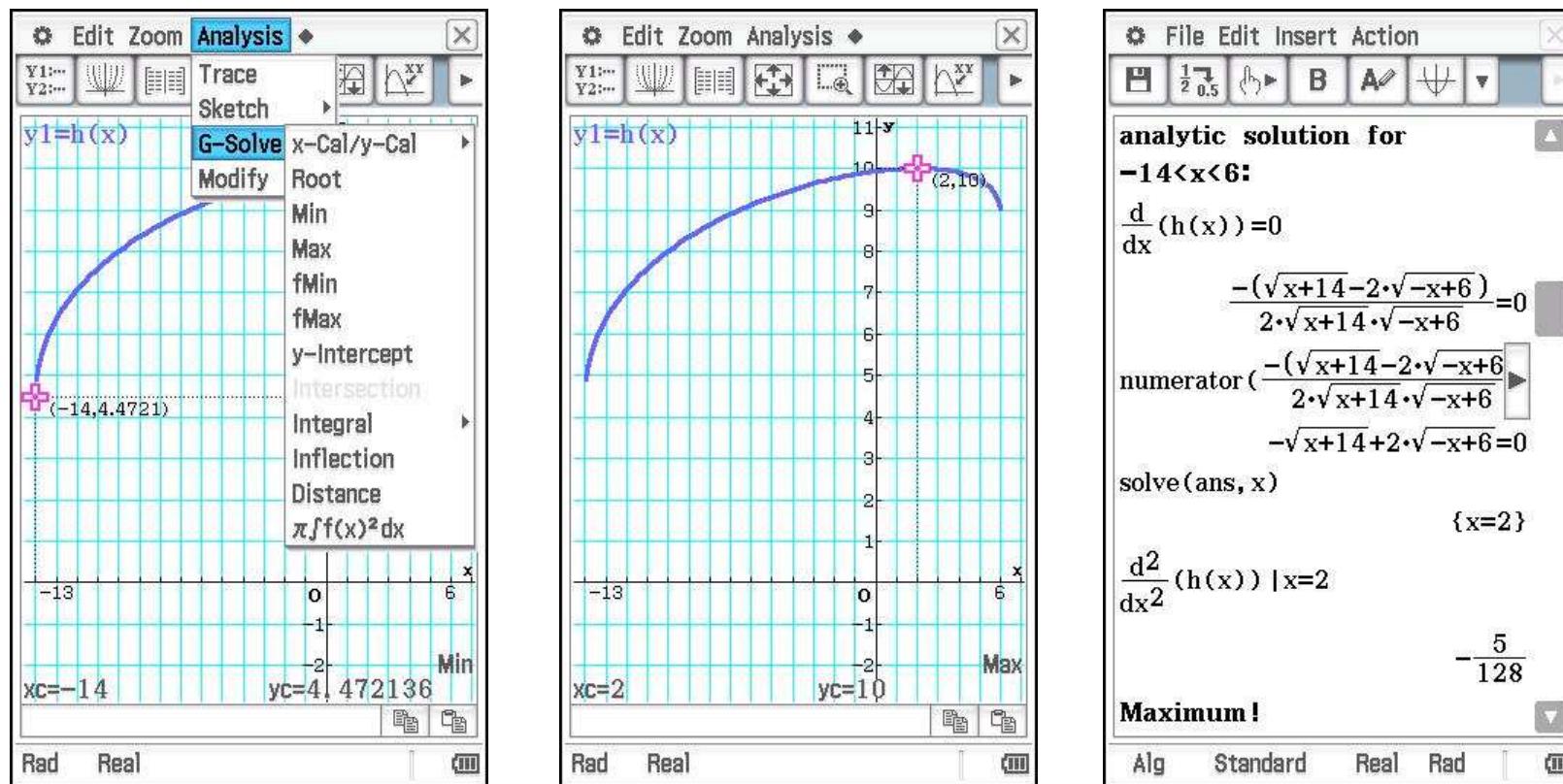
ответ: $2\sqrt{5} \leq h(x) \leq 10.$



ответ: $2\sqrt{5} \leq h(x) \leq 10$

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

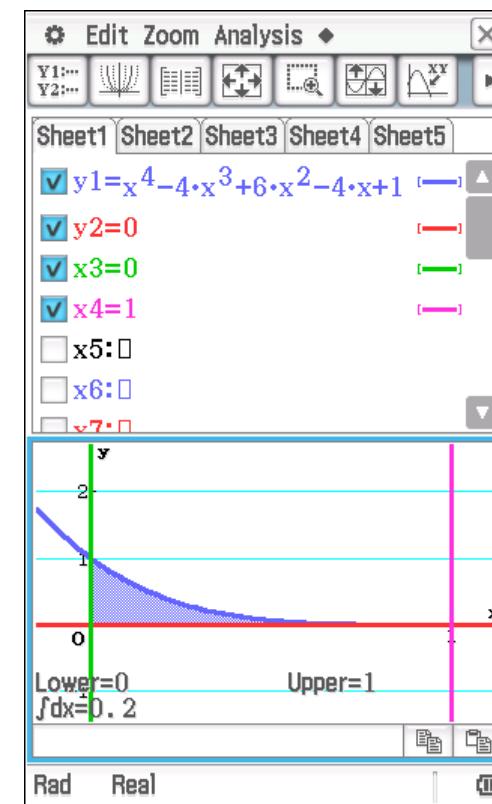
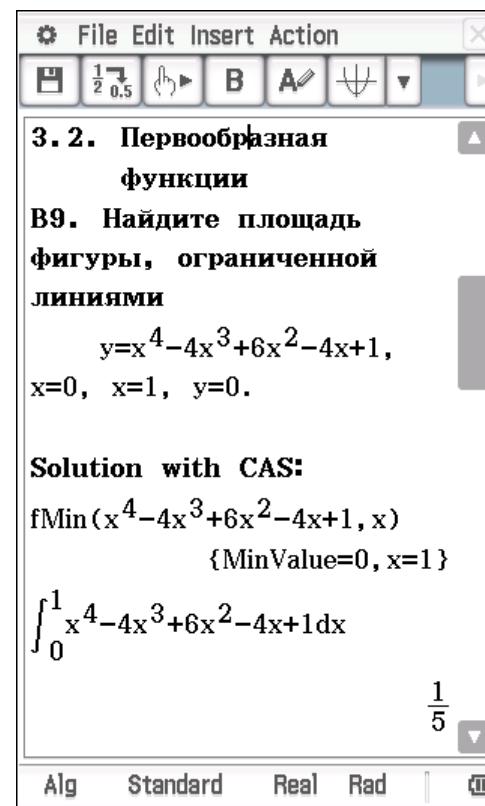
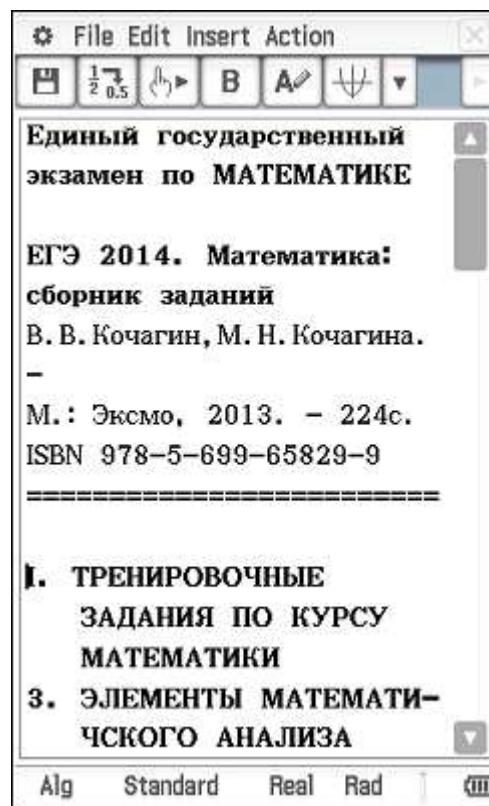
3.1. Производная функции



ответ: $4.472136 \leq h(x) \leq 10$

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

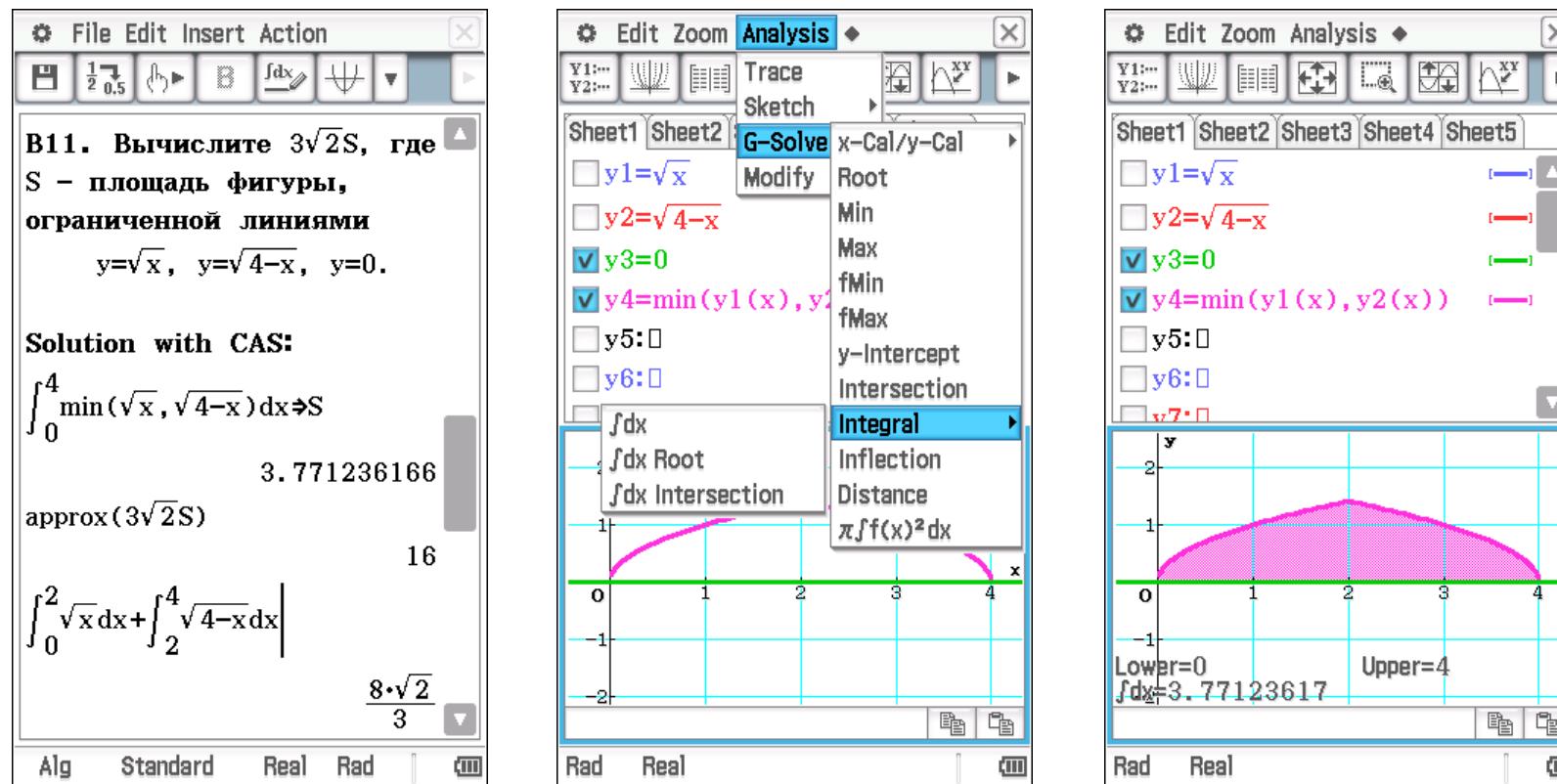
3.2. Первообразная функции



ответ: 0.2

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

3.2. Первообразная функции



ответ: 16

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

3.2. Первообразная функции

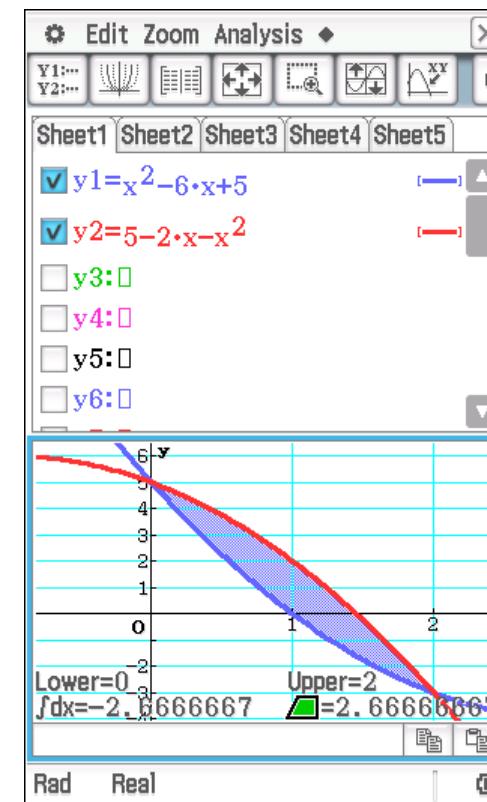
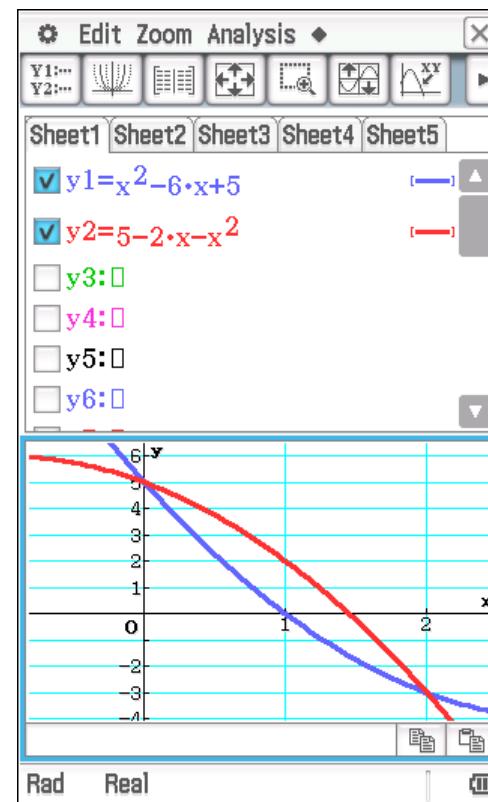
C20. Найдите площадь фигуры, ограниченной линиями
 $y=x^2-6x+5$ и $y=5-2x-x^2$.

Solution with CAS:

```
solve(x^2-6x+5=5-2x-x^2, x)
{x=0, x=2}
 $\int_0^2 (5-2x-x^2)-(x^2-6x+5) dx$ 
```

ответ: $2 + \frac{2}{3}$

graphical: Integral intersect



ответ: $2 + \frac{2}{3}$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА



1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА

B6. Решите неравенство $4x^2+4x+1 \leq 0$.

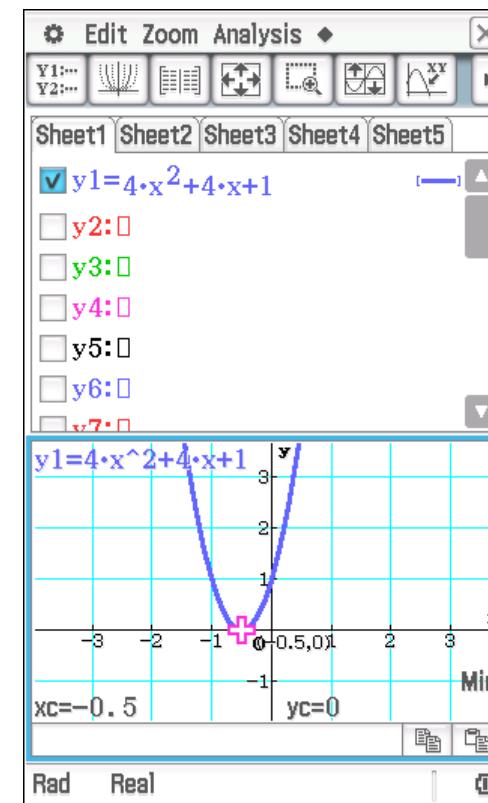
Solution with CAS:

```
solve(4x^2+4x+1≤0, x)
{x=-1/2}
```

factor(4x²+4x+1)

$$(2x+1)^2$$

graphic solution



ответ: -1/2

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА

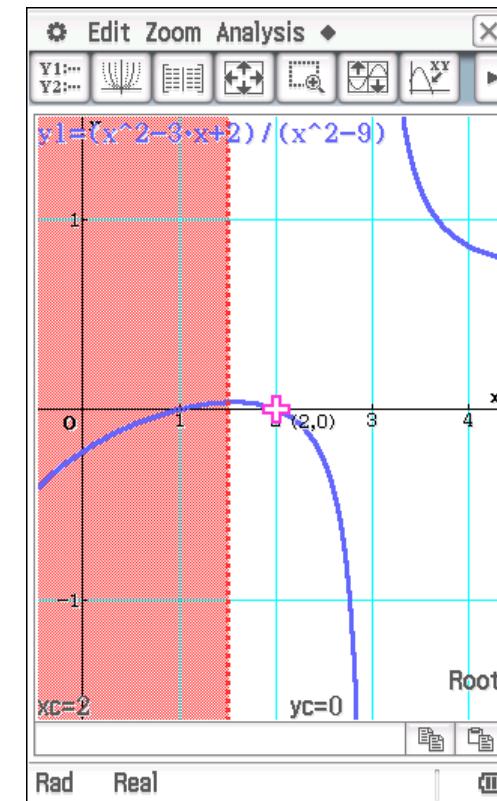
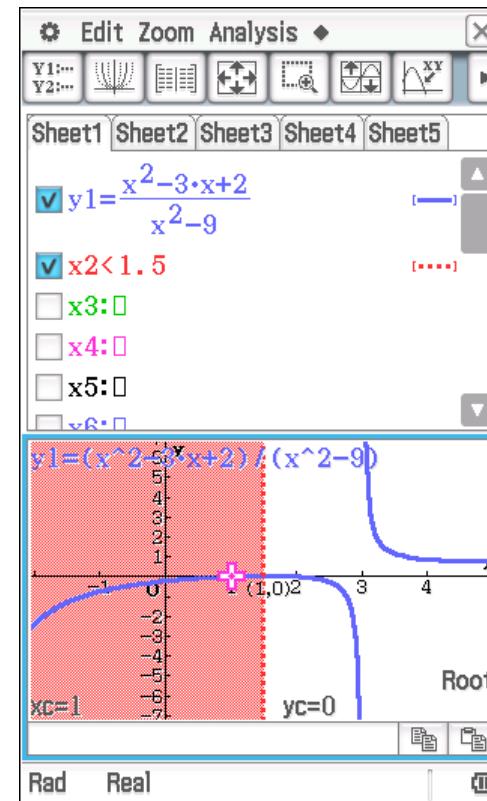
B38. Найдите корень уравнения $\frac{x^2-3x+2}{x^2-9}=0$, удовлетворяющий неравенству $-(5-2x) > -(6.5-3x)$.

Solution with CAS:

```
solve(-(5-2x) > -(6.5-3x), x)
{x < 3/2}
```

```
solve((x^2-3x+2=0, x)
{x=1, x=2}
```

```
solve((x^2-3x+2=0 | x < 3/2, x)
{x=1}
```



ответ: 1

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА

B39. Среди решений уравнения $\frac{2x-2}{x+3} + \frac{x+3}{x-3} = 5$ найдите те, которые не удовлетворяют неравенству $-x^2 - 7x + 8 < 0$.

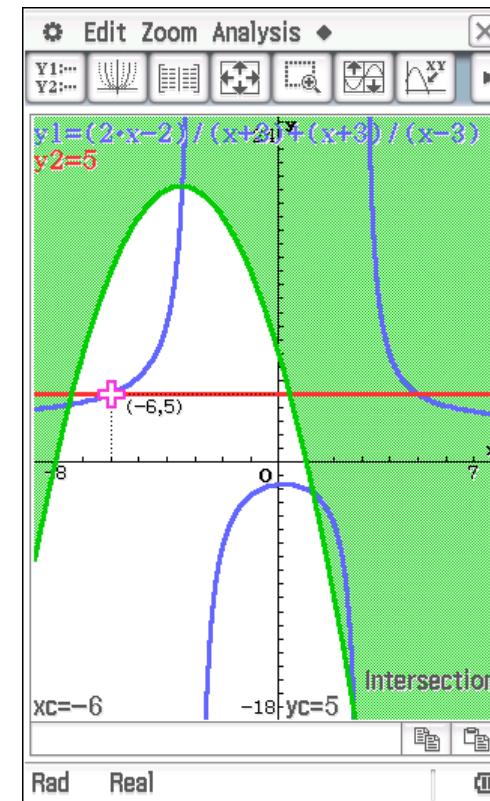
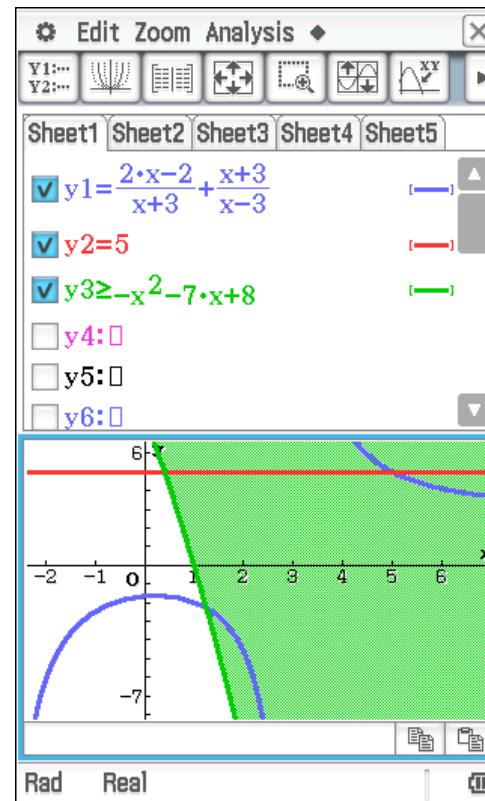
Solution with CAS:

```
solve((2x-2)/(x+3) + (x+3)/(x-3) = 5, x)
{x=-6, x=5}

judge(-x^2 - 7x + 8 < 0 | x=-6)
FALSE

judge(-x^2 - 7x + 8 < 0 | x=5)
TRUE
```

ответ: $x=-6$



ответ: -6

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА

C60. Решите неравенство

$$\frac{\sqrt{20+x-x^2}}{2x-3} \leq \frac{\sqrt{20+x-x^2}}{x-6}.$$

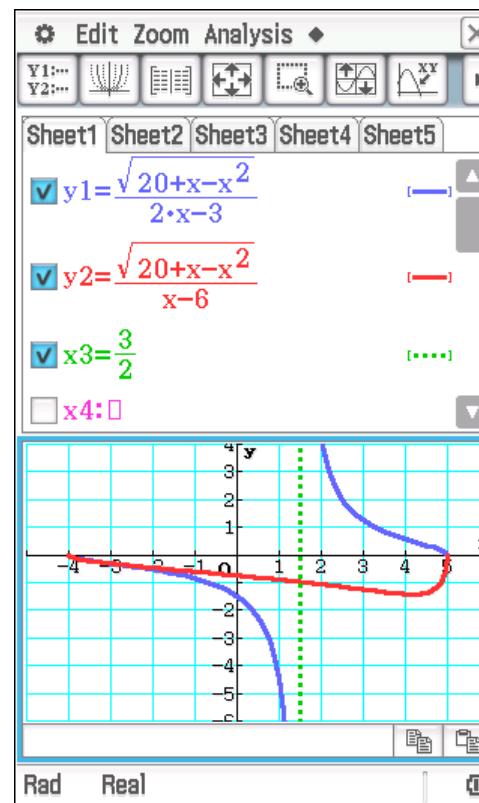
Solution with CAS:

```
solve(\frac{\sqrt{20+x-x^2}}{2x-3} \leq \frac{\sqrt{20+x-x^2}}{x-6}, x)
{x=-4, x=5, -3≤x<\frac{3}{2}}
```

```
solve(\sqrt{20+x-x^2}=0, x)
{x=-4, x=5}
```

```
solve(\frac{1}{2x-3} \leq \frac{1}{x-6}, x)
{-3≤x<\frac{3}{2}, 6< x}
```

ответ: $x=-4, x=5, -3 \leq x < \frac{3}{2}$



ответ: $x=-4, x=5, -3 \leq x < \frac{3}{2}$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА

C62. Решите неравенство

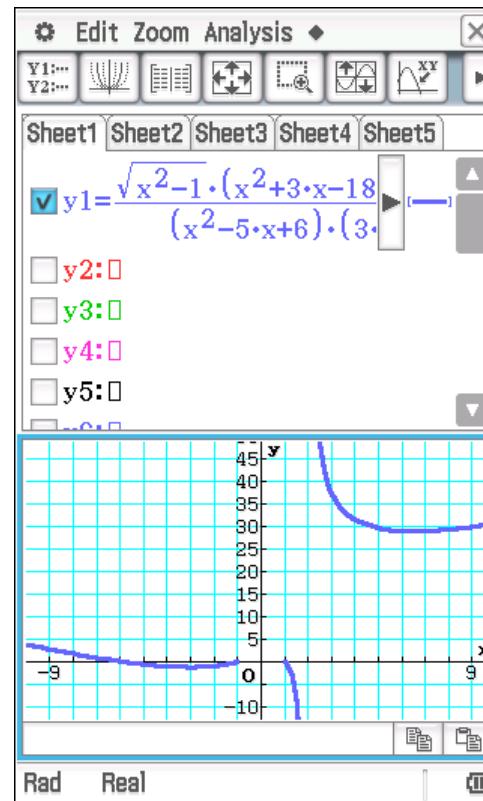
$$\sqrt{x^2-1} \cdot \frac{(x^2+3x-18)(4x^2-4x+1)}{(x^2-5x+6)(3x^2-8x+14)} \leq 0.$$

Solution with CAS:

```
solve(√(x^2-1)·(x^2+3x-18)·(4x^2-4x+1)/(x^2-5x+6)·(3x^2-8x+14), x)
```

ответ: $-6 \leq x \leq -1, 1 \leq x < 2$

graphical solution



ответ: $-6 \leq x \leq -1, 1 \leq x < 2$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

B45. Найдите наименьшее значение функции $y=\log_{\frac{1}{3}}(81-x^2)$.

Solution with CAS:

$$fMin(\log_{\frac{1}{3}}(81-x^2), x)$$

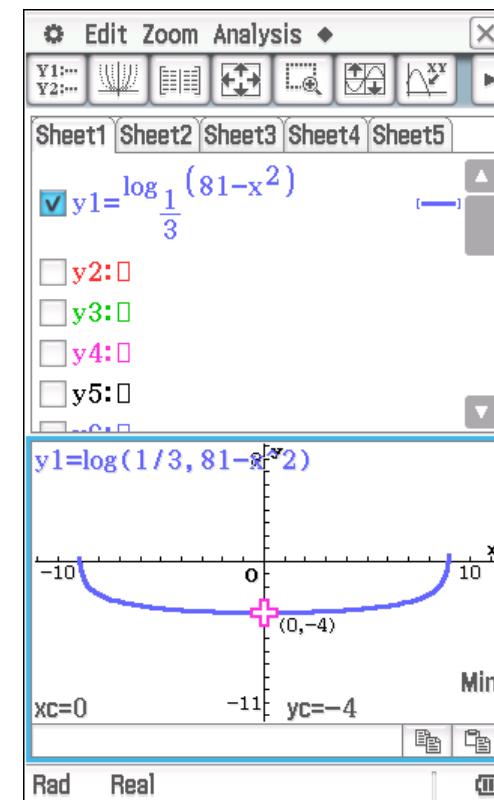
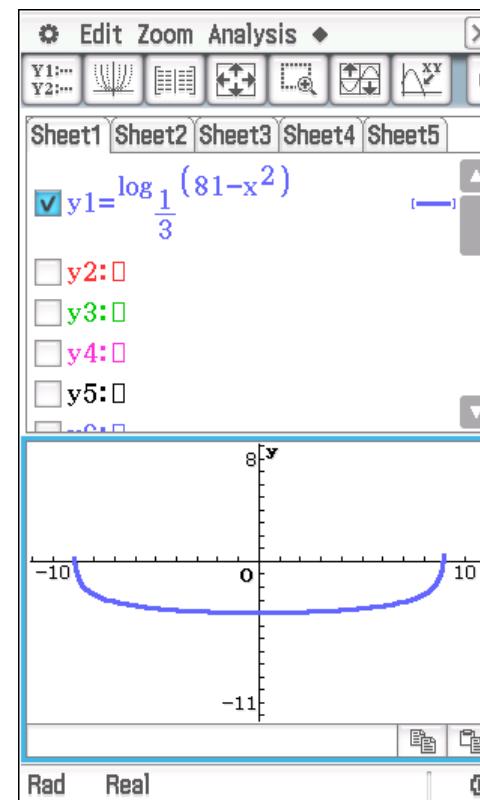
{MinValue=-4, x=0}

$$\log_{\frac{1}{3}}(81-x^2)$$

$$\frac{-\ln(-x^2+81)}{\ln(3)}$$

ответ: $y=-4$ for $x=0$

Alg Standard Real Rad



ответ: $y=-4$ for $x=0$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

C81. Найдите множество значений функции $y=\log_3(x-|x|+3)$.

Solution with CAS:

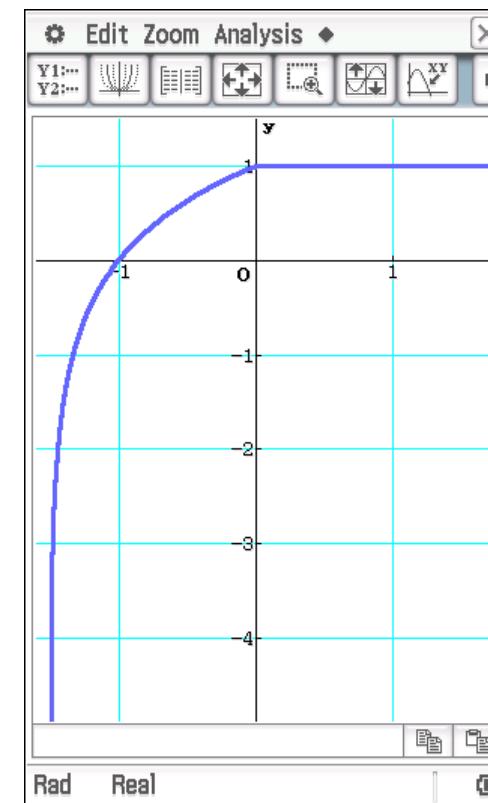
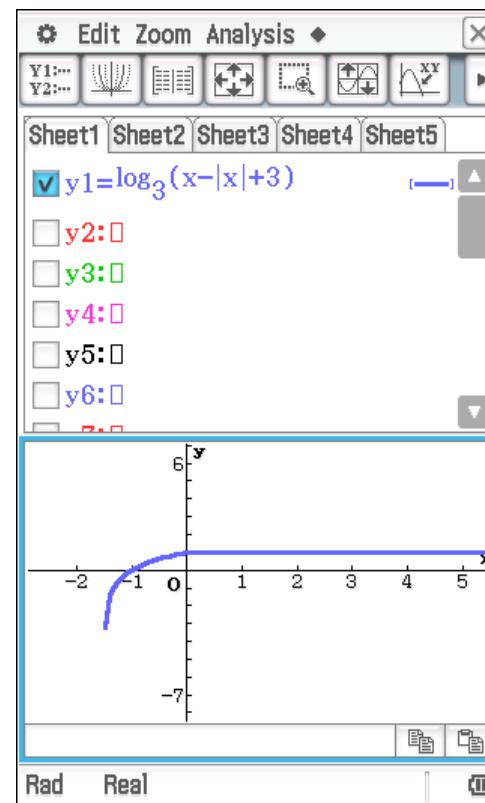
$$fMin(\log_3(x-|x|+3), x) \\ \left\{ \text{MinValue}=-\infty, x=-\frac{3}{2} \right\}$$

$$fMax(\log_3(x-|x|+3), x) \\ \left\{ \text{MaxValue}=1, x=1, x=\infty \right\}$$

Ответ: $-\infty < x \leq 1$

graphical solution

Alg Standard Real Rad



ответ: $-\infty < x \leq 1$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

C82. Найдите множество значений функции

$$y = \left(\frac{1}{2}\right)^{\frac{12}{3-\sin(x)}}.$$

Solution with CAS:

$$fMin\left(\left(\frac{1}{2}\right)^{\frac{12}{3-\sin(x)}}, x\right)$$

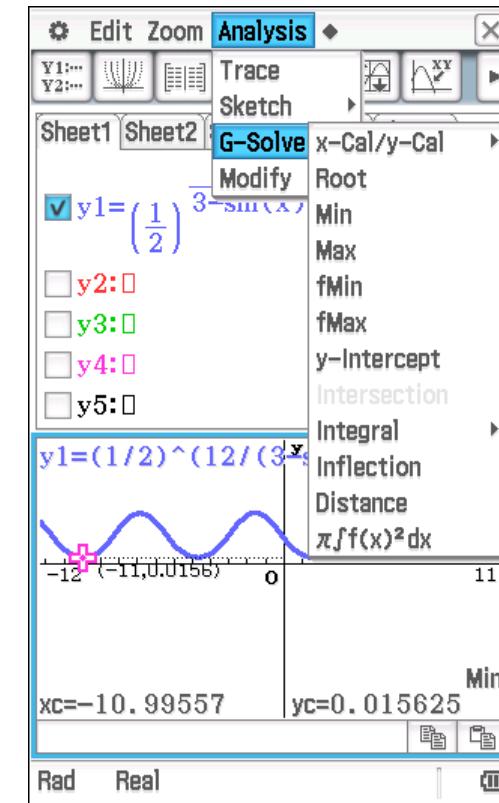
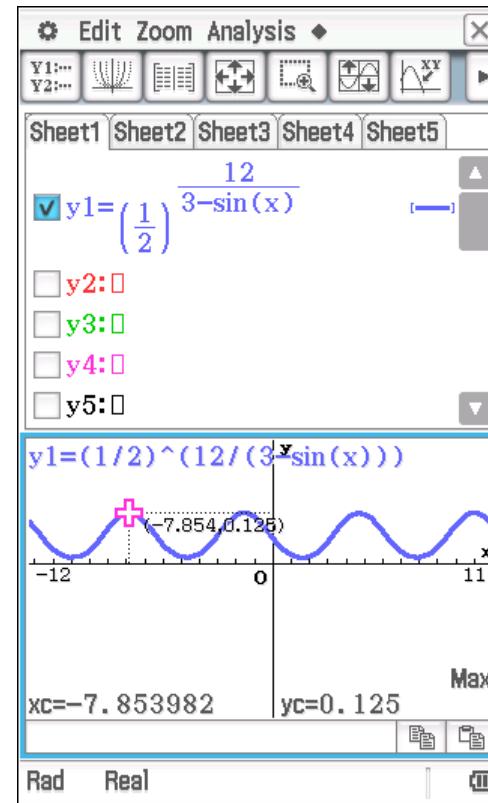
$$\left\{ \text{MinValue} = \frac{1}{64}, x = 2\pi \cdot \text{constn}() \right.$$

$$fMax\left(\left(\frac{1}{2}\right)^{\frac{12}{3-\sin(x)}}, x\right)$$

$$\left\{ \text{MaxValue} = \frac{1}{8}, x = 2\pi \cdot \text{constn}(1) \right.$$

graphical solution

Alg Standard Real Rad



ответ: $1/64 \leq x \leq 1/8$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

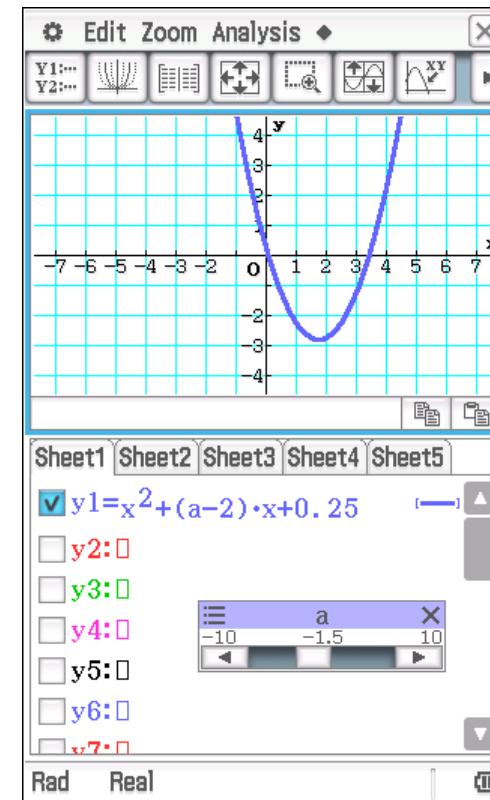
2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

C83. При каких значениях а функция $y=x^2+(a-2)x+\frac{1}{4}$ не принимает отрицательных значений?

Solution with CAS:

```
Define y(x)=x^2+(a-2)x+0.25
done
 $\frac{d}{dx}(y(x))=0$ 
 $2 \cdot x + a - 2 = 0$ 
solve(ans, x)
 $\left\{ x = \frac{-a}{2} + 1 \right\}$ 
 $y(x) | x = \frac{-a}{2} + 1$ 
 $\left( \frac{a}{2} - 1 \right)^2 - (a-2) \cdot \left( \frac{a}{2} - 1 \right) + \frac{1}{4}$ 
simplify(ans)
 $\frac{-(a-1) \cdot (a-3)}{4}$ 
solve(ans≥0, a)
 $\{1 \leq a \leq 3\}$ 
dynamic graphical solution
```

```
dx
 $2 \cdot x + a - 2 = 0$ 
solve(ans, x)
 $\left\{ x = \frac{-a}{2} + 1 \right\}$ 
 $y(x) | x = \frac{-a}{2} + 1$ 
 $\left( \frac{a}{2} - 1 \right)^2 - (a-2) \cdot \left( \frac{a}{2} - 1 \right) + \frac{1}{4}$ 
simplify(ans)
 $\frac{-(a-1) \cdot (a-3)}{4}$ 
solve(ans≥0, a)
 $\{1 \leq a \leq 3\}$ 
dynamic graphical solution
```



ответ: $1 \leq a \leq 3$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

C86. При каком значении а область определения функции

$$y = \sqrt[6]{-x^2 + 4x + a} + \sqrt{x - 3}$$

состоит из одной точки?

Solution with CAS:

$$\frac{d}{dx} (-x^2 + 4x + a) = 0$$

$$-2x + 4 = 0$$

$$\text{solve}(ans, x)$$

$$\{x=2\}$$

$$\frac{d^2}{dx^2} (-x^2 + 4x + a) < 0$$

$$-2 < 0$$

$$\text{Define } y1(x) = \sqrt[6]{-x^2 + 4x + a} + \sqrt{x - 3}$$

$$\text{done}$$

$$\text{solve}(-x^2 + 4x + a = 0 | x=3, a)$$

$$\{a=-3\}$$

ответ: a = -3

$\frac{d}{dx} (-x^2 + 4x + a) = 0$

$$-2x + 4 = 0$$

$$\text{solve}(ans, x)$$

$$\{x=2\}$$

$\frac{d^2}{dx^2} (-x^2 + 4x + a) < 0$

$$-2 < 0$$

Define $y1(x) = \sqrt[6]{-x^2 + 4x + a} + \sqrt{x - 3}$

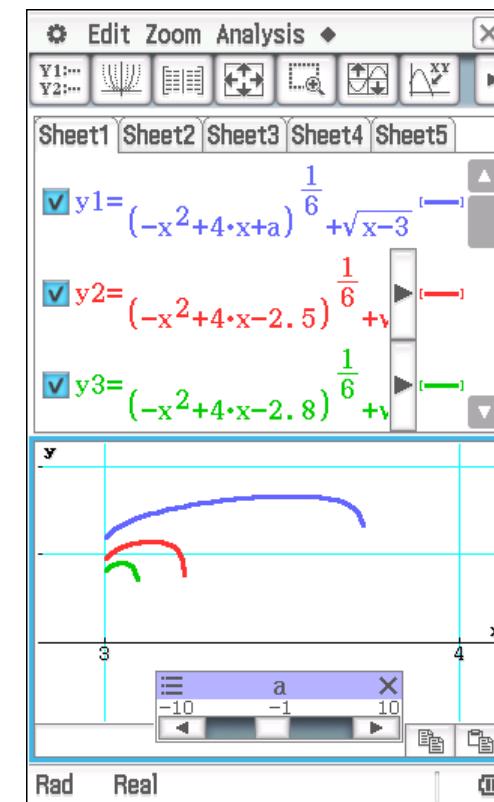
done

$\text{solve}(-x^2 + 4x + a = 0 | x=3, a)$

$$\{a=-3\}$$

ответ: a = -3

dynamic graphical solution



ответ: a = -3

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

3. ТЕКСТОВЫЕ ЗАДАЧИ

File Edit Insert Action

Единый государственный экзамен по МАТЕМАТИКЕ

ЕГЭ 2014. Математика: сборник заданий

В. В. Кочагин, М. Н. Кочагина.

М.: Эксмо, 2013. – 224с.

ISBN 978-5-699-65829-9

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II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

3. ТЕКСТОВЫЕ ЗАДАЧИ

Alg Standard Real Rad

File Edit Insert Action

B32. Сумма двух чисел равна 1100. Найдите наибольшее из них, если 6% одного из них равны 5% другого.

Solution with CAS:

$$\begin{cases} x+y=1100 \\ 0.06x=0.05y \end{cases} \quad |_{x, y}$$

{x=500, y=600}

solve({x+y=1100, 0.06x=0.05y})

{x=500, y=600}

ответ: 600

Alg Standard Real Rad

ответ: 600

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ

File Edit Insert Action

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4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ

Alg Standard Real Deg

B6. Решите уравнение $|2\sin(x)-1|=3$. В ответе укажите наименьший положительный корень (в градусах).

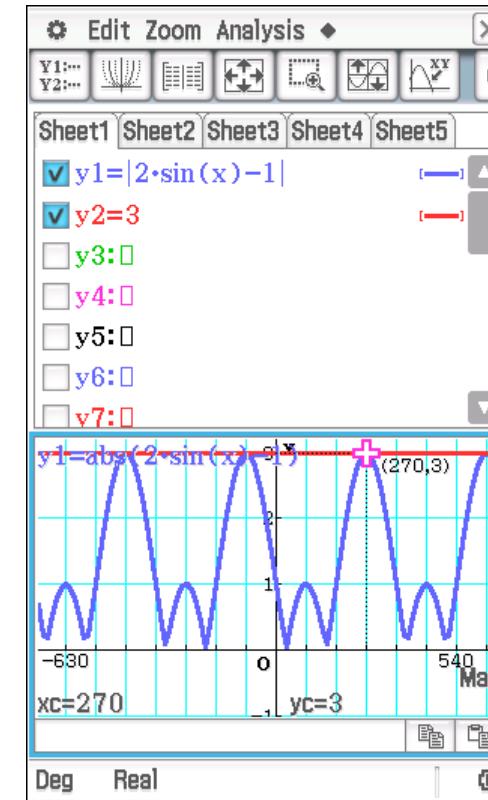
Solution with CAS:

```
solve(|2sin(x)-1|=3, x)
{x=360·constn(1)-90, x=360·
seq(360·k-90, k, -3, 3, 1)
{-1170, -810, -450, -90, 270
seq(360·k+270, k, -3, 3, 1)
{-810, -450, -90, 270, 630, 9}
```

ответ: 270

graphical solution

Alg Standard Real Deg



ответ: 270°

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ

B25. Укажите середину промежутка, являющегося решением неравенства

$$\left(\frac{1}{2}\right)^{|4x+1|} \geq \frac{1}{8}.$$

Solution with CAS:

```
solve((1/2)^|4x+1| >= 1/8, x)
      {-1 <= x <= -1/2}
```

$$\frac{-1+0.5}{2} = -\frac{1}{4}$$

ответ: -0.25

Sheet1

$y_1 = \left(\frac{1}{2}\right)^{|4x+1|}$

$y_2 \geq \frac{1}{8}$

$y_3: \square$

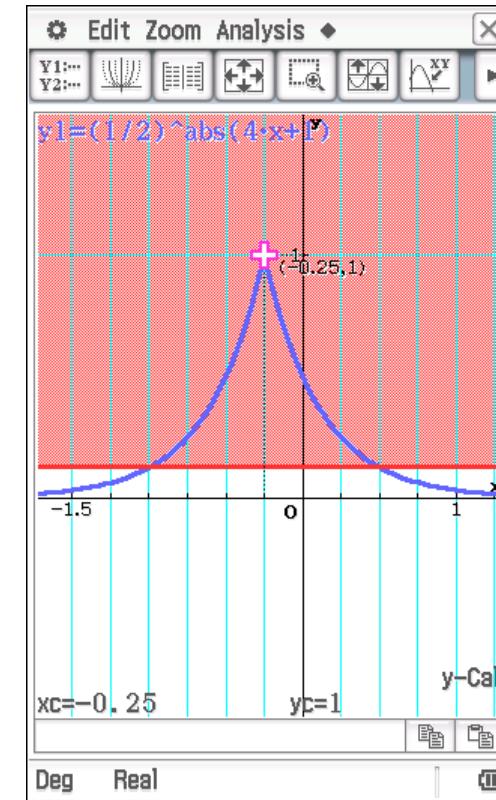
$y_4: \square$

$y_5: \square$

$y_1 = (1/2)^{\text{abs}(4 \cdot x + 1)}$

$x_c = -0.25$

$y_c = 1$



ответ: -0.25

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ

C71. Решите неравенство

$$|x+1| + |x+2| + |x-1| + |x-2| < 8x - 32.$$

Solution with CAS:

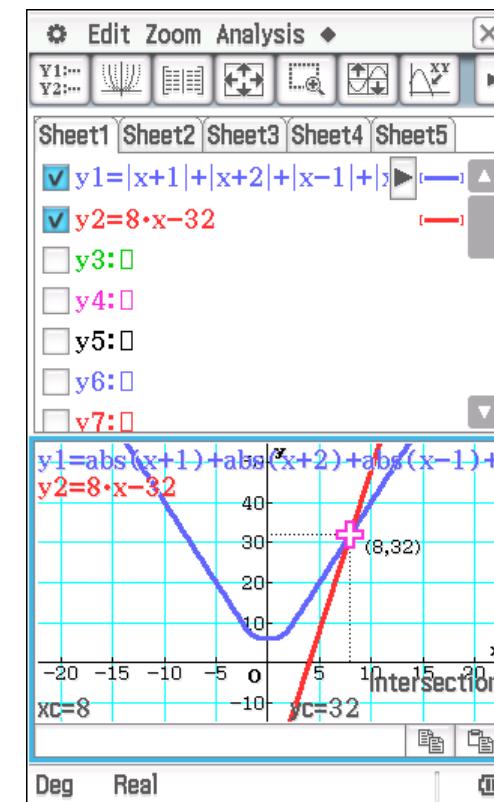
```
solve(|x+1| + |x+2| + |x-1| + |x-2| < 8x)
{x > 8}
```

graphical solution

critical points: $x = -2, x = -1, x = 1, x = 2$

```
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
-4*x < 8*x - 32
solve(ans | x < -2, x)
No Solution
```

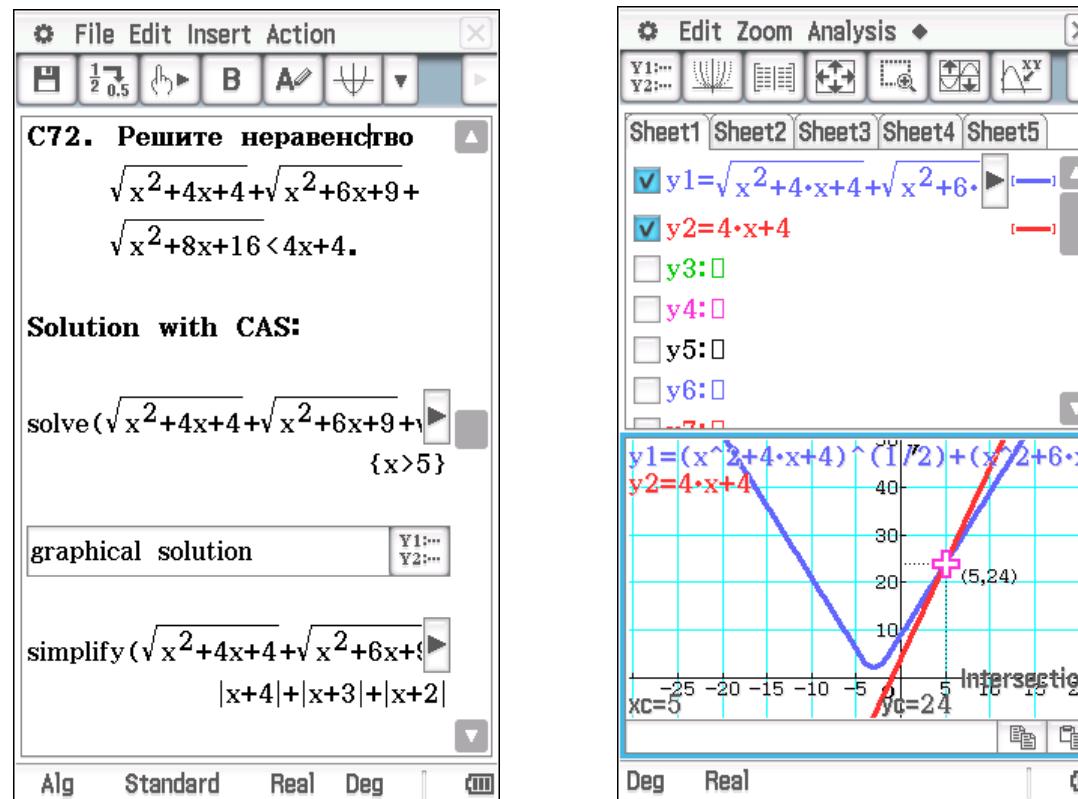
```
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
-2*x + 4 < 8*x - 32
solve(ans | x < -1 and x ≥ -2, x)
No Solution
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
6 < 8*x - 32
solve(ans | x < 1 and x ≥ -1, x)
No Solution
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
2*x + 4 < 8*x - 32
solve(ans | x < 2 and x ≥ 1, x)
No Solution
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
4*x < 8*x - 32
solve(ans | x ≥ 2, x)
{x > 8}
```



ответ: $x > 8$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ



ответ: $x > 5$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ

The figure consists of three side-by-side screenshots of the ClassPad software interface, showing the step-by-step solution of the inequality $|\log_3(x^2-16)| + x - 5 = |\log_3(x^2-16)| + |x-5|$.

Screenshot 1 (Left): Shows the initial inequality and a note about the triangle inequality $|a+b| \leq |a| + |b|$, leading to the conclusion that $x=5$ is the only possible solution.

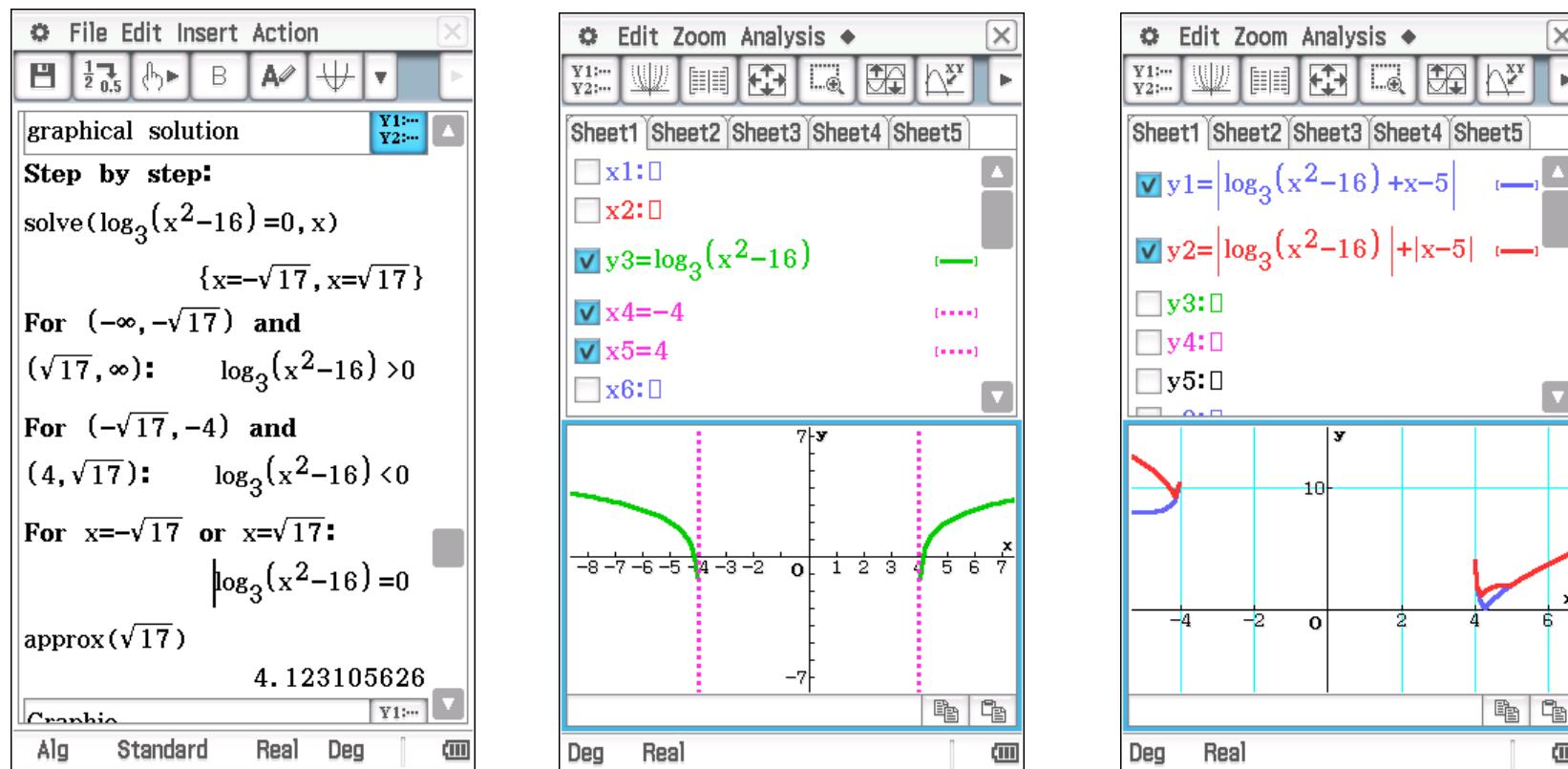
Screenshot 2 (Middle): Shows the CAS command `solve(|log_3(x^2-16)| + x - 5 = |log_3(x^2-16)| + |x-5|)`. A warning message "CAS: WARNING! More solutions may exist" appears. The output shows multiple solutions: $x=6, x=6.006246748, x=6.0$. It also shows the command `numerical solve for (-sqrt(17), sqrt(17))` and the result 4.123105626 .

Screenshot 3 (Right): Shows the Solve screen with the equation $|\log_3(x^2-16)| + x - 5 = |\log_3(x^2-16)| + |x-5|$. The result is listed as $x=4.1231056$ and $Left-Right=0$. The numerical solve result 4.123105626 is highlighted.

ответ: $x \in [-\sqrt{17}, -4] \cup (4, \sqrt{17}] \cup [5, \infty)$

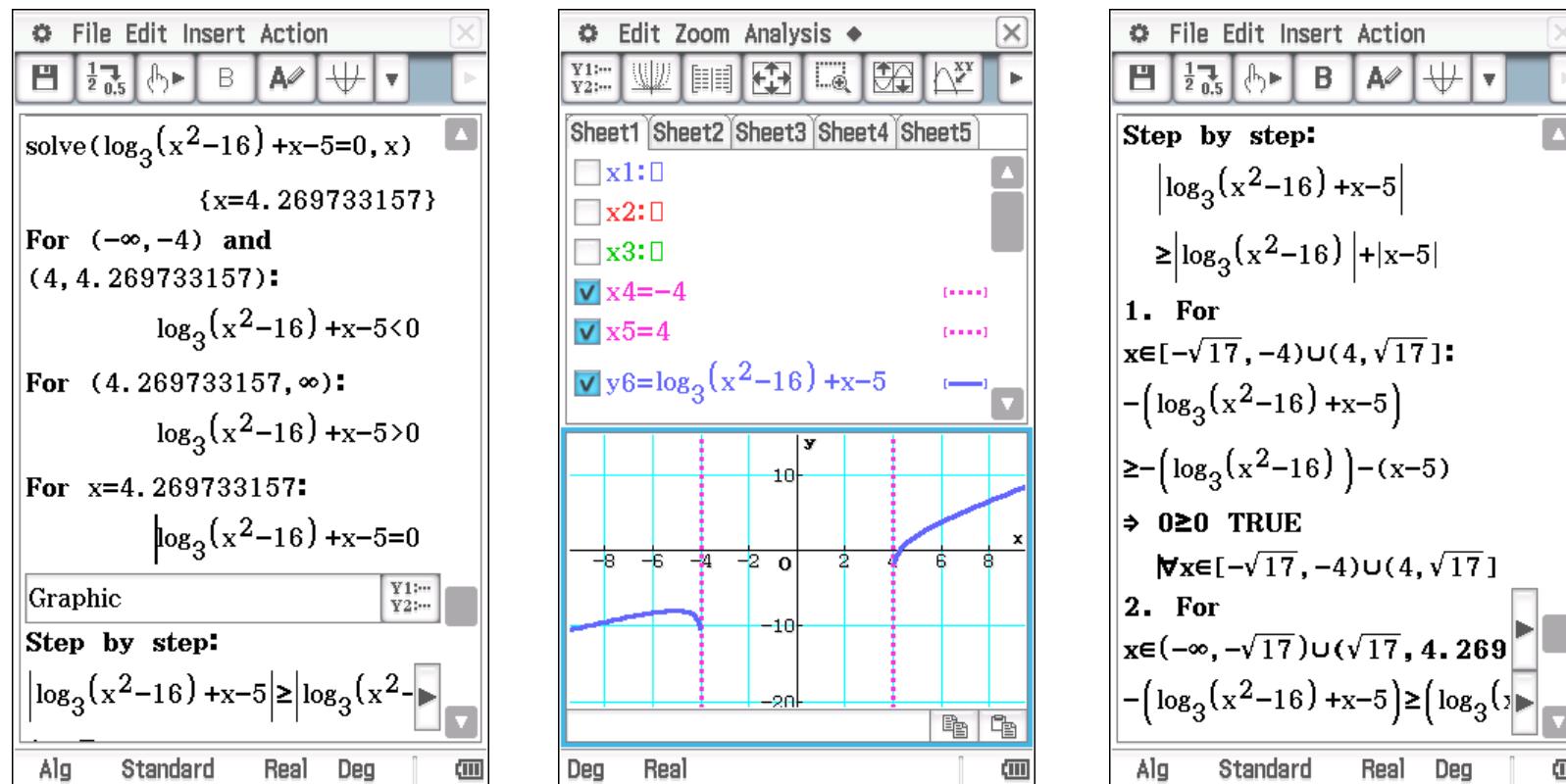
eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ



eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ



ответ: $x \in [-\sqrt{17}, -4] \cup (4, \sqrt{17}] \cup [5, \infty)$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

The screenshot shows a ClassPad calculator interface with a menu bar (File, Edit, Insert, Action) and a toolbar with various icons. The main window displays a step-by-step solution for the inequality $|\log_3(x^2-16)| + x - 5 \geq |\log_3(x^2-16)| + |x-5|$.

The solution is broken down into four cases:

- For $x \in [-\sqrt{17}, -4) \cup (4, \sqrt{17}]$:
 $-(\log_3(x^2-16)) + x - 5 \geq -(\log_3(x^2-16)) - (x-5)$
 $\Rightarrow 0 \geq 0$ TRUE
 $\forall x \in [-\sqrt{17}, -4) \cup (4, \sqrt{17}]$
- For $x \in (-\infty, -\sqrt{17}) \cup (\sqrt{17}, 4.269733157]$:
 $-(\log_3(x^2-16)) + x - 5 \geq (\log_3(x^2-16)) - (x-5)$
 $\Rightarrow -2\log_3(x^2-16) \geq 0$
 $\text{solve}(-2\log_3(x^2-16) \geq 0, x)$
 $\Rightarrow \text{no solution}$
- For $x \in (4.269733157, 5)$:
 $(\log_3(x^2-16)) + x - 5 \geq (\log_3(x^2-16)) - (x-5)$
 $\Rightarrow 2x - 10 \geq 0 \Rightarrow x \geq 5$ no solution
- For $x \in [5, \infty)$:
 $(\log_3(x^2-16)) + x - 5 \geq (\log_3(x^2-16)) + (x-5)$
 $\Rightarrow 0 \geq 0$ TRUE $\forall x \in [5, \infty)$

The final answer is given as **ответ: $x \in [-\sqrt{17}, -4) \cup (4, \sqrt{17}] \cup [5, \infty)$** . The calculator also shows mode settings at the bottom: Alg, Standard, Real, Deg.

ответ: $x \in [-\sqrt{17}, -4) \cup (4, \sqrt{17}] \cup [5, \infty)$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

File Edit Insert Action

1 2 0.5 B A $\frac{d}{dx}$

Единый государственный экзамен по МАТЕМАТИКЕ

ЕГЭ 2014. Математика: сборник заданий

В. В. Кочагин, М. Н. Кочагина. |

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II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

Alg Standard Real Deg

File Edit Insert Action

1 2 0.5 B A $\frac{d}{dx}$

1. В 11 классе 30 человек.
18 человек изучают английский язык,
16 – немецкий,
9 – оба языка.

Сколько человек изучают:
а) только английский язык,
б) только немецкий язык,
в) не изучают ни одного языка?

Solution with CAS:

Alg Standard Real Deg

File Edit Insert Action

1 2 0.5 B A $\frac{d}{dx}$

```
seq(x, x, 1, 30, 1)⇒Ω
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}▶
seq(x, x, 1, 18, 1)⇒A
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}▶
seq(x, x, 10, 25, 1)⇒B
{10, 11, 12, 13, 14, 15, 16, 17}▶
[[dim(Ω), dim(A), dim(B)]]
[30 18 16]
```

Set theory: Intersection A∩B
A∩B

No computation of A∩B –
only in text mode possibel.
Students wrote a programm
for set theory.
(see another vcf-file)

Alg Standard Real Deg

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

The image shows two side-by-side screenshots of the ClassPad software interface.

Left Screenshot:

- Top Bar:** File, Edit, Insert, Action.
- Toolbar:** Includes icons for fraction, decimal, matrix, and other mathematical operations.
- Title:** 4-fields-table
- Table:**

$$\begin{bmatrix} & B & \bar{B} \\ A & A \cap B & A \cap \bar{B} \\ \bar{A} & \bar{A} \cap B & \bar{A} \cap \bar{B} \end{matrix}$$
- Text:** given:

$$\begin{bmatrix} & B & \bar{B} \\ A & 9 & 18 \\ \bar{A} & 16 & 30 \end{bmatrix}$$
- Text:** fill the table:

$$\begin{bmatrix} & B & \bar{B} \\ A & 9 & 9 & 18 \\ \bar{A} & 7 & 16 & 30 \end{bmatrix}$$
- Bottom Bar:** Alg, Standard, Real, Deg, Unit.

Right Screenshot:

- Top Bar:** File, Edit, View, Draw.
- Toolbar:** Includes icons for selection, text, shape, and other drawing tools.
- Table:**

$$\begin{array}{|c|c|c|c|} \hline & A & \bar{A} & \Omega \\ \hline A & 9 & 5 & 18 \\ \hline \bar{A} & 7 & 12 & 16 \\ \hline \Omega & 16 & 30 & 30 \\ \hline \end{array}$$
- Venn-diagram:** A Venn diagram with two overlapping circles labeled A and B. Circle A contains the value 9, circle B contains the value 7, and their intersection contains the value 9. The universal set Ω is represented by a rectangle containing the values 5, 9, and 7.

ответ: а)9 б)7 в)5

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

File Edit Insert Action

31. Три стрелка независимо друг от друга стреляют по мишени.

Вероятность попадания первого стрелка равна 0,7; второго – 0,6; третьего – 0,4.

Какова вероятность:
 а) хотя бы одного попадания,
 б) трех попаданий, если каждый сделал по одному выстрелу.

Alg Standard Real Rad

Solution: $P(A)=0.7$,
 $P(B)=0.6$, $P(C)=0.4$

a)
 $P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C}) =$
 $1 - 0.3 \cdot 0.4 \cdot 0.6 = \frac{116}{125}$

approx(ans) 0.928

b)
 $P(A \cap B \cap C) = P(A)P(B)P(C) =$
 $0.7 \cdot 0.6 \cdot 0.4 = \frac{21}{125}$

approx(ans) 0.168

Alg Standard Real Rad

$P(A \cap B \cap C) = P(A)P(B)P(C) =$
 $0.7 \cdot 0.6 \cdot 0.4 = \frac{21}{125}$

approx(ans) 0.168

Venn-diagram

ответ: а)0.928 б)0.168

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ – Venn-diagram with 4 sets $A, B, C, D \subset \Omega$

The figure consists of three side-by-side screenshots of the ClassPad software interface, showing the process of creating and executing a Venn diagram with four sets.

Left Window: Shows the input code for generating four sets A, B, C, and D from the sequence of numbers 1 to 100. The code includes:

- `Venn-diagram with 4 sets A, B, C, D ⊂ Ω`
- `seq(x, x, 0, 100, 1) ⇒ Ω`
- `{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, }`
- `seq(x, x, 0, 100, 2) ⇒ A`
- `{0, 2, 4, 6, 8, 10, 12, 14, 16, 1 }`
- `seq(x, x, 0, 100, 3) ⇒ B`
- `{0, 3, 6, 9, 12, 15, 18, 21, 24, }`
- `seq(x, x, 0, 100, 5) ⇒ C`
- `{0, 5, 10, 15, 20, 25, 30, 35, 4 }`
- `seq(x, x, 0, 100, 7) ⇒ D`
- `{0, 7, 14, 21, 28, 35, 42, 49, 5 }`

Middle Window: Shows the execution of the code to generate sets A, B, C, and D, followed by the generation of a Venn diagram with 4 regions. The output shows:

- `seq(x, x, 0, 100, 7) ⇒ D`
- `{49, 56, 63, 70, 77, 84, 91, 98}`
- `expToStr Ω, Ω`
- `done`
- `expToStr A, A`
- `done`
- `expToStr B, B`
- `done`
- `expToStr C, C`
- `done`
- `expToStr D, D`
- `done`
- `StrOVenn(Ω, A, B, C, D, 4, 2, 1)`
- `done`
- `stop`

Right Window: Shows the resulting Venn diagram with four overlapping circles labeled A, B, C, and D. The regions are numbered as follows:

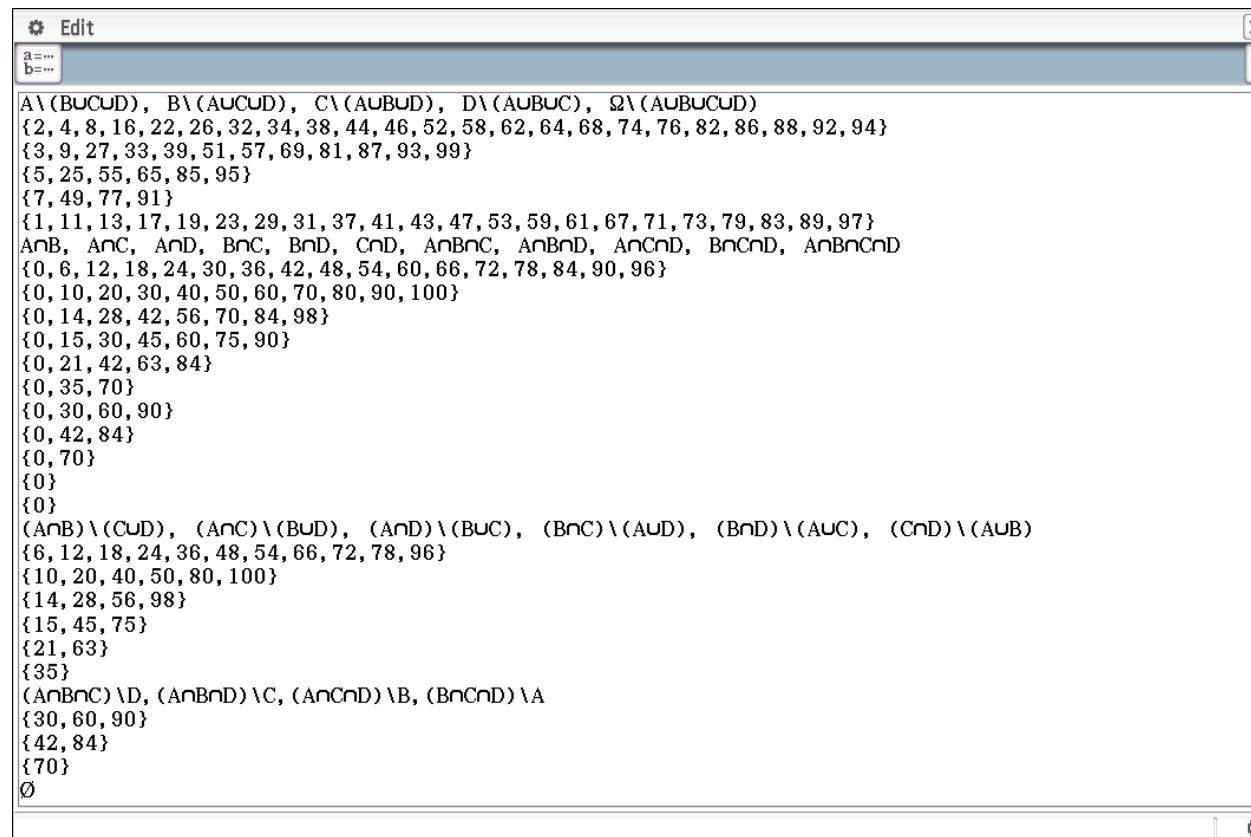
Region	Value
Region A only	23
Region B only	12
Region C only	6
Region D only	4
Region A ∩ B	11
Region A ∩ C	3
Region A ∩ D	2
Region B ∩ C	3
Region B ∩ D	1
Region C ∩ D	1
Region A ∩ B ∩ C	2
Region A ∩ B ∩ D	0
Region A ∩ C ∩ D	1
Region B ∩ C ∩ D	1
Region A ∩ B ∩ C ∩ D	4

The window also displays the following set operations and their results:

- `Ω:101`
- `A \ (B ∪ C ∪ D)`
- `B \ (A ∪ C ∪ D)`
- `C \ (A ∪ B ∪ D)`
- `D \ (A ∪ B ∪ C)`
- `{2, 4, 8, 16, 22, 26, 32, 34, 38, 4 }`
- `{3, 9, 27, 33, 39, 51, 57, 69, 81, }`
- `{5, 25, 55, 65, 85, 95}`
- `{7, 49, 77, 91}`
- `{1, 11, 13, 17, 19, 23, 29, 31, 37}`
- `A ∩ B, A ∩ C, A ∩ D, B ∩ C, B ∩ D, C ∩ D`
- `{0, 6, 12, 18, 24, 30, 36, 42, 48, }`
- `{0, 10, 20, 30, 40, 50, 60, 70, 80 }`

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ – Venn-diagram with 4 sets $A, B, C, D \subset \Omega$



```

    a=...
    b=...
    A\B\{C\D}, B\{A\B\{C\D\}}, C\{A\B\{C\D\}}, D\{A\B\{C\D\}}, \Omega\{A\B\{C\D\}}
    {2, 4, 8, 16, 22, 26, 32, 34, 38, 44, 46, 52, 58, 62, 64, 68, 74, 76, 82, 86, 88, 92, 94}
    {3, 9, 27, 33, 39, 51, 57, 69, 81, 87, 93, 99}
    {5, 25, 55, 65, 85, 95}
    {7, 49, 77, 91}
    {1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}
    A\B, A\B\{C\}, A\B\{C\}\D, A\B\{C\}\{D\}, A\B\{C\}\{D\}\Omega, A\B\{C\}\{D\}\{C\}, A\B\{C\}\{D\}\{C\}\D, A\B\{C\}\{D\}\{C\}\{D\}, A\B\{C\}\{D\}\{C\}\{D\}\Omega, A\B\{C\}\{D\}\{C\}\{D\}\{D\}, A\B\{C\}\{D\}\{C\}\{D\}\{D\}\Omega
    {0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96}
    {0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100}
    {0, 14, 28, 42, 56, 70, 84, 98}
    {0, 15, 30, 45, 60, 75, 90}
    {0, 21, 42, 63, 84}
    {0, 35, 70}
    {0, 30, 60, 90}
    {0, 42, 84}
    {0, 70}
    {0}
    {0}
    (A\B)\{C\}D, (A\B)\{C\}\{D\}, (A\B)\{C\}\{D\}\Omega, (A\B)\{C\}\{D\}\{C\}, (A\B)\{C\}\{D\}\{C\}\D, (A\B)\{C\}\{D\}\{C\}\{D\}, (A\B)\{C\}\{D\}\{C\}\{D\}\Omega, (A\B)\{C\}\{D\}\{C\}\{D\}\{D\}, (A\B)\{C\}\{D\}\{C\}\{D\}\{D\}\Omega
    {6, 12, 18, 24, 36, 48, 54, 66, 72, 78, 96}
    {10, 20, 40, 50, 80, 100}
    {14, 28, 56, 98}
    {15, 45, 75}
    {21, 63}
    {35}
    (A\B\{C\})\D, (A\B\{C\})\{C\}, (A\B\{C\})\{C\}\D, (A\B\{C\})\{C\}\{D\}, (A\B\{C\})\{C\}\{D\}\Omega, (A\B\{C\})\{C\}\{D\}\{C\}, (A\B\{C\})\{C\}\{D\}\{C\}\D, (A\B\{C\})\{C\}\{D\}\{C\}\{D\}, (A\B\{C\})\{C\}\{D\}\{C\}\{D\}\Omega, (A\B\{C\})\{C\}\{D\}\{C\}\{D\}\{D\}, (A\B\{C\})\{C\}\{D\}\{C\}\{D\}\{D\}\Omega
    {30, 60, 90}
    {42, 84}
    {70}
    \emptyset
  
```

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ – Compute $\Omega \setminus (A \cup B \cup C \cup D)$ with 4 sets $A, B, C, D \subset \Omega$

```

File Edit Insert Action
[File] [Edit] [Insert] [Action]
[Stop] [1/2] [0.5] [Back] [Forward] [B] [A] [Delete] [Down]
StrOVenn(Ω, A, B, C, D, 4, 2, 1)
done

stop
Compute Ω \ (A ∪ B ∪ C ∪ D):
Menge(A, "∪", B)
done

Menge(Result, "∪", C)
done

Menge(Result, "∪", D)
done

Menge(Ω, "-", Result)
done

Result
" {1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97} "
Ω \ (A ∪ B ∪ C ∪ D) contains 1 and all prime numbers (except 2, 3, 5, 7)

Alg Standard Real Deg

```

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

File Edit Insert Action

35. Рассмотрите ряд чисел:
40, 90, 50, 20, 20, 50.

Найдите:

- размах,
- среднее арифметическое,
- медиану,
- моду.

Solution with CAS:

```
{40, 90, 50, 20, 20, 50}⇒list
{40, 90, 50, 20, 20, 50}
dim(list)
6
```

Alg Standard Real Rad

File Edit Insert Action

размах (range)|
max(list)-min(list) 70

среднее арифметическое (mean)
mean(list) 45

sortA(list)
{20, 20, 40, 50, 50, 90}

медиан (median)
median(list) 45

мода (mode)
mode(list) {50, 20}

OneVariable list

Alg Standard Real Rad

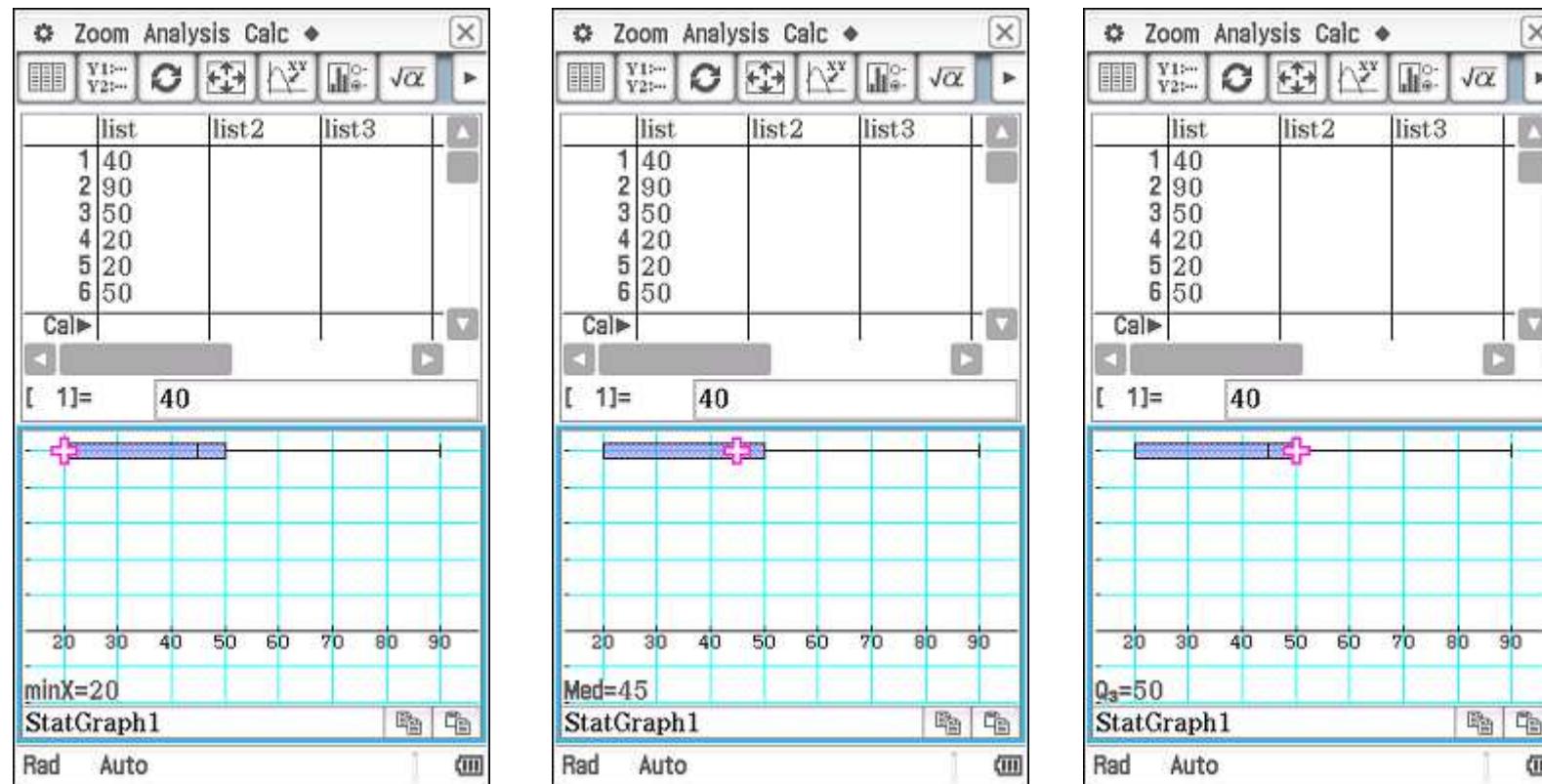
File Edit Insert Action

OneVariable|
x = 45
 $\Sigma x = 270$
 $\Sigma x^2 = 15500$
 $\sigma_x = 23.629078$
 $s_x = 25.884358$
n = 6
minX = 20
 $Q_1 = 20$
Med = 45
 $Q_3 = 50$
maxX = 90
Mode = 20
Mode = 50
ModeN = 2
ModeF = 2

Alg Standard Real Rad

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ (Boxplot)



eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

File Edit Insert Action

40. В таблице записаны результаты ежедневного измерения температуры воздуха (в градусах Цельсия) на метеостанции в полдень в первую декаде февраля.

Число месяца	1	2
Температура, °C	-8	-6

Найдите среднюю температуру в полдень в эту декаду.

Alg Standard Real Rad

File Edit Insert Action

40. В таблице записаны результаты ежедневного измерения температуры воздуха (в градусах Цельсия) на метеостанции в полдень в первую декаде февраля.

	3	4	5	6	7	8	9	10
—	-2	0	-2	-2	-1	1	0	0

Найдите среднюю температуру в полдень в эту декаду.

Alg Standard Real Rad

File Edit Insert Action

Solution with CAS:

```
trn([-8 -6 -2 0 -2 -2 -1])  

[-8  

-6  

-2  

0  

-2  

-1  

1  

0  

0]
```

```
matToList(ans, 1)⇒list  

{-8, -6, -2, 0, -2, -2, -1, 1, 0}  

mean(ans)  

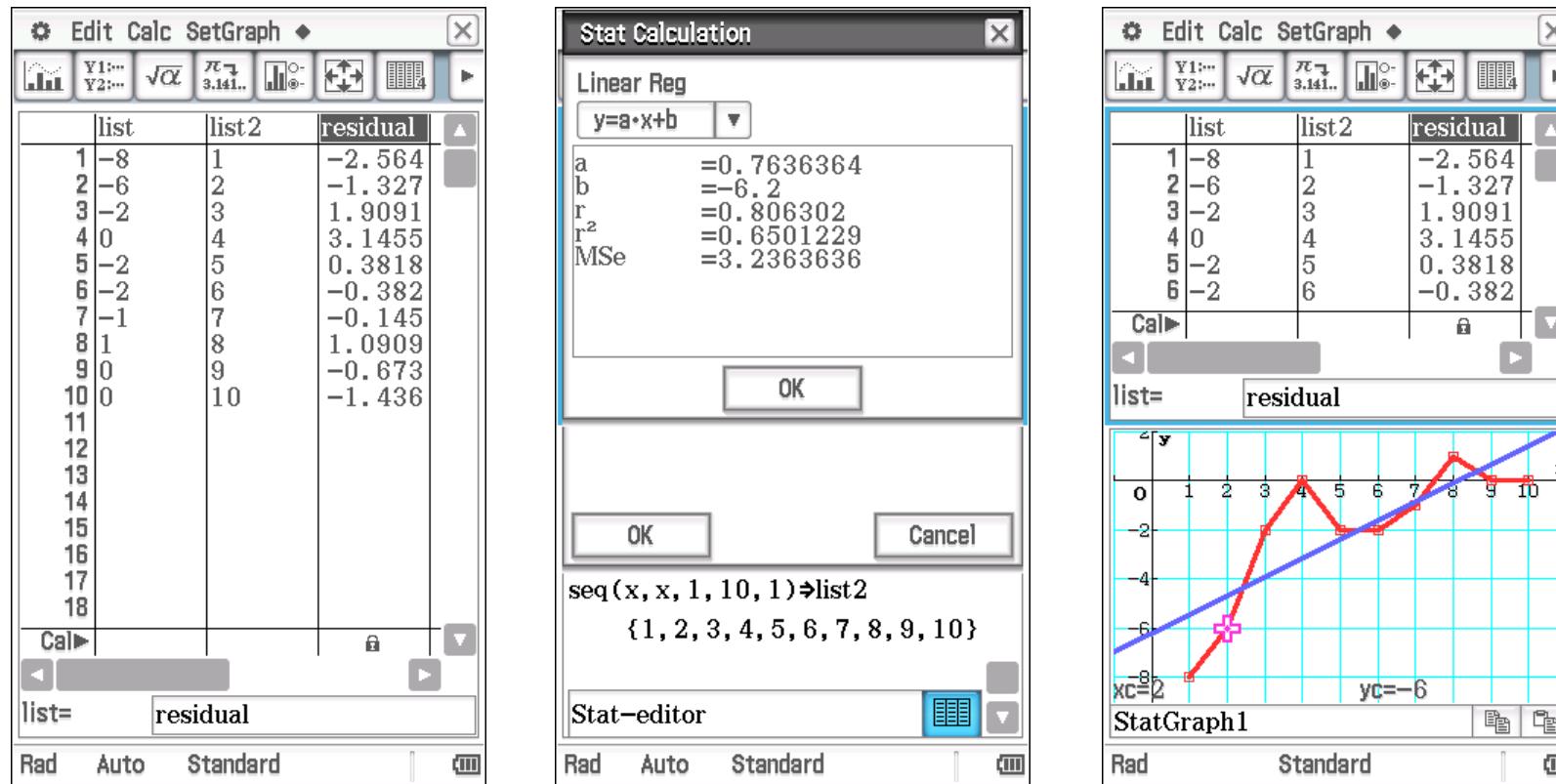
-2
```

Alg Standard Real Rad

ответ: -2°

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ



eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

Method of least squares:
Define $y(x)=a*x+b$
done
Define $F(a, b) = \sum_{k=1}^{10} ((y(k)-list[k])^2)$
done
 $\frac{d}{da}(F(a, b)) = 0 \Rightarrow Equ1$
 $770 \cdot a + 110 \cdot b + 94 = 0$
 $\frac{d}{db}(F(a, b)) = 0 \Rightarrow Equ2$
 $110 \cdot a + 20 \cdot b + 40 = 0$
 $\begin{cases} Equ1 \\ Equ2 \end{cases} \Big|_{a, b}$
 $\left\{ a = \frac{42}{55}, b = -\frac{31}{5} \right\}$

Method of least squares:
Define $y(x)=a*x+b$
done
 $\begin{pmatrix} a, b \end{pmatrix} = \sum_{k=1}^{10} ((y(k)-list[k])^2)$
done
 $\frac{d}{da}(F(a, b)) = 0 \Rightarrow Equ1$
 $770 \cdot a + 110 \cdot b + 94 = 0$
 $\frac{d}{db}(F(a, b)) = 0 \Rightarrow Equ2$
 $110 \cdot a + 20 \cdot b + 40 = 0$
 $\begin{cases} Equ1 \\ Equ2 \end{cases} \Big|_{a, b}$
 $\left\{ a = \frac{42}{55}, b = -\frac{31}{5} \right\}$

Define $F(a, b) = \sum_{k=1}^{10} ((y(k)-list[k])^2)$
done
 $\frac{d}{da}(F(a, b)) = 0 \Rightarrow Equ1$
 $770 \cdot a + 110 \cdot b + 94 = 0$
 $\frac{d}{db}(F(a, b)) = 0 \Rightarrow Equ2$
 $110 \cdot a + 20 \cdot b + 40 = 0$
 $\begin{cases} Equ1 \\ Equ2 \end{cases} \Big|_{a, b}$
 $\left\{ a = \frac{42}{55}, b = -\frac{31}{5} \right\}$
approx(ans)
 $\{a=0.7636363636, b=-6.2\}$

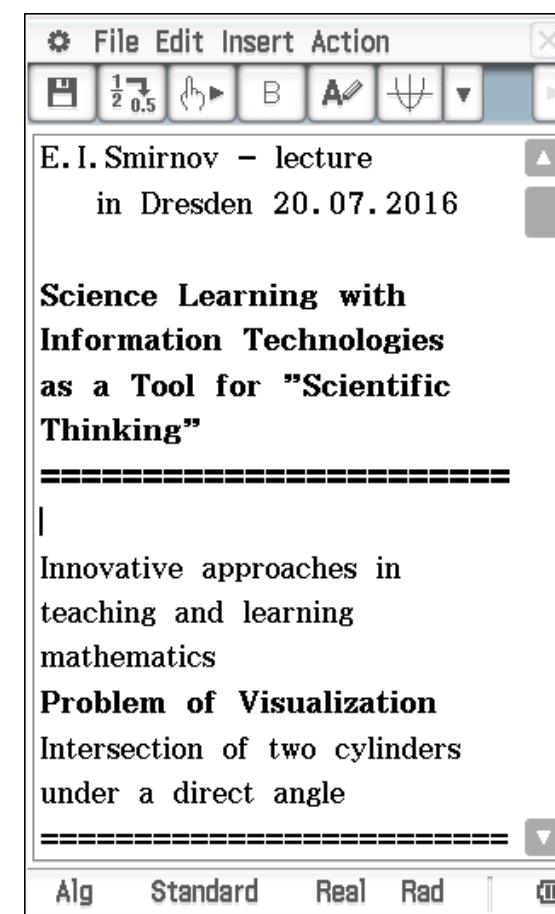
eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

**Innovative approaches in teaching
and learning mathematics**

Problem of Visualization

**Intersection of two cylinders
under a direct angle**



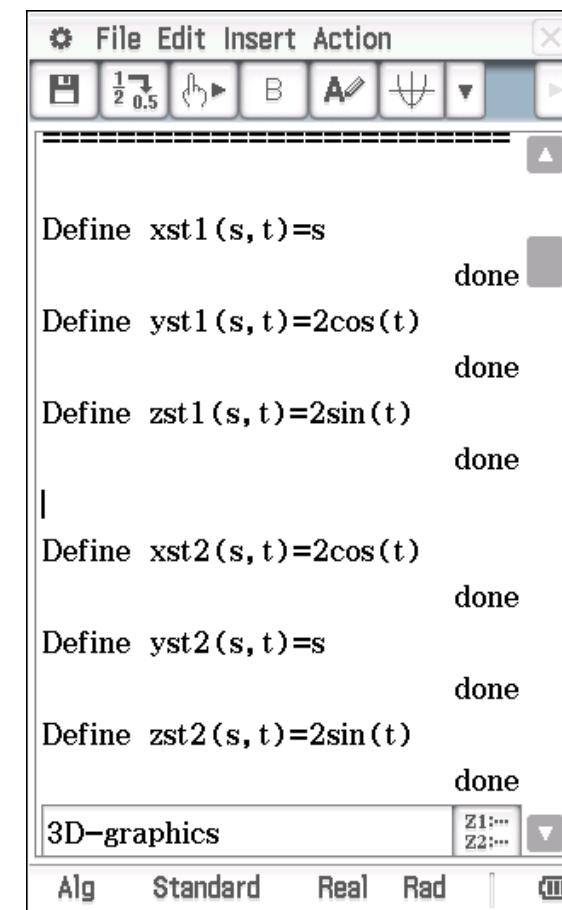
eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

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and learning mathematics**

Problem of Visualization

**Intersection of two cylinders
under a direct angle**



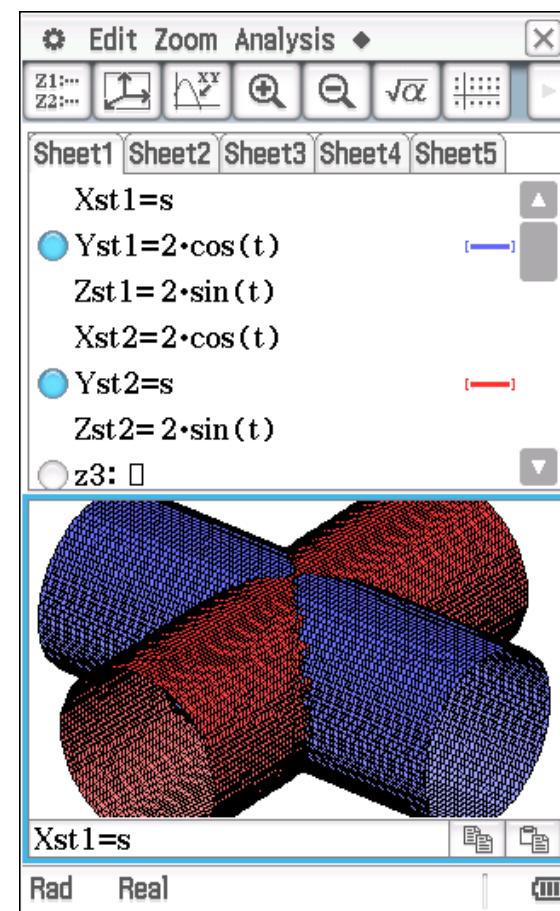
eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

**Innovative approaches in teaching
and learning mathematics**

Problem of Visualization

**Intersection of two cylinders
under a direct angle**



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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

The image displays three consecutive screenshots of the ClassPad software interface, showing a sequence of mathematical steps:

- Screenshot 1:** Shows the input of a rotation matrix (z-axis) and its multiplication by a vector. The result is a vector with components involving trigonometric functions of r and t .
- Screenshot 2:** Shows the simplification of the resulting vector components using the given value $r = \frac{\pi}{6}$. The simplified vector components are shown as fractions involving s and $\cos(t)$.
- Screenshot 3:** Shows the definition of four variables: $xst3(s, t) = \frac{-s + \sqrt{3} \cdot \cos(t)}{2}$, $yst3(s, t) = \frac{\sqrt{3} \cdot s + \cos(t)}{2}$, $zst3(s, t) = 2 \cdot \sin(t)$, and $xst4(s, t) = -s$. The final message "3D-graphics with rotation" is visible at the bottom.

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The image displays three windows of the ClassPad software:

- Edit Sheet** window (left): Shows a list of equations defined on Sheet1:
 - Xst1=s
 - Yst1=2·cos(t)
 - Zst1=2·sin(t)
 - Xst2=2·s
 - Yst2=0
 - Zst2=0.01·t
 - Xst3= $\frac{-s}{2} + \sqrt{3} \cdot \cos(t)$
 - Yst3= $\frac{\sqrt{3} \cdot s}{2} + \cos(t)$
 - Zst3=2·sin(t)
 - Xst4=-s
 - Yst4= $\sqrt{3} \cdot s$
 - Zst4=0.01·t
 - z5: □
- View Window** window (center): A dialog box titled "Memory" showing view parameters:

xmin	: -6
max	: 6
grid	: 90
ymin	: -6
max	: 6
grid	: 90
zmin	: -6
max	: 6
- Edit Zoom Analysis** window (right): Displays a 3D plot of a hyperbolic paraboloid surface, colored green and blue.

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

File Edit Insert Action

Recognition problem

To find the volume of intersection of the two cylinders
 $r=0$ (no rotation)

Define $xst1(s, t) = s$
done

Define $yst1(s, t) = 2\cos(t)$
done

Define $zst1(s, t) = 2\sin(t)$
done

Define $xst2(s, t) = 2\cos(t)$
done

Define $yst2(s, t) = s$
done

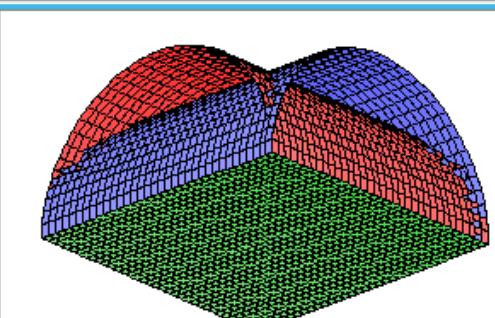
Define $zst2(s, t) = 2\sin(t)$
done

Alg Standard Real Rad

Edit Zoom Analysis

Sheet1 Sheet2 Sheet3 Sheet4 Sheet5

Xst1=s
Yst1=2·cos(t)
Zst1=2·sin(t)
Xst2=2·cos(t)
Yst2=s
Zst2=2·sin(t)
z3=0



Xst1=s

Rad Real

File Edit Insert Action

Define $z3(x, y) = 0$
done

3D-graphics without rota
done

radius R=2:

$V := 8 * \int_0^2 \int_{-x}^x \int_0^{\sqrt{4-x^2}} 1 dz dy dx$

$\frac{128}{3}$

$V := 8 * \int_0^R \int_{-x}^x \int_0^{\sqrt{R^2-x^2}} 1 dz dy dx$

$\frac{16 \cdot R^3}{3}$

Alg Standard Real Rad

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

The image displays three windows of the ClassPad software:

- Left Window (Calculator View):**

Rotation matrix (z-axis):

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ with angle } \gamma$$

$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2\cos(t) \\ s \\ 2\sin(t) \end{bmatrix}$

$$\begin{bmatrix} -s\cdot\sin(\gamma)+2\cdot\cos(t)\cdot\cos(\gamma) \\ s\cdot\cos(\gamma)+2\cdot\cos(t)\cdot\sin(\gamma) \\ 2\cdot\sin(t) \end{bmatrix}$$

ans | $r = -\frac{\pi}{6}$

$$\begin{bmatrix} \frac{s}{2}+\sqrt{3}\cdot\cos(t) \\ \frac{\sqrt{3}\cdot s}{2}-\cos(t) \end{bmatrix}$$
- Middle Window (Edit Sheet):**

Sheet1 Sheet2 Sheet3 Sheet4 Sheet5

 - Xst1=s
 - Yst1=2·cos(t)
 - Zst1=2·sin(t)
 - Xst2=2·s
 - Yst2=0
 - Zst2=0.01·t
 - Xst3= $\frac{s}{2}+\sqrt{3}\cdot\cos(t)$
 - Yst3= $\frac{\sqrt{3}\cdot s}{2}-\cos(t)$
 - Zst3=2·sin(t)
 - Xst4=s
 - Yst4= $\sqrt{3}\cdot s$
 - Zst4=0.01·t
 - z5: □
- Right Window (View Window):**

Memory

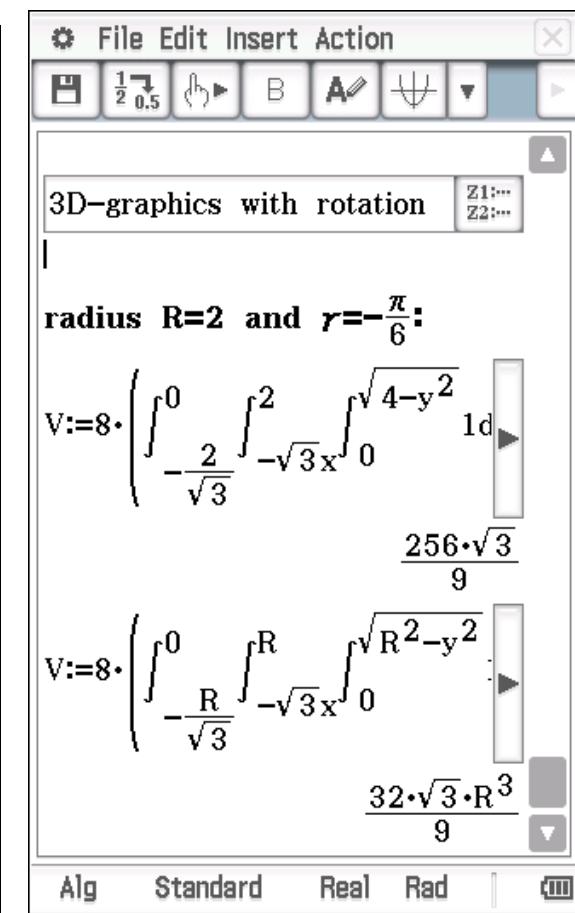
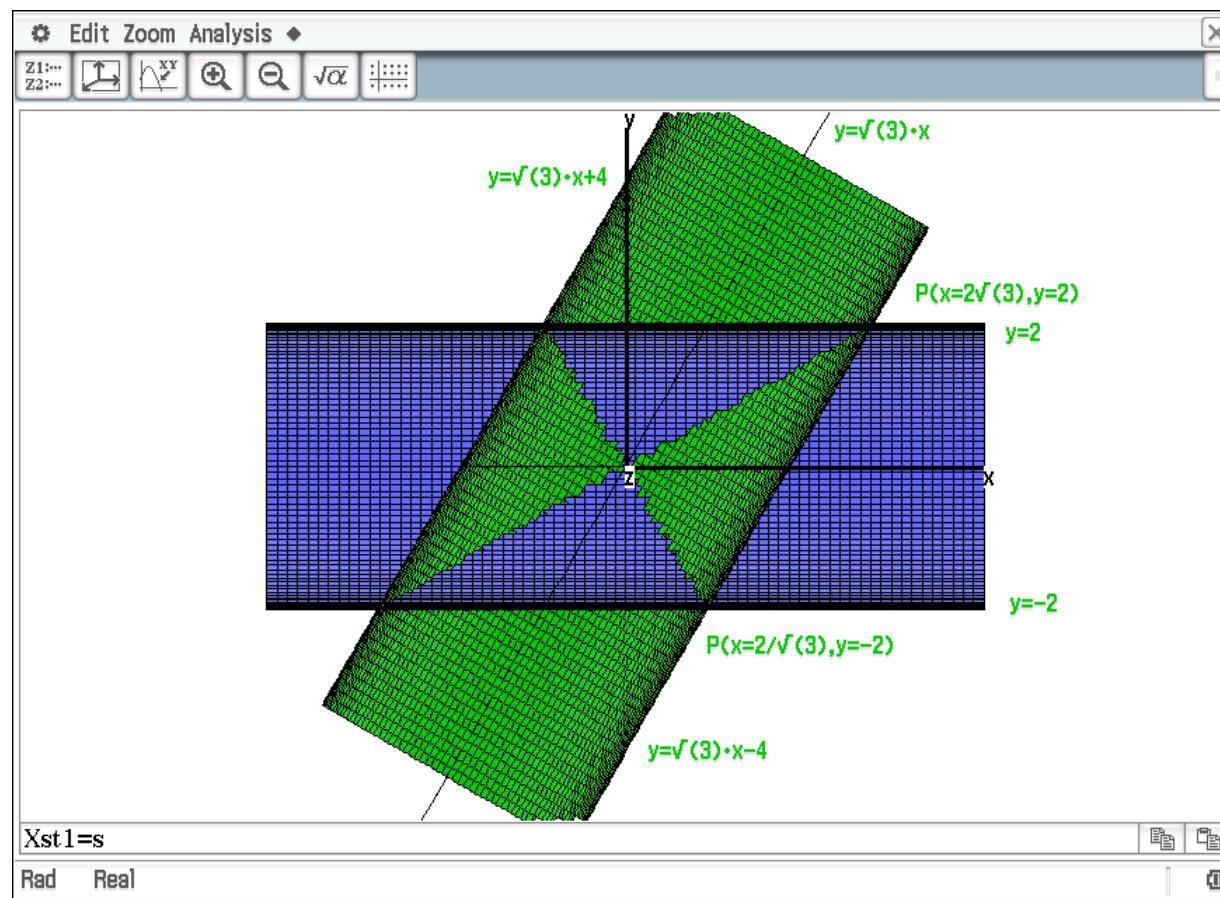
 - zmin : -5
 - max : 5
 - angle θ : -90
 - angle ϕ : 0
 - smin : -5
 - max : 5
 - tmin : 0
 - max : π

OK Cancel Default

3D Plot View: A 3D surface plot showing a green and blue shaded surface.

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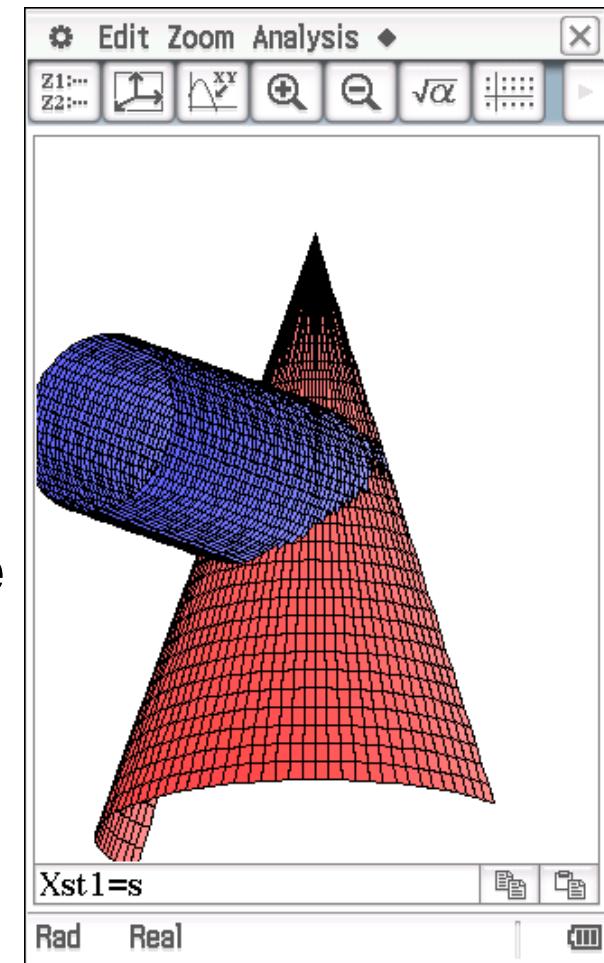
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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

Innovative approaches in teaching
and learning mathematics

Problem of Visualization

Intersection of a cylinder and a cone



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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

E. I. Smirnov – lecture
in Dresden 20.07.2016

Science Learning with
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Problem of Visualization
Intersection of a cylinder and a
cone

Alg Standard Real Rad

solve($\sqrt{1-x^2} = -c*x+3, x$)

$$\left\{ x = \frac{3 \cdot c - \sqrt{c^2 - 8}}{c^2 + 1}, x = \frac{3 \cdot c + \sqrt{c^2 - 8}}{c^2 + 1} \right.$$

solve($c^2 - 8, c$)

$$\{ c = -2\sqrt{2}, c = 2\sqrt{2} \}$$

Define xst1(s, t) = s
done

Define yst1(s, t) = cos(t)
done

Define zst1(s, t) = sin(t)
done

Define xst2(s, t) = s * cos(t/2)
done

Define vst2(s, t) = s * sin(t/2)

Alg Standard Real Rad

Define zst2(s, t) = $3 - 2\sqrt{2}|s|$
done

3D-graphics Z1:... Z2:...

R=1 (cylinder)

$V := 2 \cdot \int_{-1}^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_0^{\sqrt{\left(\frac{z-3}{2\sqrt{2}}\right)^2 - y^2}} 1 dx dz dy$

$$5.762207211$$

$\int_0^{\sqrt{\left(\frac{z-3}{2\sqrt{2}}\right)^2 - y^2}} 1 dx$

$\sqrt{-2 \cdot (8 \cdot y^2 - z^2 + 6 \cdot z - 9)}$

Alg Standard Real Rad

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

The left window shows the ClassPad interface with the following equations:

- $Yst1 = \cos(t)$
- $Zst1 = \sin(t)$
- $Xst2 = s \cdot \cos\left(\frac{t}{2}\right)$
- $Yst2 = s \cdot \sin\left(\frac{t}{2}\right)$
- $Zst2 = 3 - 2 \cdot \sqrt{2} \cdot |s|$

At the bottom, it shows $Xst1 = s$ and has Rad and Real buttons.

The right window shows the derivation of the volume of a cylinder:

$$R=1 \text{ (cylinder)}$$

$$V := 2 \cdot \int_{-1}^1 \left(\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \left(\int_0^{\sqrt{\left(\frac{z-3}{2\sqrt{2}}\right)^2 - y^2}} 1 dx \right) dy \right) dz$$

The result is displayed as 5.762207211 .

Below, the integral is shown again:

$$\int_0^{\sqrt{\left(\frac{z-3}{2\sqrt{2}}\right)^2 - y^2}} 1 dx$$

The final result is given as:

$$\frac{\sqrt{-2 \cdot (8 \cdot y^2 - z^2 + 6 \cdot z - 9)}}{4}$$

Further down, the integral is shown again:

$$\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \frac{\sqrt{-2 \cdot (8 \cdot y^2 - z^2 + 6 \cdot z - 9)}}{4} dy$$

The result is given as:

$$\frac{2 \cdot z^2 \cdot \sin^{-1}\left(\frac{2 \cdot \sqrt{2} \cdot \sqrt{-z^2 + 1}}{\sqrt{z^2 - 6 \cdot z + 9}}\right)}{8} - \frac{3 \cdot z \cdot \sin^{-1}\left(\frac{2 \cdot \sqrt{2} \cdot \sqrt{-z^2 + 1}}{\sqrt{z^2 - 6 \cdot z + 9}}\right)}{4} + \frac{|3 \cdot z - 1| \cdot \sqrt{-2 \cdot (z^2 - 1)}}{4} + \frac{9 \cdot \sin^{-1}\left(\frac{2 \cdot \sqrt{2} \cdot \sqrt{-z^2 + 1}}{\sqrt{z^2 - 6 \cdot z + 9}}\right)}{8}$$

At the bottom, it shows $2 \cdot \int_{-1}^1 ansdz$ and the result 5.762207211 .

At the very bottom, it shows Alg, Standard, Real, Rad buttons.

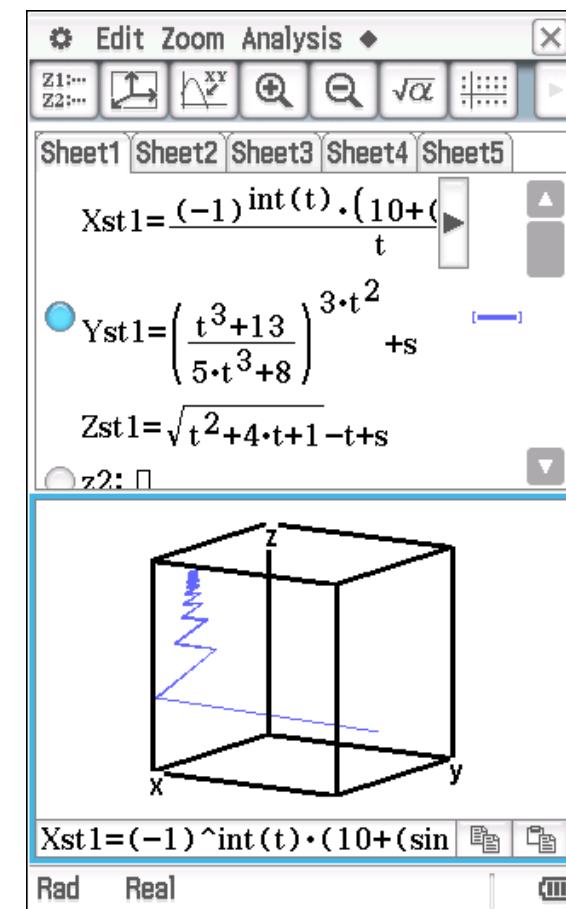
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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

**Innovative approaches in teaching
and learning mathematics**

Problem of subject's Essence

To find the limit of a sequence M_n



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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

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teaching and learning
mathematics

Problem of subject's
Essence

Alg Standard Real Rad

To find the limit of a sequence M_n

$\lim (M_n) = M^*$,

$M_n = \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix}, M^* = \begin{bmatrix} x^* \\ y^* \\ z^* \end{bmatrix}$

Define $x(n) = (-1)^n \cdot \frac{10 + \sin(n)}{n}$

$x(n)$

$$\frac{(-1)^n \cdot ((\sin(n))^2 + 10)}{n}$$

Alg Standard Real Rad

$\lim (x(n))$
 $n \rightarrow \infty$

Undefined

$\lim (|x(n)|)$
 $n \rightarrow \infty$

0

=====

Define $y(n) = \left(\frac{n^3 + 13}{5n^3 + 8} \right)^{3n^2}$

done

$\lim (y(n))$
 $n \rightarrow \infty$

0

=====

Define $z(n) = \sqrt{n^2 + 4n + 1} - n$

Alg Standard Real Rad

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

The figure consists of three side-by-side screenshots of the ClassPad software interface, which is a handheld graphing calculator.

Screenshot 1:

- Top bar: File, Edit, Insert, Action.
- Toolbar: Save, $\frac{1}{2} 0.5$, $\leftarrow \rightarrow$, B , A , $\frac{\partial}{\partial}$.
- Text area:

$$\lim_{n \rightarrow \infty} (z(n)) = 2$$

$$x^* = 0, y^* = 0, z^* = 2.$$

Compute the norm of

$$\begin{bmatrix} x(n) \\ y(n) \\ z(n) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}:$$

$$\text{norm}\left(\begin{bmatrix} x(n) \\ y(n) \\ z(n) \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}\right)$$

$$\sqrt{(\sin(n))^4 + n^2 \cdot \left(\left(\frac{n^3 + 13}{5 \cdot n^3 + 8}\right)^2\right)}$$
- Bottom bar: Alg, Standard, Real, Rad.

Screenshot 2:

- Top bar: File, Edit, Insert, Action.
- Toolbar: Save, $\frac{1}{2} 0.5$, $\leftarrow \rightarrow$, B , $f \ddot{x}$, $\frac{\partial}{\partial}$.
- Text area:

$$\text{Define } M(n) = \begin{bmatrix} x(n) \\ y(n) \\ z(n) \end{bmatrix}$$

done

$$M(n) = \begin{bmatrix} (-1)^n \cdot ((\sin(n))^2 + 10) \\ \left(\frac{n^3 + 13}{5 \cdot n^3 + 8}\right)^{3 \cdot n^2} \\ -n + \sqrt{n^2 + 4 \cdot n + 1} \end{bmatrix}$$

$$M^* := \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$$
- Bottom bar: Alg, Standard, Real, Rad.

Screenshot 3:

- Top bar: File, Edit, Insert, Action.
- Toolbar: Save, $\frac{1}{2} 0.5$, $\leftarrow \rightarrow$, B , $f \ddot{x}$, $\frac{\partial}{\partial}$.
- Text area:

$$\sqrt{(\sin(n))^4 + n^2 \cdot \left(\left(\frac{n^3 + 13}{5 \cdot n^3 + 8}\right)^2\right)}$$

$$\text{approx}(\text{seq}(M(n), n, 1, 3, 1))$$

$$\left[\begin{array}{c} 0.5 \\ -3.339971619 \\ -37e-5 \\ 75 \end{array} \right], \left[\begin{array}{c} 1.152250099e-15 \\ 1.69041576 \end{array} \right] \}$$

$$\text{approx}(M(10))$$

$$\left[\begin{array}{c} 1.029595897 \\ 6.074877816e-209 \\ 1.874342087 \end{array} \right]$$

$$\text{approx}(M(100))$$

$$\left[\begin{array}{c} 0.1025640616 \\ 0 \\ 1.985293057 \end{array} \right]$$
- Bottom bar: Alg, Standard, Real, Rad.

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

The image displays three side-by-side screenshots of the ClassPad software interface, illustrating its use for scientific calculations and 3D visualization.

Screenshot 1 (Left): Shows a sequence of approximations for matrices M(n) for n=1, 3, 10, and 100. The results are presented as lists of lists. For example, approx(M(10)) is shown as:

$$\begin{bmatrix} 1.029595897 \\ 6.074877816 \times 10^{-209} \\ 1.874342087 \end{bmatrix}$$

Screenshot 2 (Middle): Shows approximations for large matrices M(10^8) and M(10^8) again. It also contains definitions for 3D plot components:

```

Define xst1(s,t)=(-1)^int(t).►
done
Define yst1(s,t)=((t^3+13)/(5t^3+8))^3t.►
done
Define zst1(s,t)=sqrt(t^2+4t+1)-t+s
done

```

Screenshot 3 (Right): Displays a 3D plot of a helix-like curve within a unit cube. The axes are labeled x, y, and z. Below the plot, the following parameters are listed:

- zc = 1.4503897
- xc = -10.70717
- sc = 9e-4
- tc = 1
- Xst1 = (-1)^int(t) * (10 + (sin

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

File Edit Insert Action

3D-graphic

=====

Computing the limits step by step:

x(n):

$$\left| (-1)^n \cdot \frac{10 + (\sin(n))^2}{n} \right| = \frac{10 + (\sin(n))^2}{n} < \frac{10+1}{n} = \frac{11}{n} \rightarrow 0, \quad n \rightarrow \infty$$

Alg Standard Real Rad

File Edit Insert Action

y(n):

$$\left(\frac{n^3 + 13}{5n^3 + 8} \right)^{3n^2} \rightarrow \left(\frac{1}{5} \right)^\infty = 0, \quad n \rightarrow \infty$$

approx(seq($\frac{n^3 + 13}{5n^3 + 8}$, n, 1, 10, 1))

{1.076923077, 0.4375, 0.27}

$$\frac{n^3 + 13}{5n^3 + 8} < 1, \text{ if } n > 1.$$

$$\frac{n^3 + 13}{5n^3 + 8} = \frac{1}{5} \cdot \frac{n^3 + 13}{n^3 + 8/5}$$

$$= \frac{1}{5} \cdot \left(\frac{n^3 + 1.6}{n^3 + 1.6} + \frac{11.4}{n^3 + 1.6} \right)$$

$$= \frac{1}{5} \cdot \left(1 + \frac{11.4}{n^3 + 1.6} \right)$$

Alg Standard Real Rad

File Edit Insert Action

$$= \frac{1}{5} \cdot \left(\frac{n^3 + 1.6}{n^3 + 1.6} + \frac{11.4}{n^3 + 1.6} \right)$$

$$= \frac{1}{5} \cdot \left(1 + \frac{11.4}{n^3 + 1.6} \right)$$

z(n):

$$\sqrt{n^2 + 4n + 1} - n = \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n}$$

$$= \frac{4 + 1/n}{\frac{1}{n} \sqrt{n^2 + 4n + 1} + n/n}$$

$$= \frac{4 + 1/n}{\sqrt{1 + 4/n + 1/n^2} + 1} \rightarrow \frac{4}{1+1} = 2,$$

for $n \rightarrow \infty$

Alg Standard Real Rad

Opportunities and methods of using ClassPad



Большое спасибо за ваше внимание!