



Something about myself: **Paul Ernst Ludwig Paditz**

Education:

Diploma mathematics, TU Dresden, Germany, 1974.

Doctor Natural Science (Dr.rer.nat.), TU, Dresden, Germany, 1977.

Postgrad., State University, St. Petersburg, Russia, 1984-1985.

Habilitation (Dr.rer.nat.habil.), HfV Dresden, Germany, 1989.

Working:

Assistant, department mathematics, TU Dresden, 1977-1979.

Head assistant, HfV Dresden, Institute of Mathematics, 1979-1991,
applied professor, HfV Dresden, 1992.

Professor, Faculty informatics/mathematics, Univ. of Applied Sciences,
Dresden, since 1992.

Levenberg-Marquardt Algorithm using ClassPad



Using CAS in Math Education – first experience in Germany with the newest ClassPad technology CP400

Preface:

Ministry of Education in Saxony/Germany introduced 2004:
Modern Math Education with CAS, DGS and TC
beginning in the 8th class upto 12th class,
using graphic calculators (GTR)

CAS – Computer Algebra Systems

DGS – Dynamic Geometry Software

TC – Spreadsheet (Table Calculation)

8th class:

- Knowledge of the use of CAS when forming more complex terms and equations
- Investigating the influence of parameters in the function equation to trace the graph with DGS, TC, GTR or CAS
- Finding equations for measurement series with the help of linear regression with GTR, CAS or TC
- Solving linear systems of equations with more complex coefficients with GTR or CAS (two equations with two unknown variables)

9th class:

Functions and Powers

Mastered of determining zero quadratic functions, graphical solving quadratic equations and solving with GTR or CAS

10th class:

Obtaining the inverse function with CAS, graphical interpretation

Use of CAS to demonstrate the properties of functions
Obtaining illustrative of the limit concept

Know of parametric representation and polar coordinates to describe curves with GTR and CAS

11/12th class:

Differential calculus

The use of CAS in particular, should promote discovery learning, and support for substantive tasks, the reflection on the facts and the interpretation of the result.

Integral Calculus

The use of CAS in particular, should promote discovery learning, and support for substantive tasks, the reflection on the facts and the interpretation of the result.

worksheets:

- ☞ The use of worksheets in mathematics instruction has a long tradition.
- ☞ The use of a worksheet should guide the students to a structured work.
- ☞ Instead of an oral instruction, which requires a synchronous work of all students, the worksheet individually and with their own timing can be processed next.
- ☞ The sequence of the work orders in the worksheet helps to recognize the logical structure of a problem; the work procedures help to penetrate the question.

worksheets:

The disadvantage of a sheet of paper with work orders is seen, that the tools which can be used must be made available about. The students do not know always, how to carry out its solution steps in detail. Moreover, in a classic worksheet are missing the self check of the results, a feedback of the partial steps and also the visualization of the results.

The new developed **eActivity in the ClassPad** represents an extremely rich extension of the worksheet. The eActivity combines the written representation of the setting of tasks of a worksheet with the tool level of the ClassPad.

These tools are the individual menus or modules, which the ClassPad offers: Computer algebra system, dynamic geometry software, computer statistics, curve plotter, and much more.

eActivity:

Thus, all tasks can be worked on with the possibilities of the computer.

At the same time, the documentation of the work can be entered directly.

The found results can be visualized immediately or the results may be in a hidden file, can be viewed.

It is an interactive work of the students, between setting of tasks and the results and control of the results themselves.

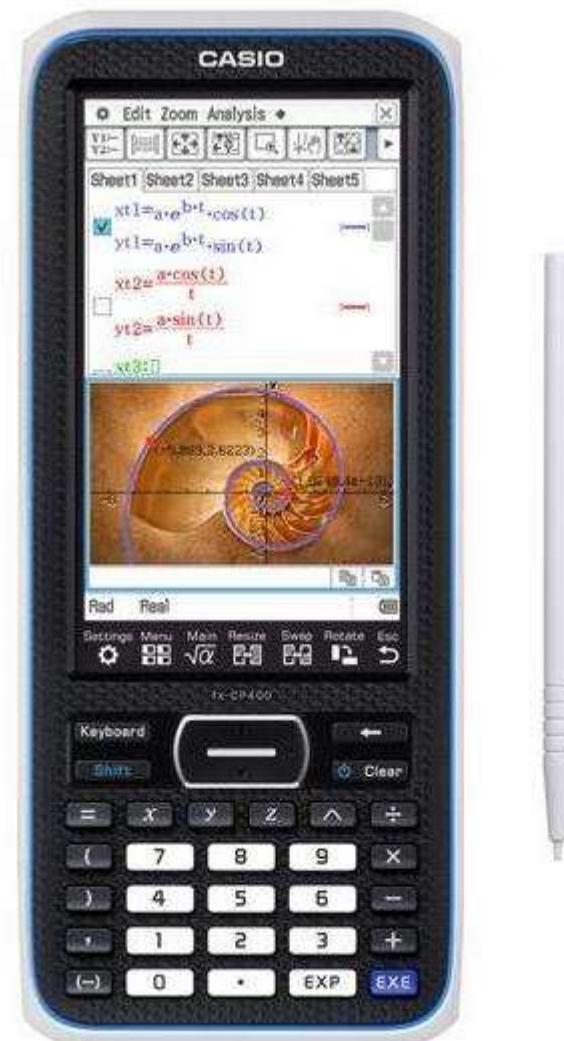
The tool – ClassPad 400:

Графический калькулятор
с сенсорным дисплеем

Fx-CP400 обладает рядом функций,
помогающих школьникам лучше
усваивать материал:

USB поддержка для быстрой и
легкой передачи данных и сов-
местимость с **проектором CASIO**
для отображения информации на
доске.

<http://edu.casio.ru/fx-cp400/>



The tool – Beamer XJ-A146:

<http://casio-projectors.ru/products/xja146/>



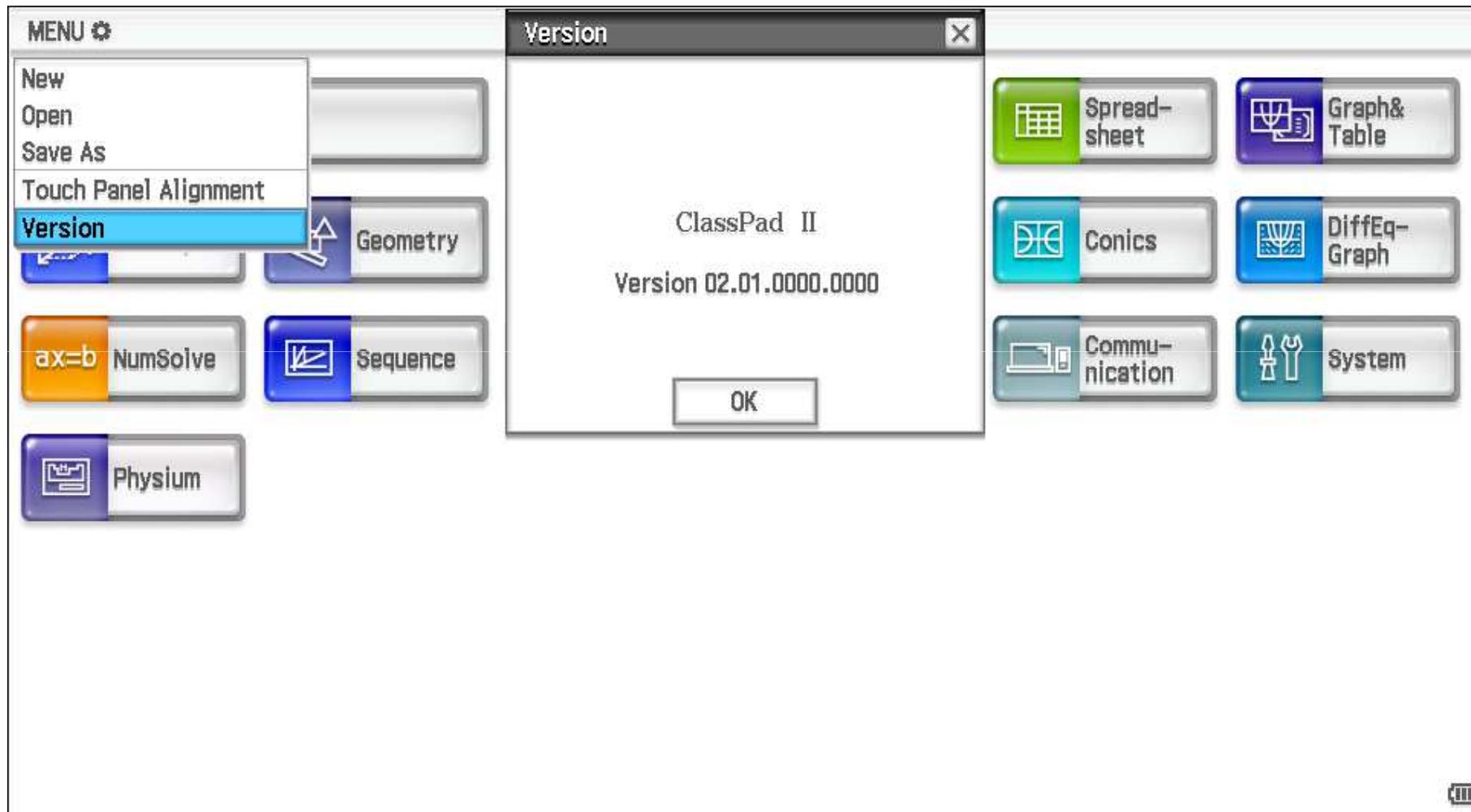
The **ClassPad-software by CASIO** was introduced in 2003 for a new generation of handheld calculators and simultaneously for the Windows PC:
emulator version of the handheld calculator.

Now we have the ClassPad 400 (ClassPad II) with the **new software version 02.01.2000** (13.06.2017), the emulator version is named:

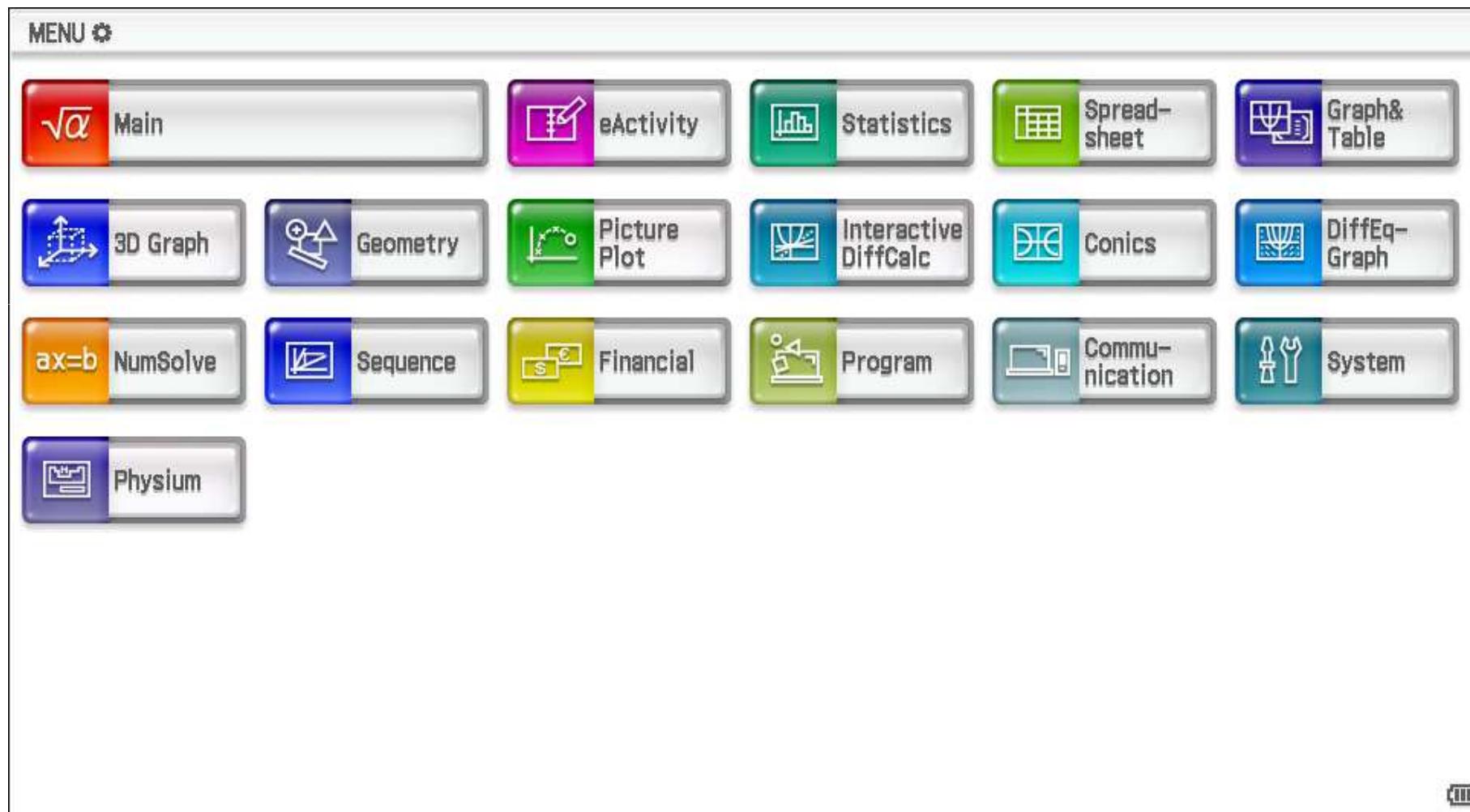
ClassPad Manager Subscription for ClassPad II Series v2.01, cp. <https://edu.casio.com/>

Have a look in the emulator version:

Levenberg-Marquardt Algorithm using ClassPad



The desktop in the PC emulator version: full screen



An eActivity in the PC emulator version: full screen

The screenshot shows a ClassPad emulator interface with the following content:

File Edit Insert Action

Prof. Dr. Ludwig Paditz 14.05.2017

internationale Klimafinanzierung (Deutschland 2005–2015)

=====

deutsche Ausgaben für den internationalen Klimaschutz in Millionen-EUR/Jahr

{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}+2000⇒listx
{2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012, 2013, 2014, 2015}
{471, 515, 648, 881, 1063, 1431, 1563, 1664, 1950, 2344, 2684}⇒listy
{471, 515, 648, 881, 1063, 1431, 1563, 1664, 1950, 2344, 2684}

=====

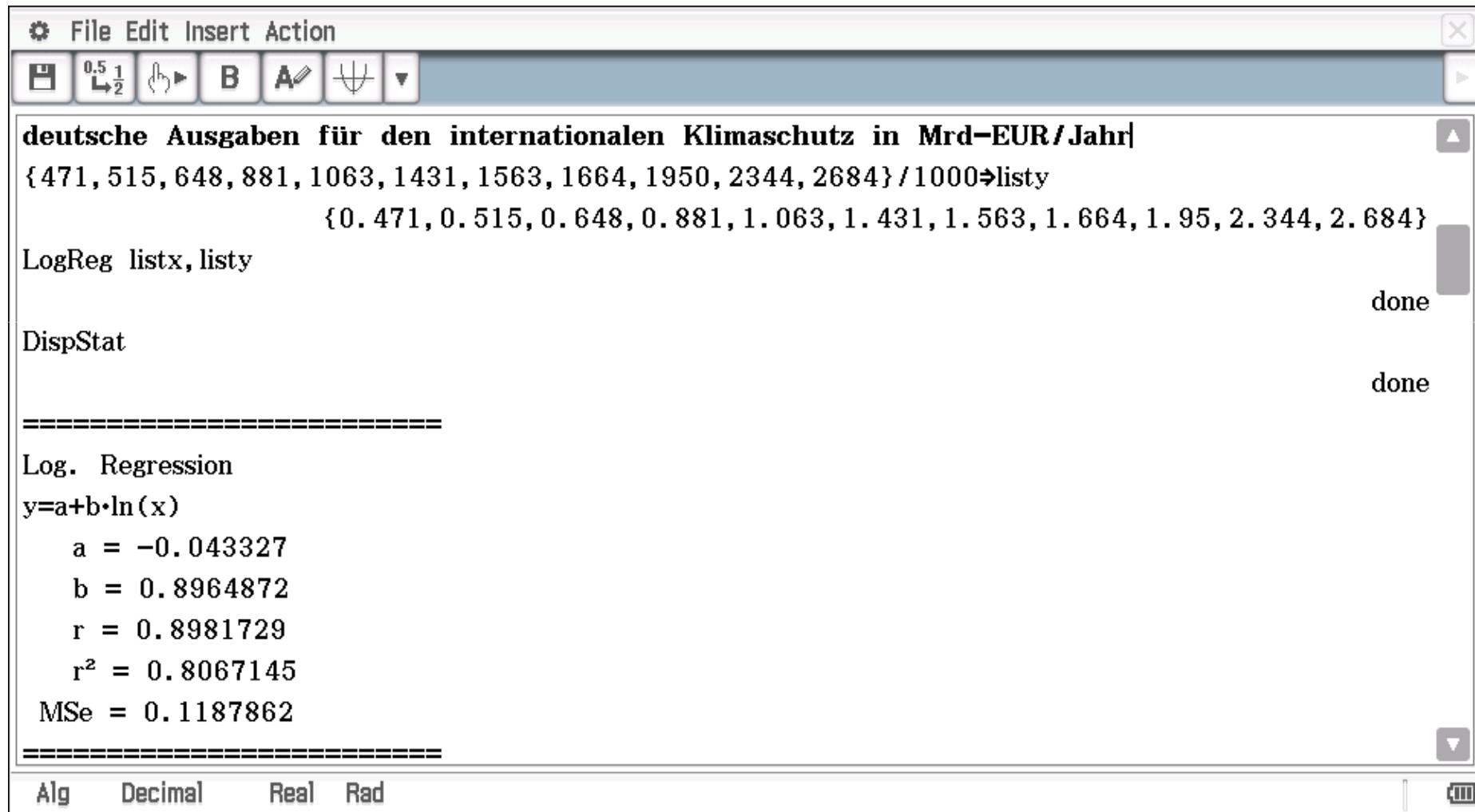
Jahreszählung (Datentransformation)

seq(x, x, 1, 11, 1)⇒listx
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

deutsche Ausgaben für den internationalen Klimaschutz in Mrd-EUR/Jahr

Alg Decimal Real Rad

The Logarithmic Regression (quasilinear): full screen



The screenshot shows a handheld calculator interface with a blue header bar containing icons for File, Edit, Insert, and Action. Below the header is a toolbar with icons for file operations, zoom (0.5, 1, 2), and text styles (B, A, A/̄). The main display area contains the following text:

deutsche Ausgaben für den internationalen Klimaschutz in Mrd-EUR/Jahr
{471, 515, 648, 881, 1063, 1431, 1563, 1664, 1950, 2344, 2684}/1000⇒listy
{0.471, 0.515, 0.648, 0.881, 1.063, 1.431, 1.563, 1.664, 1.95, 2.344, 2.684}
LogReg listx, listy
done
DispStat
done
=====

Log. Regression

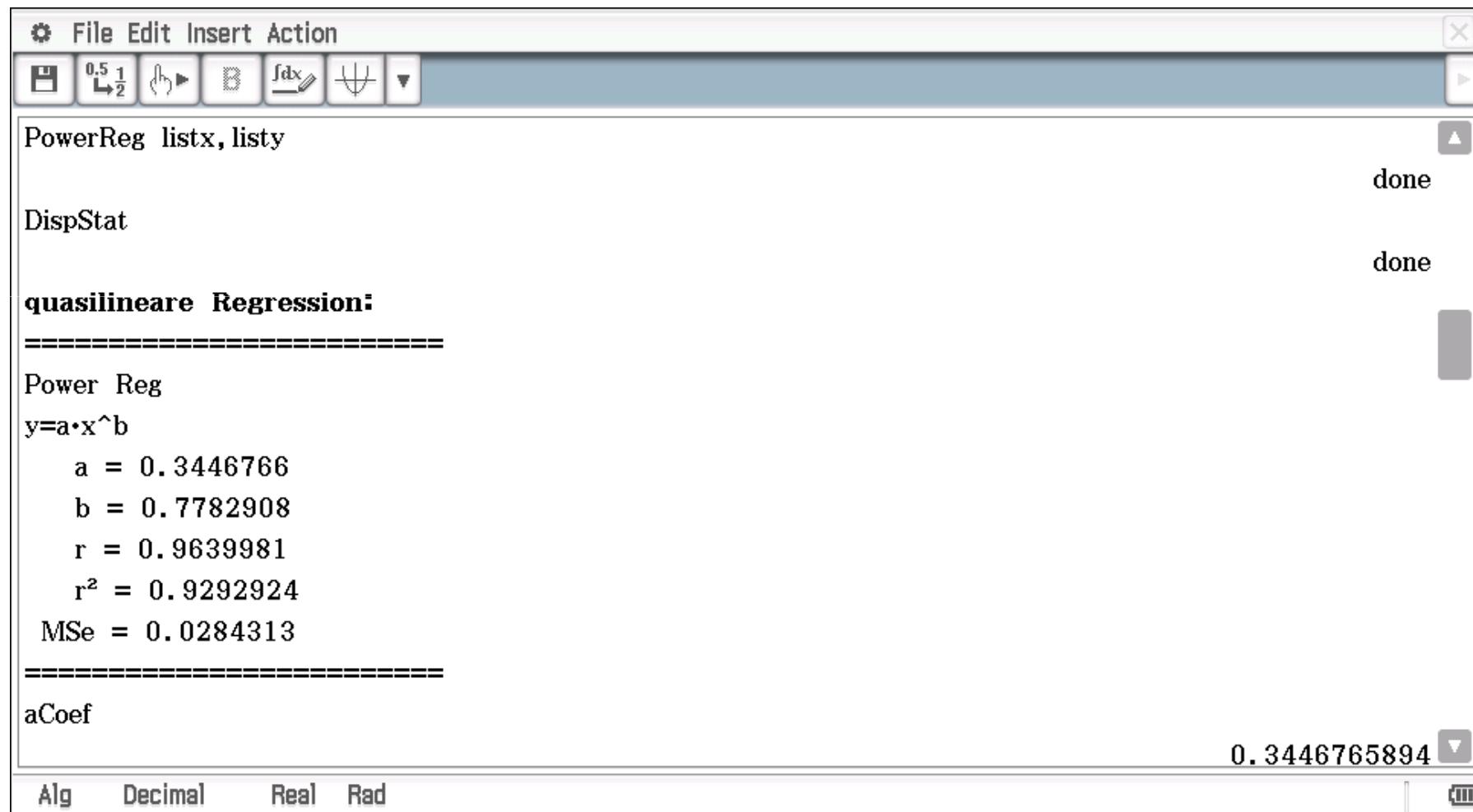
y=a+b·ln(x)

a = -0.043327
b = 0.8964872
r = 0.8981729
r² = 0.8067145
MSe = 0.1187862

=====

Alg Decimal Real Rad

The Power Regression (quasilinear): full screen



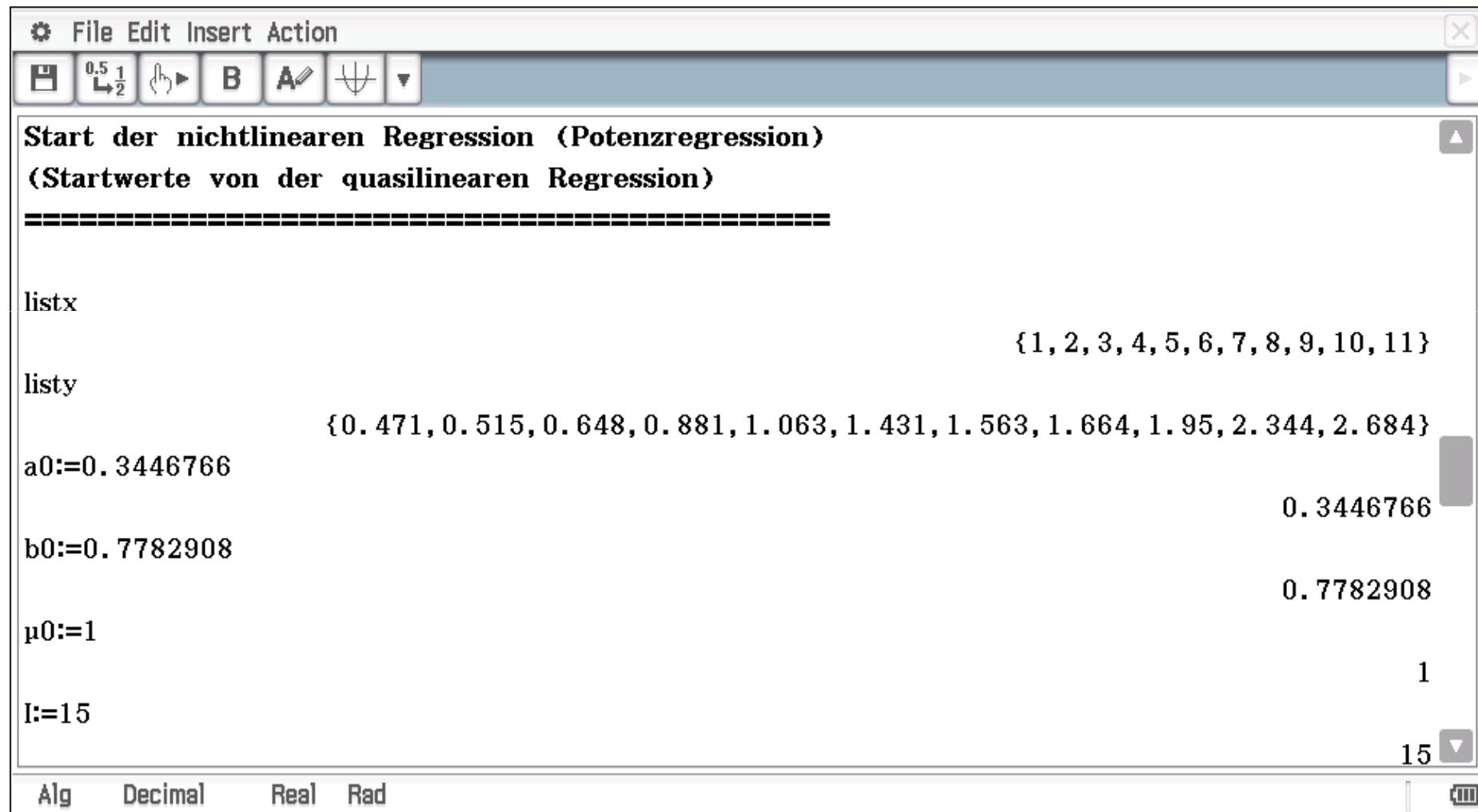
The screenshot shows a ClassPad calculator interface with the following content:

- Top menu bar: File, Edit, Insert, Action.
- Toolbar icons: Save, Print, Undo, Redo, Copy, Paste, etc.
- Text area:
 - PowerReg listx, listy
 - DispStat
 - quasilineare Regression:**

Power Reg
 $y=a \cdot x^b$
a = 0.3446766
b = 0.7782908
r = 0.9639981
 $r^2 = 0.9292924$
MSe = 0.0284313

aCoef
- Bottom status bar: Alg, Decimal, Real, Rad.
- Bottom right corner: A numeric value 0.3446765894 with a dropdown arrow.

The Power Regression (new, nonlinear): full screen



The screenshot shows a ClassPad calculator interface with the following content:

File Edit Insert Action

Start der nichtlinearen Regression (Potenzregression)
(Startwerte von der quasilinearen Regression)

listx {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

listy {0.471, 0.515, 0.648, 0.881, 1.063, 1.431, 1.563, 1.664, 1.95, 2.344, 2.684}

a0:=0.3446766 0.3446766

b0:=0.7782908 0.7782908

μ 0:=1 1

I:=15 15

Alg Decimal Real Rad

The Power Regression (new, nonlinear): PowRegLM

File Edit Insert Action
0.5 1/2 ↺ ▶ B fdx ▾

PowRegLM(listx, listy, a0, b0, μ0, I)| done

Nach 8 Iterationsschritten ist die Rechengenauigkeit erreicht:

[i, ρμ₀, μ, ε] [8 1.110619469 7.8125E-3 3.15E-13]

vecs [-1.533571961E-7 3.341276212E-7]

vocab [0.2172525584 1.023999599]

MSerr 0.01613769519

ε 3.15E-13

Alg Decimal Real Rad

The regression functions and the MSe values:

LogReg $y(x) = a + b * \ln(x)$: $MSe = 0.1187862$

PowerReg $y(x) = a * x^b$: $MSe = 0.0284313$
(quasilinear: $\ln(y) = \ln(a) + b * \ln(x)$)

now real MSe: $0.3446766 \Rightarrow a$ (optimal, quasilinear)
 $0.7782908 \Rightarrow b$ (optimal, quasilinear)

$MSe = \text{sum}((\text{listy} - a * \text{listx}^b)^2) / 9 = 0.0421417077$

PowRegLM $y(x) = a * x^b$: $MSerr = 0.01613769519$

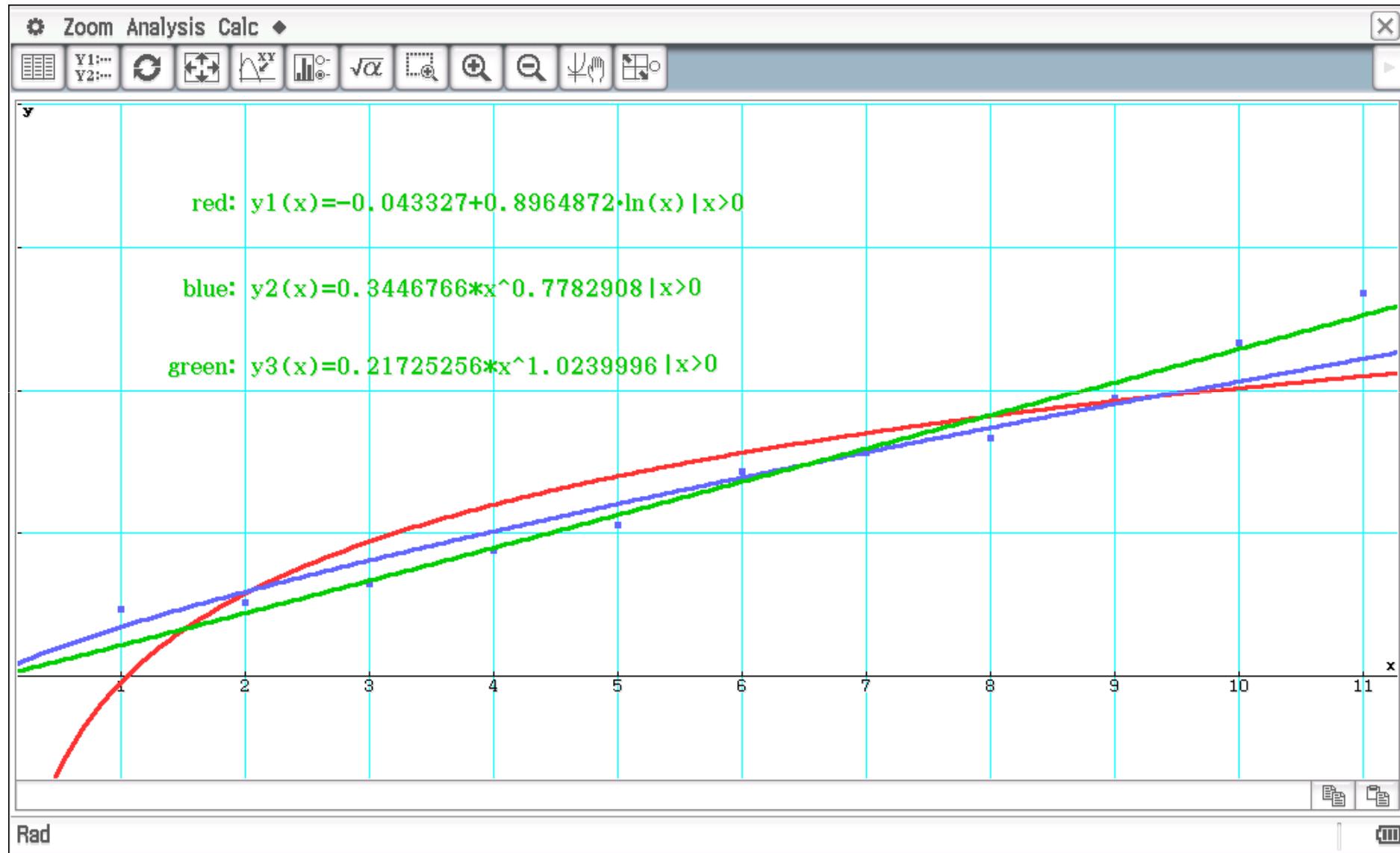
Levenberg-Marquardt Algorithm using ClassPad

The screenshot shows the Casio ClassPad interface. At the top, there is a menu bar with "File", "Edit", and "Type". Below the menu is a toolbar with icons for graph, list, and equation entry. A table is displayed with columns labeled "listx", "listy", "list3", "list4", "list5", and "list6". The data in the "listx" column is 1 through 11, and the "listy" column contains values 0.471, 0.515, 0.648, 0.881, 1.063, 1.431, 1.563, 1.664, 1.95, 2.344, and 2.684 respectively. Below the table is a "Cal▶" button. In the bottom left, there is a command input field with "[7]=". The bottom section shows a list of equations under "Sheet1":

- $y_1 = -0.043327 + 0.8964872 \cdot \ln(x) \mid x > 0$ (checked)
- $y_2 = 0.3446766 \cdot x^{0.7782908} \mid x > 0$ (checked)
- $y_3 = 0.2172525584 \cdot x^{1.023999599} \mid x > 0$ (checked)
- $y_4: \square$ (unchecked)
- $y_5: \square$ (unchecked)
- $y_6: \square$ (unchecked)
- $y_7: \square$ (unchecked)
- $y_8: \square$ (unchecked)
- $y_9: \square$ (unchecked)
- $v_{10}: \square$ (unchecked)

At the bottom, there are buttons for "Rad" and "Real".

Levenberg-Marquardt Algorithm using ClassPad



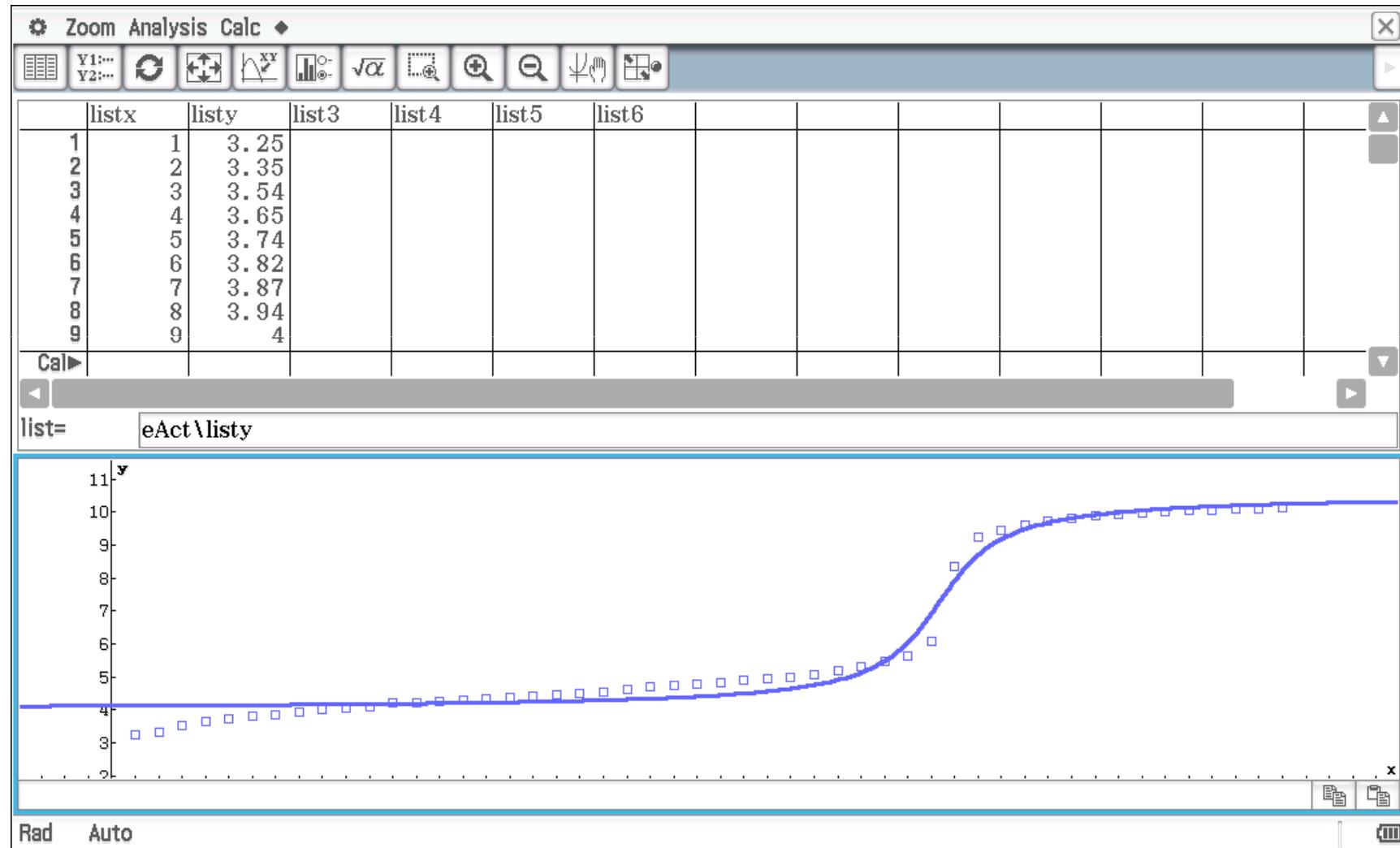
The general **logistic** regress. $y(x) = c/(1+a \cdot e^{(-b \cdot x)})+d$ and the **arctan**-regression $y(x) = a \cdot \arctan(c \cdot (x - b)) + d$ respectively we discuss with a **real data set**:

`listx:={1, 2, ..., 50}` contains time points (with step 1),

`listy:={3.25,3.35,3.54,3.65,3.74,3.82,3.87,3.94,4,4.06,
4.11,4.22,4.22,4.27,4.32,4.34,4.39,4.44,4.46,4.52,
4.56,4.62,4.7,4.73,4.77,4.82,4.89,4.93,5,5.09,5.19,
5.31,5.47,5.65,6.08,8.33,9.22,9.44,9.61,9.74,9.8,
9.88,9.92,9.96,10.01,10.05,10.06,10.08,10.1,10.13}`

contains chemical data generated during an experiment in the class room (chemical reaction, pH values).

What is a good regression function?



The result with LogiDReg-program: Mserr=0.176675

```

File Edit Insert Action
LogiDReg(listx, listy, A0, b0, c0, d0, p0, I)
[i, p, pμ₀, ε]
done
vecs
[ -4.46372939E-6
  -1.24851645E-7
   1.039374255E-7
  -4.662524305E-8 ]
vocab
[ 20.4915612
   0.5827483528
   5.825679601
   4.266081287 ]
MSerr
0.1766749421
Zum Vergleich:
TI: MSerr=0.1766749421
Alg Decimal Real Rad

```

The result with **ArctDReg**-program: Mserr=0.112791

```

File Edit Insert Action
ArctDReg(listx, listy, a0, b0, c0, d0, μ0, I)
done
vecs

$$\begin{bmatrix} 8.393259409 \times 10^{-8} \\ -8.488923858 \times 10^{-8} \\ -1.432446392 \times 10^{-7} \\ 1.885617414 \times 10^{-8} \end{bmatrix}$$

vocab

$$\begin{bmatrix} 2.073000403 \\ 35.35123402 \\ 0.4965009396 \\ 7.266764848 \end{bmatrix}$$

MSerr
0.1127910421
[i, μ, ρμ₀, ε]
[18 6.103515625E-5 1.434262948 0]
stop

```

Alg Decimal Real Rad

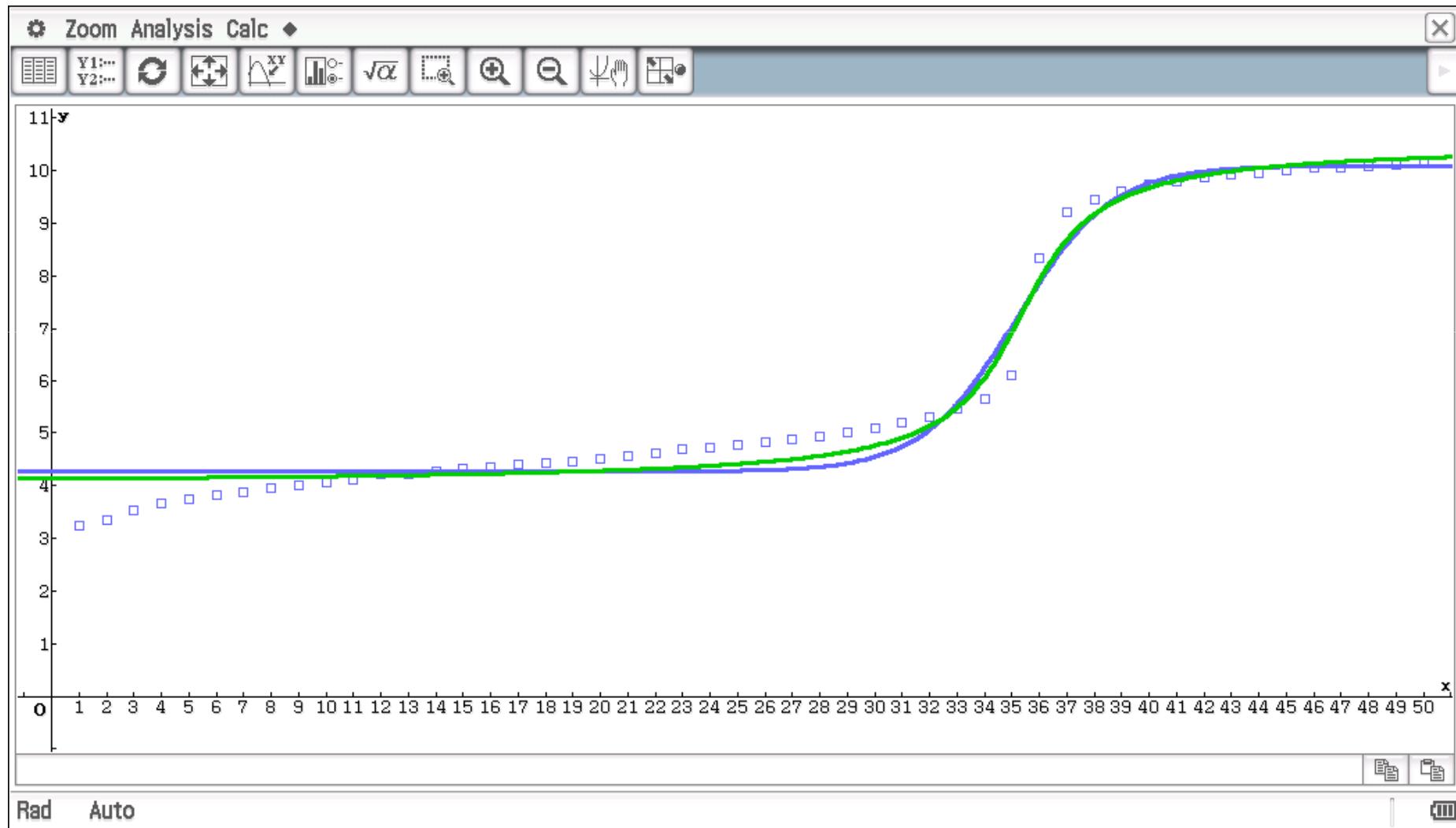
The logistic and arctan-regression in comparision:

The screenshot shows the Casio ClassPad interface with the following details:

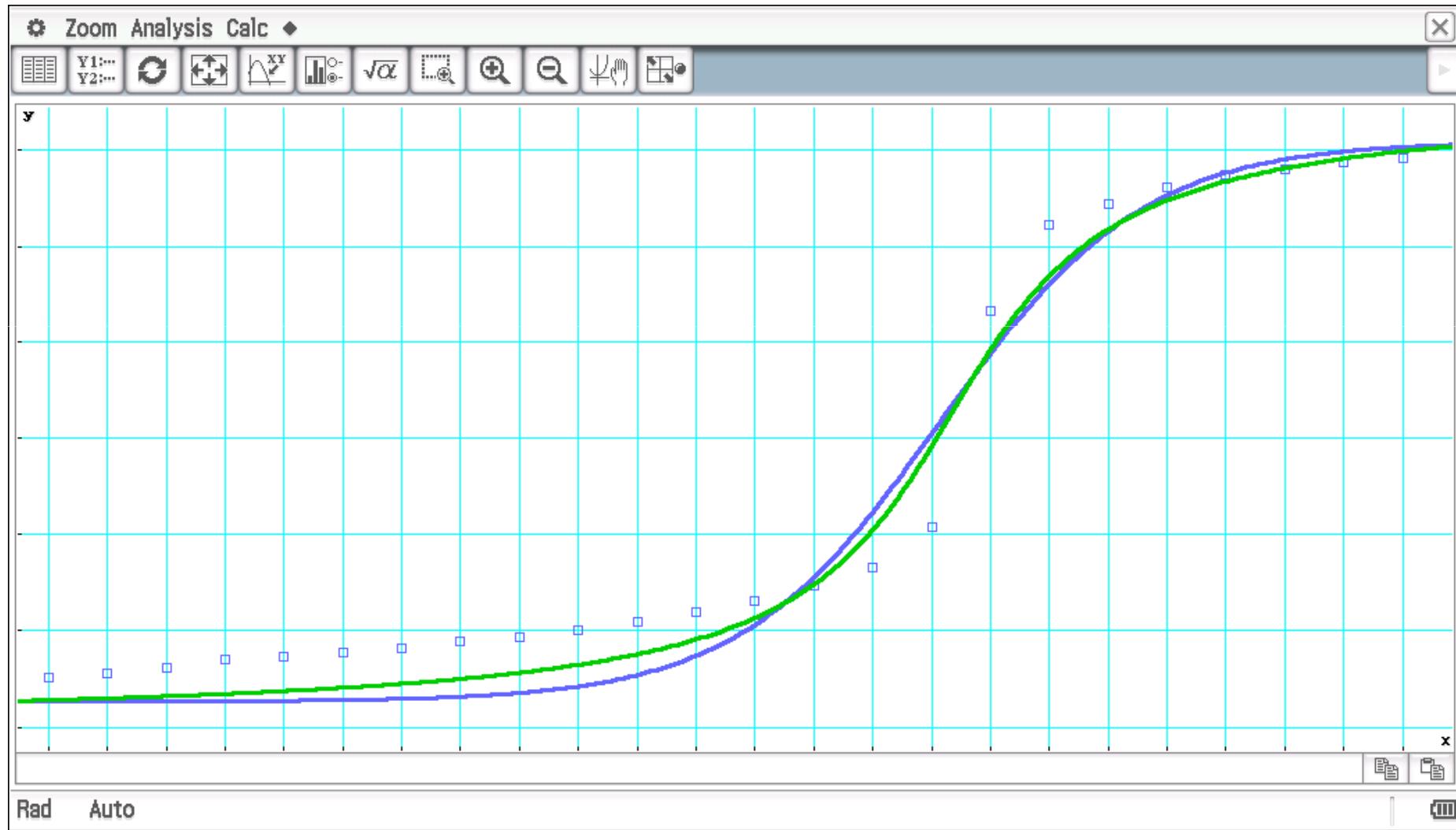
- Data List View:** A table titled "Cal▶" displays data points in columns labeled "listx" and "listy". The data points are as follows:

| | listx | listy |
|---|-------|-------|
| 1 | 1 | 3.25 |
| 2 | 2 | 3.35 |
| 3 | 3 | 3.54 |
| 4 | 4 | 3.65 |
| 5 | 5 | 3.74 |
| 6 | 6 | 3.82 |
| 7 | 7 | 3.87 |
| 8 | 8 | 3.94 |
- Equation View:** The bottom section shows the following regression equations:
 - y1:** $y_1 = \frac{5.825679928}{1+793169457.6 \cdot e^{-0.5827479673 \cdot x}} + 4.266081147$ (blue checked)
 - y2:** $y_2 = 2.072999762 \cdot \tan^{-1}(0.4965020338 \cdot (x - 35.35123467)) + 7.266764704$ (green checked)
 - y3:** \square
 - y4:** \square
 - y5:** \square
 - y6:** \square
 - y7:** \square
- Mode:** The mode is set to "Real".

The logistic and arctan-regression in comparision:

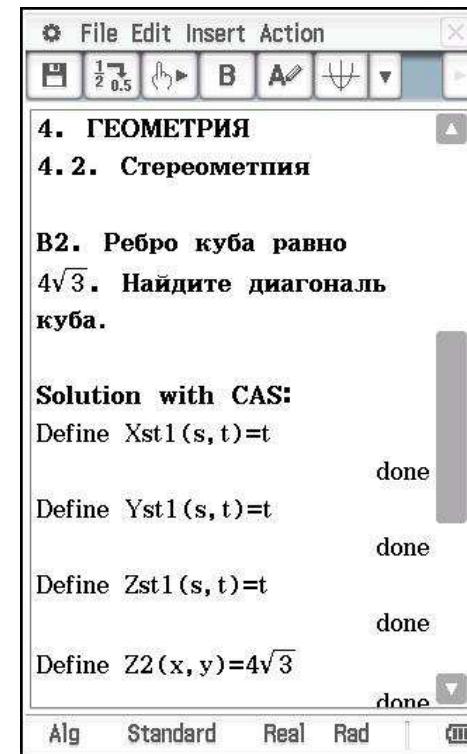
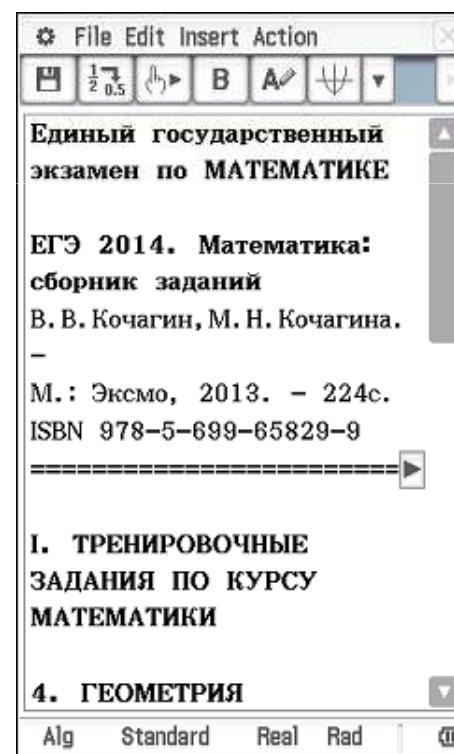
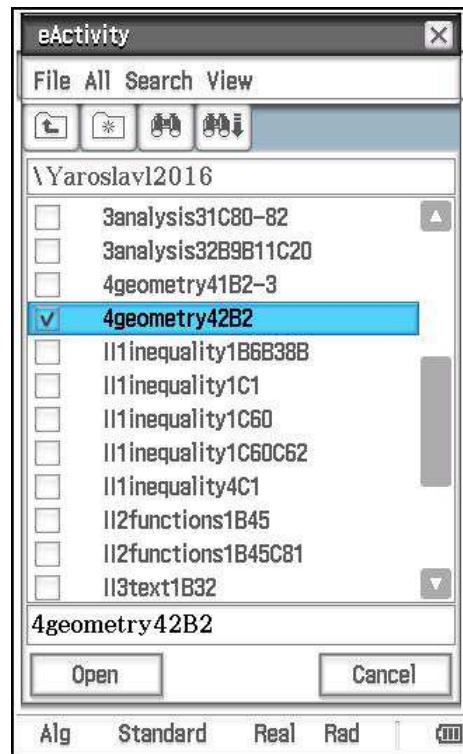


The logistic and arctan-regression in comparision:



eActivity: 4. ГЕОМЕТРИЯ 4.2. Стереометрия

Ребро куба равно $4\sqrt{3}$. Найдите диагональ куба.



eActivity: 4. ГЕОМЕТРИЯ 4.2. Стереометрия

Ответ: диагональ куба равно 12.

File Edit Insert Action

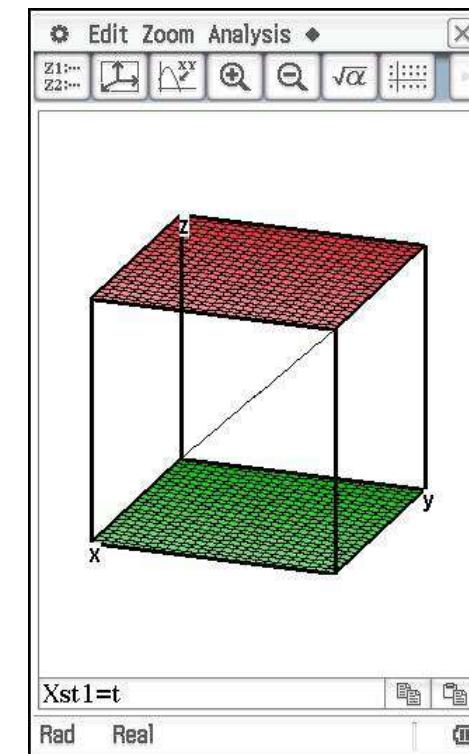
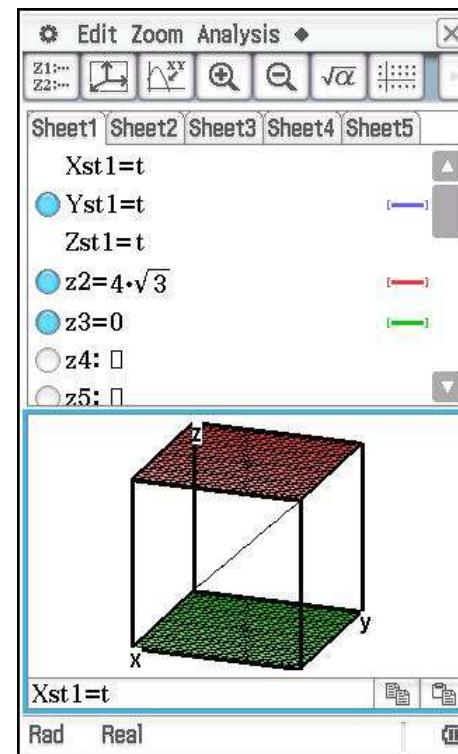
```

Define Zst1(s,t)=t
done
Define Z2(x,y)=4√3
done
Define Z3(x,y)=0
done
solve(Z2(x,y)=Zst1(s,t),t)
{t=4·√3}
norm([Xst1(s,t)]
      [Yst1(s,t)]
      [Zst1(s,t)])
| t=4·√3
12

```

3D - graphic solution

Alg Standard Real Rad



eActivity: 4. ГЕОМЕТРИЯ 4.1. Планиметрия

Найдите диаметр окружности, описанной около квадрата с стороной $8\sqrt{2}$.

File Edit Insert Action

Единый государственный экзамен по МАТЕМАТИКЕ

ЕГЭ 2014. Математика: сборник заданий

Б. В. Кочагин, М. Н. Кочагина.

М.: Эксмо, 2013. – 224с.

ISBN 978-5-699-65829-9

I. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ПО КУРСУ МАТЕМАТИКИ

4. ГЕОМЕТРИЯ

Alg Standard Real Rad

File Edit Insert Action

4.1. Планиметрия

B2. Найдите диаметр окружности, описанной около квадрата с стороной $8\sqrt{2}$.

Solution with CAS:

$$(8\sqrt{2})^2 + (8\sqrt{2})^2 = d^2$$

$$256 = d^2$$

$$\text{solve}(ans, d)$$

$$\{d=-16, d=16\}$$

ответ: $d=16$

graphic construction

Alg Standard Real Rad

File Edit View Draw

16

$$(8\sqrt{2})^2 + (8\sqrt{2})^2 = d^2$$

$$256 = d^2$$

$$\text{solve}(ans, d)$$

$$\{d=-16, d=16\}$$

ответ: $d=16$

graphic construction

Alg Standard Real Rad

eActivity: 4. ГЕОМЕТРИЯ 4.1. Планиметрия

Найдите площадь круга (S), вписанного в прямоугольный треугольник с катетами, равными 24 и 10. В ответе укажите S/π .

B3. Найдите площадь круга (S), вписанного в прямоугольный треугольник с катетами, равными 24 и 10. В ответе укажите $\frac{S}{\pi}$.

Solution with CAS:

A(0, 0), B(24, 0), C(0, 10)
 Center M(r, r)
 $D(r, 0)$, $E(0, r)$, $F(x, y)$ with
 $y = -\frac{10}{24}x + 10$
 $|MD|^2 = r^2$, $|ME|^2 = r^2$, $|MF|^2 =$
 $(x-r)^2 + ((-\frac{10}{24}x+10)-r)^2 = r^2$

$|MD|^2 = r^2$, $|ME|^2 = r^2$, $|MF|^2 =$
 $(x-r)^2 + ((-\frac{10}{24}x+10)-r)^2 = r^2$
 $\text{solve}((r-x)^2 + (r - (-\frac{10}{24}x+10))^2 = r^2)$
 $\left\{ x = \frac{12 \cdot (7 \cdot r - 2 \cdot \sqrt{30} \cdot \sqrt{-r^2 + 34 \cdot r})}{169} \right.$

only one solution for x :

$\text{solve}(-r^2 + 34 \cdot r - 120 = 0, r)$
 $\{r=4, r=30\}$

thus $r=4$:
 $S=r^2\pi=16\pi \Rightarrow \frac{S}{\pi}=16$.

geometry window:

graphic construction

$\text{solve}(-r^2 + 34 \cdot r - 120 = 0, r)$
 $\{r=4, r=30\}$

thus $r=4$:
 $S=r^2\pi=16\pi \Rightarrow \frac{S}{\pi}=16$.

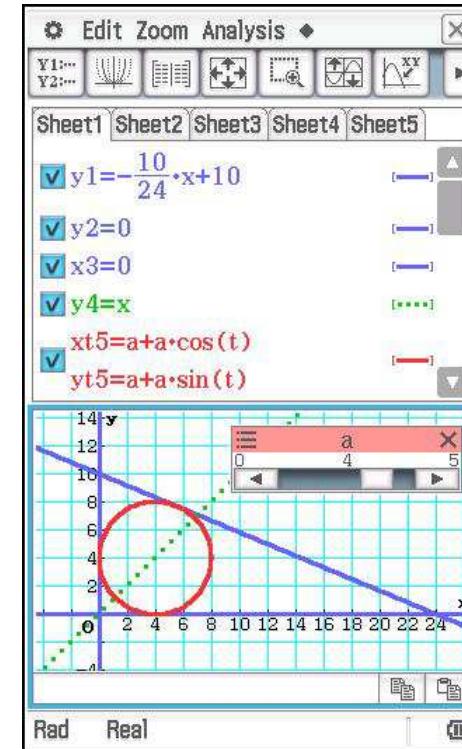
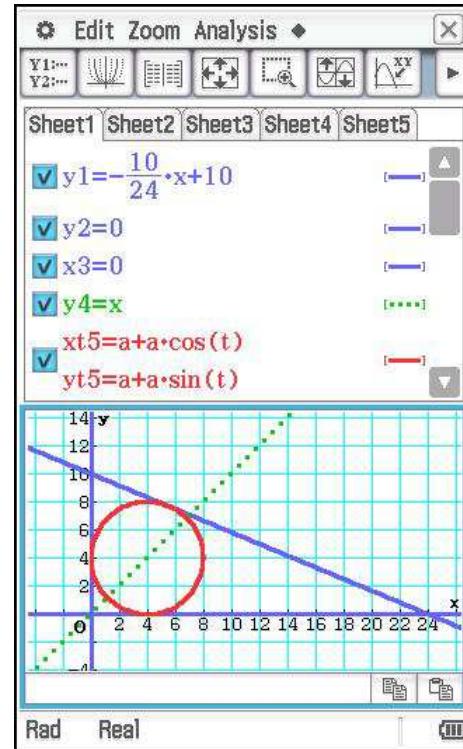
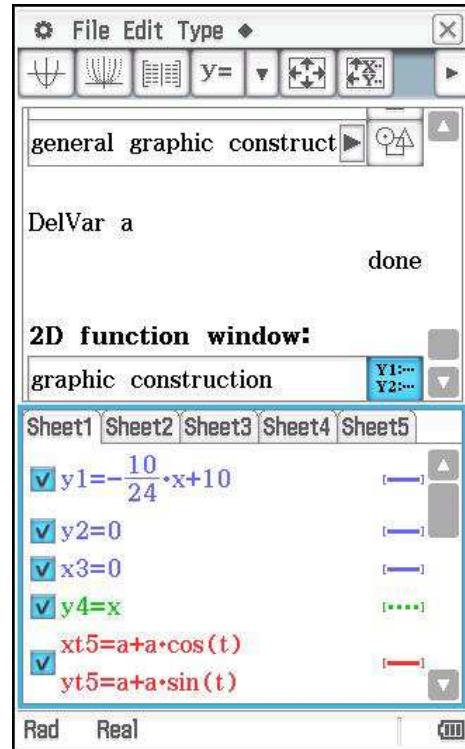
geometry window:

graphic construction

graphic construction

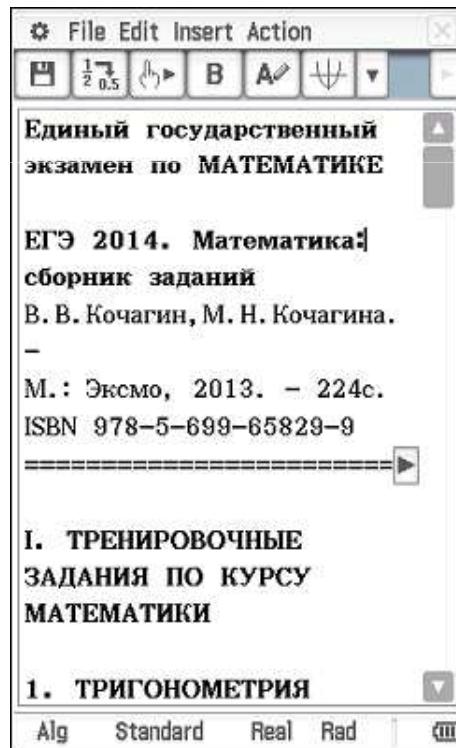
eActivity: 4. ГЕОМЕТРИЯ 4.1. Планиметрия

Найдите площадь круга (S), вписанного в прямоугольный треугольник с катетами, равными 24 и 10.
В ответе укажите S/π .



eActivity: 1. ТРИГОНОМЕТРИЯ

1.1. Преобразования тригонометрических выражений



File Edit Insert Action

1.1. Преобразования тригонометрических выражений

B19. Найдите значение выражения

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\tan(5\pi+\alpha)}{\sin(\pi-\alpha)-1}, \text{ если } \operatorname{ctg}(\alpha)=0.25$$

Solution with CAS without condition:

(remark: $\operatorname{ctg}(\alpha)$ must be written as $1/\tan(\alpha)$)

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\tan(5\pi+\alpha)}{\sin(\pi-\alpha)-1}$$

Alg Standard Real Rad

File Edit Insert Action

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\tan(5\pi+\alpha)}{\sin(\pi-\alpha)-1}$$

$$\frac{\cos(\alpha)-\frac{1}{\tan(\alpha)}}{\sin(\alpha)-1}$$

simplify(ans)

$$\frac{1}{\tan(\alpha)}$$

now with condition

$$\tan(\alpha)=0.25$$

$$\frac{1}{\tan(\alpha)} | \tan(\alpha)=0.25$$

4

ответ: $\operatorname{ctg}(\alpha)=4$

Alg Standard Real Rad

eActivity: 1. ТРИГОНОМЕТРИЯ

1.1. Преобразования тригонометрических выражений

File Edit Insert Action

direct computation with condition $\tan(\alpha)=0.25$:

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\frac{1}{\tan(5\pi+\alpha)}}{\sin(\pi-\alpha)-1} \quad | \tan(\alpha)=0.25$$

$$\frac{\cos(\alpha)-4}{\sin(\alpha)-1}$$

direct computation with condition $\alpha=\arctan(0.25)$:

(here in the CAS $\tan^{-1}(0.25)$ means $\arctan(0.25)$)

$$\frac{\sin\left(\frac{9}{2}\pi-\alpha\right)-\frac{1}{\tan(5\pi+\alpha)}}{\sin(\pi-\alpha)-1} \quad | \alpha=\tan^{-1}(0.25)$$

$$\frac{-4\sqrt{17}+4}{17}$$

$$\frac{-\sqrt{17}+1}{17}$$

Alg Standard Real Rad

File Edit Insert Action

$\frac{\tan(\pi-\alpha)-1}{\sin(\pi-\alpha)-1} \quad | \alpha=\tan^{-1}(0.25)$

$$\frac{-4\sqrt{17}+4}{17}$$

$$\frac{-\sqrt{17}+1}{17}$$

simplify(ans)

remark:

$$\tan^{-1}(0.25) \quad 4$$

approx(ans)

$$\tan^{-1}\left(\frac{1}{4}\right) \quad 0.2449786631$$

$$\sin\left(\frac{9}{2}\pi-\alpha\right)-\frac{1}{\tan(5\pi+\alpha)} \quad | \alpha=\text{ans}$$

Alg Standard Real Rad

File Edit Action

$\frac{\sin\left(\frac{9}{2}\cdot\pi-\alpha\right)-\frac{1}{\tan(5\cdot\pi+\alpha)}}{\sin(\pi-\alpha)-1}$

$$= \frac{\sin\left(\frac{9}{2}\cdot\pi-\alpha\right)-\frac{1}{\tan(\alpha)}}{\sin(\pi-\alpha)-1}$$

$$= \frac{\sin\left(\frac{1}{2}\cdot\pi-\alpha\right)-\frac{1}{\tan(\alpha)}}{\sin(\pi-\alpha)-1}$$

$$= \frac{\cos(-\alpha)-\frac{1}{\tan(\alpha)}}{\sin(\pi-\alpha)-1}$$

$$= \frac{\cos(\alpha)-\frac{1}{\tan(\alpha)}}{\sin(\pi-\alpha)-1}$$

$$= \frac{\cos(\alpha)-\frac{1}{\tan(\alpha)}}{-\sin(-\alpha)-1}$$

$$= \frac{\cos(\alpha)-\frac{1}{\tan(\alpha)}}{\cos(\alpha)-\frac{1}{\tan(\alpha)}} \quad | \text{ERROR!}$$

Sorry, not equivalent

OK

$\sin(\alpha)-1$

$$= \frac{\cos(\alpha)\cdot\sin(\alpha)-1}{\sin(\alpha)-1}$$

$$= \frac{\cos(\alpha)}{\sin(\alpha)}$$

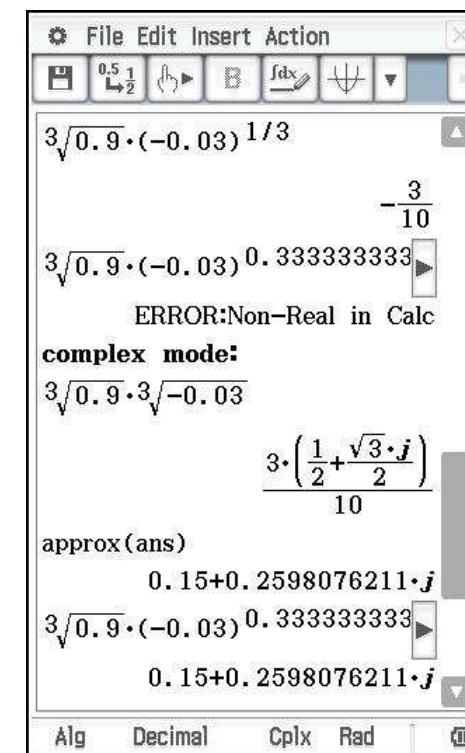
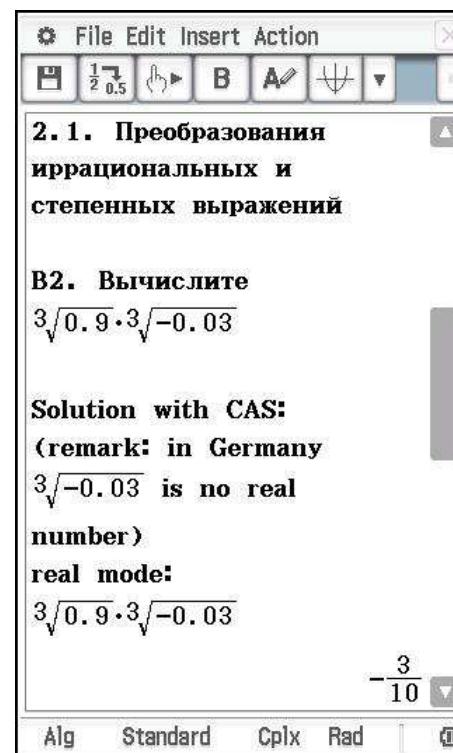
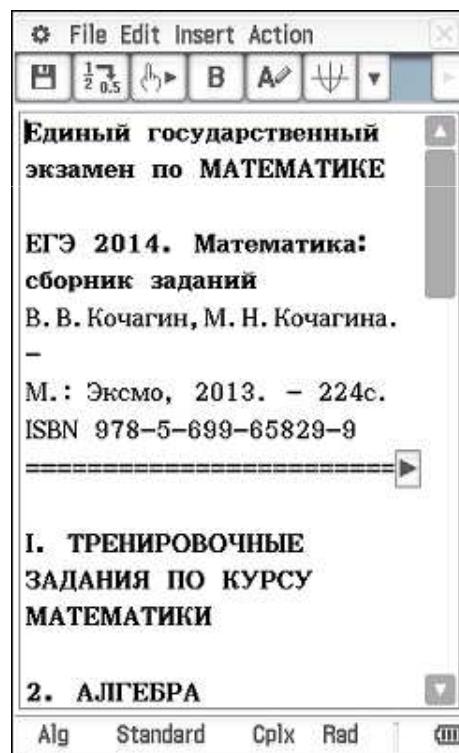
$$= \frac{1}{\tan(\alpha)}$$

$$= 4$$

Exp:((sin(((9)/(2))\cdot\pi-\alpha)-((1)/(\tan(\alpha))))/(cos(((9)/(2))\cdot\pi-\alpha)-((1)/(\tan(\alpha)))))

eActivity: 2. АЛГЕБРА

2.1. Преобразования иррациональных и степенных выражений



eActivity: 2. АЛГЕБРА

2.1. Преобразования иррациональных и степенных выражений

File Edit Insert Action

2.1. Преобразования иррациональных и степенных выражений

B47. Упростите выражение $\frac{3(\sqrt{a}-3\cdot 4\sqrt{a\cdot b})}{4\sqrt{a\cdot b}-3\cdot \sqrt{b}}$, если $\frac{a}{b}=7\frac{58}{81}$

Solution with CAS:
(remark: in CAS write $7+\frac{58}{81}$)
suppose $a \geq 0$ and $b > 0$

Alg Standard Cplx Rad

File Edit Insert Action

$\frac{3(\sqrt{a}-3\cdot 4\sqrt{a\cdot b})}{4\sqrt{a\cdot b}-3\cdot \sqrt{b}}$

$\frac{-3\cdot (\sqrt{a}-3\cdot (a\cdot b)^{\frac{1}{4}})}{3\cdot \sqrt{b}-(a\cdot b)^{\frac{1}{4}}}$

$\frac{3(\sqrt{a}-3\cdot 4\sqrt{a\cdot b})}{4\sqrt{a\cdot b}-3\cdot \sqrt{b}} | a=(7+\frac{58}{81})\cdot b$

$\frac{-3\cdot (\frac{25\cdot \sqrt{b}}{9}-5\cdot (b^2)^{\frac{1}{4}})}{3\cdot \sqrt{b}-5\cdot (b^2)^{\frac{1}{4}}}$

simplify(ans|b>0)

5

Alg Standard Real Rad

File Edit Insert Action

$\frac{-3\cdot (\frac{25\cdot \sqrt{b}}{9}-5\cdot (b^2)^{\frac{1}{4}})}{3\cdot \sqrt{b}-5\cdot (b^2)^{\frac{1}{4}}}$

simplify(ans|b>0)

5

ответ: 5

remark:

$7+\frac{58}{81}$

$\frac{625}{81}$

$7\frac{58}{81}=7\cdot\frac{58}{81}$

$\frac{406}{81}=\frac{406}{81}$

Alg Standard Real Rad

eActivity: 2. АЛГЕБРА

2.2. Иррациональные уравнения

2.3. Преобразования логарифмических выражений

File Edit Insert Action

2.2. Иррациональные уравнения

B19. Решите уравнение

$$\sqrt{x^2+3x-4} + \sqrt{x^3+12x^2-11x-2} = 0.$$

Solution with CAS:

```
solve(x^2+3x-4=0, x)
{x=-4, x=1}

solve(x^3+12x^2-11x-2=0, x)
{x=1, x=-sqrt(161)/2 - 13/2, x=sqrt(161)/2}

solve(sqrt(x^2+3x-4)+sqrt(x^3+12x^2-11x-2)=0, x)
{x=1}
```

ответ: x=1

Alg Standard Real Rad

File Edit Insert Action

2.3. Преобразования логарифмических выражений

B17. Найдите значение выражения

$$(\log_5(36) + \log_5(2) - \log_5(8)) \cdot \log_9\left(\frac{1}{25}\right)$$

Solution with CAS:

$$\begin{aligned} & (\log_5(36) + \log_5(2) - \log_5(8)) \cdot \log_9\left(\frac{1}{25}\right) \\ & - \left(\frac{2 \cdot \ln(3) + 2 \cdot \ln(2)}{\ln(5)} - \frac{2 \cdot \ln(2)}{\ln(5)} \right) \cdot \frac{-2}{\ln(3)} \end{aligned}$$

simplify(ans)

Alg Standard Real Rad

File Edit Action

$$\begin{aligned} & (\log_5(36) + \log_5(2) - \log_5(8)) \\ & = \log_5\left(36 \cdot \frac{2}{8}\right) \cdot \log_9\left(\frac{1}{25}\right) \\ & = \log_5(9) \cdot \log_9\left(\frac{1}{25}\right) \\ & = \log_5(9) \cdot (-\text{smiley face}) \\ & = \log_5(9) \cdot (-\log_9(5^2)) \\ & = \log_5(9) \cdot (-2) \cdot \log_9(5) \\ & = \frac{\ln(9)}{\ln(5)} \cdot (-2) \cdot \log_9(5) \\ & = \frac{\ln(9)}{\ln(5)} \cdot (-2) \cdot \frac{\ln(5)}{\ln(9)} \\ & = -2 \end{aligned}$$

Exp:(log(5,36)+ log(5,2)- log(5,8))

eActivity: 2. АЛГЕБРА

2.4. Логарифмические уравнения и неравенства

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2.4. Логарифмические уравнения и неравенства

C87. Решение неравенство

$$\frac{\log_{0.1x}(10) \cdot \log(10x)}{\log_{0.01x}(10)} \leq 2.$$

Solution with CAS:

(Remark: $\lg(\dots) = \log(\dots)$)

$$\frac{\log_{0.1x}(10) \cdot \log(10x)}{\log_{0.01x}(10)} \leq 2$$

$$\frac{1}{x} \cdot (\ln(x) - 2 \cdot \ln(5) - 2 \cdot \ln(2)) \leq 2$$

simplify(ans)

$$\left(\frac{\ln(x)}{x} + 1 \right) \cdot \ln\left(\frac{x}{100}\right) \leq 2$$

Alg Standard Real Rad

File Edit Insert Action

File Edit Insert Action

2.4. Логарифмические уравнения и неравенства

сimplify(ans)

$$\left(\frac{\ln(x)}{\ln(10)} + 1 \right) \cdot \ln\left(\frac{x}{100}\right) \leq 2$$

$$\ln\left(\frac{x}{10}\right)$$

solve(ans, x)

$$\{0 < x \leq 1, 10 < x \leq 1000\}$$

Remark: $x \neq 100$,

$\log_{0.01x}(10)$ for $x=100$ is not defined!

$\log_{0.01x}(10) | x=100$

Undefined

Alg Standard Real Rad

File Edit Insert Action

2.4. Логарифмические уравнения и неравенства

$\log_{0.01x}(10)$ for $x=100$ is not defined!

$\log_{0.01x}(10) | x=100$

Undefined

$\log_{0.1x}(10) \cdot \log(10x)$

$\log_{0.01x}(10) | x=100$

Undefined

$\lim_{x \rightarrow 100^+} (\log_{0.01x}(10))$

$\lim_{x \rightarrow 100^-} (\log_{0.01x}(10))$

Alg Standard Real Rad

eActivity: 2. АЛГЕБРА

2.5. Показательные уравнения и неравенства

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2.5. Показательные уравнения и неравенства

B22. Сколько целых чисел входит в область определения функции

$$y = \sqrt{\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} - 3} ?$$

Solution with CAS:

Define $y1(x) = \sqrt{\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} - 3}$

done

2D-graphic

Alg Standard Real Rad

Edit Zoom Analysis

Sheet1 Sheet2 Sheet3 Sheet4 Sheet5

y1: $\sqrt{\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} - 3}$

y2: 0

y3: 0

y4: 0

y5: 0

Rad Real

File Edit Insert Action

$\lim_{x \rightarrow 2^-} \sqrt{\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} - 3} = \infty$

$y1(0) = 0$

$y1(1) = 2\sqrt{6}$

$\text{solve}\left(\left(\frac{1}{3}\right)^{\frac{x+2}{x-2}} \geq 3, x\right) \quad \{x \geq 0\}$

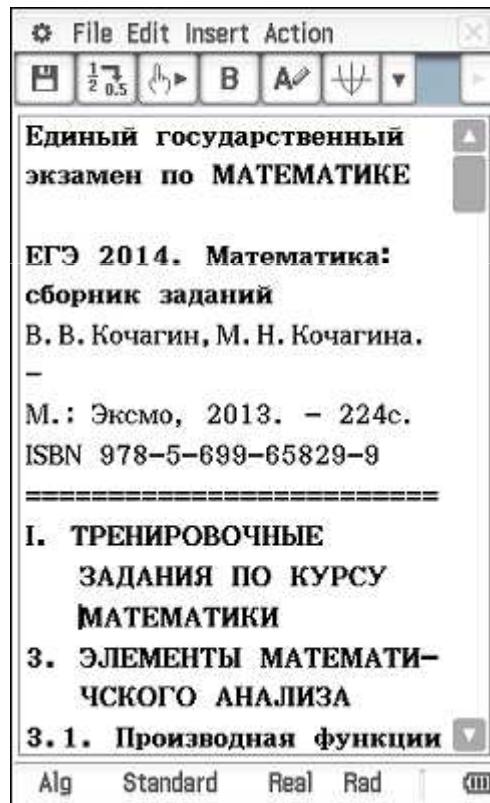
$\text{solve}\left(\frac{x+2}{2-x} \geq 1, x\right) \quad \{0 \leq x < 2\}$

Alg Standard Real Rad

ответ: 2 ($x=0, x=1$)

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

3.1. Производная функции



C80. Найдите множество значений функции

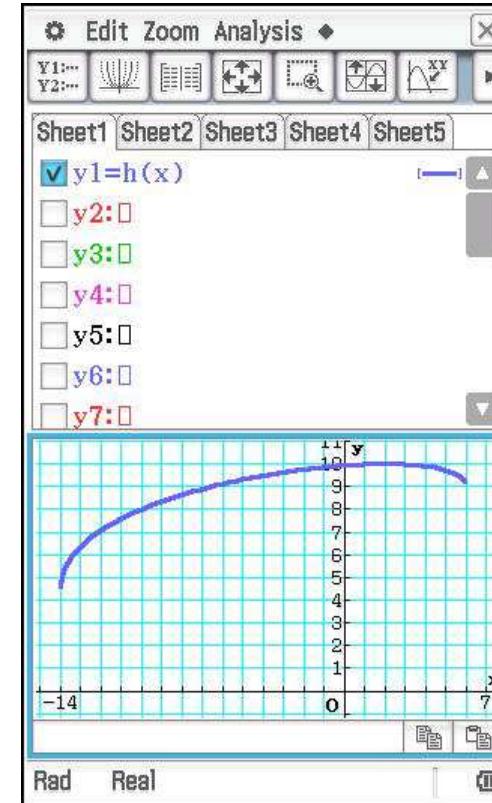
$$h(x)=2\sqrt{x+14}+\sqrt{6-x}.$$

Solution with CAS:

```
Define h(x)=2*sqrt(x+14)+sqrt(6-x)
done
fMin(h(x),x)
{.MinValue=2*sqrt(5), x=-14}
fMax(h(x),x)
{.MaxValue=10, x=2}

graphical solution
```

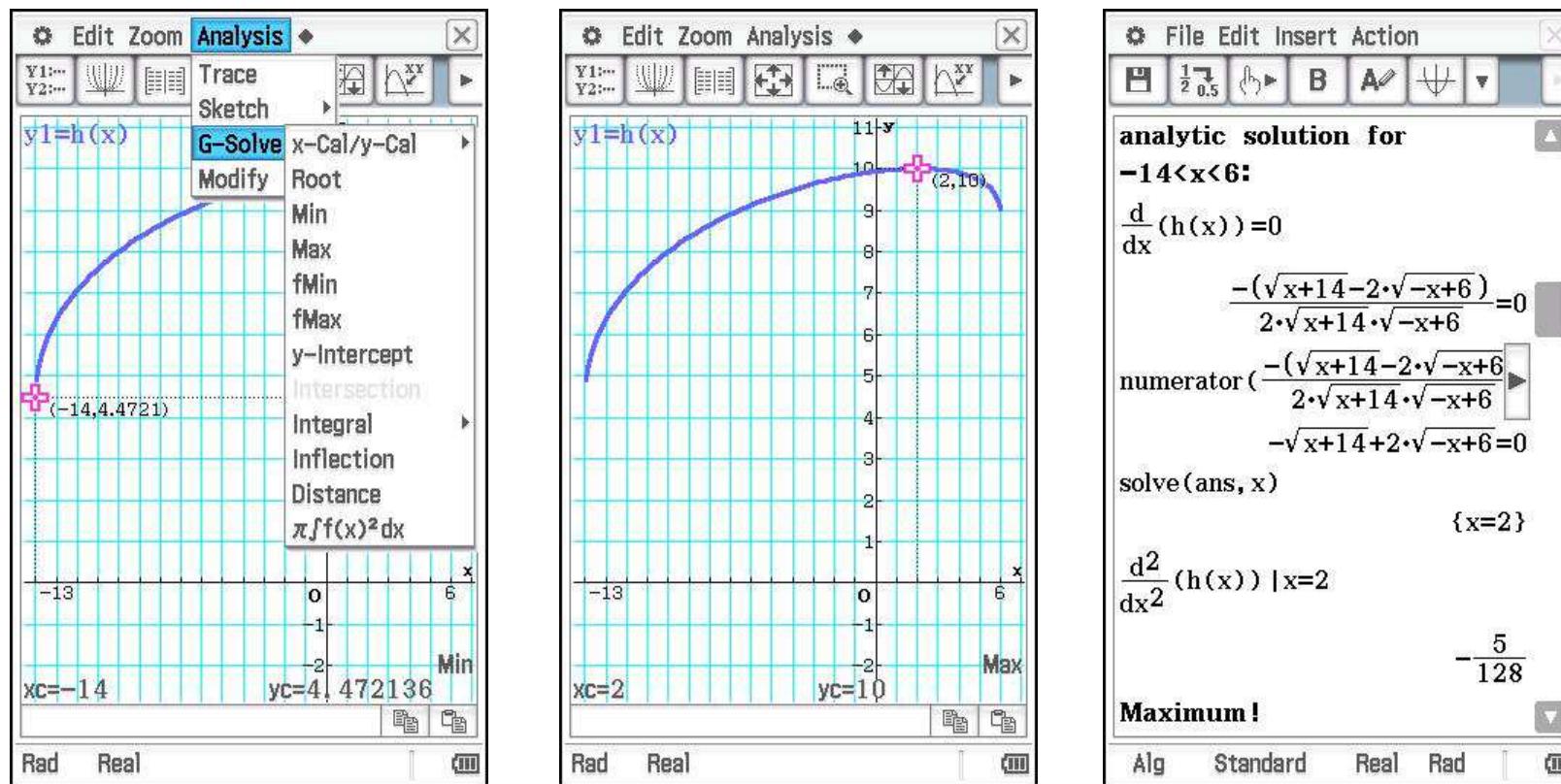
ответ: $2\sqrt{5} \leq h(x) \leq 10.$



ответ: $2\sqrt{5} \leq h(x) \leq 10$

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

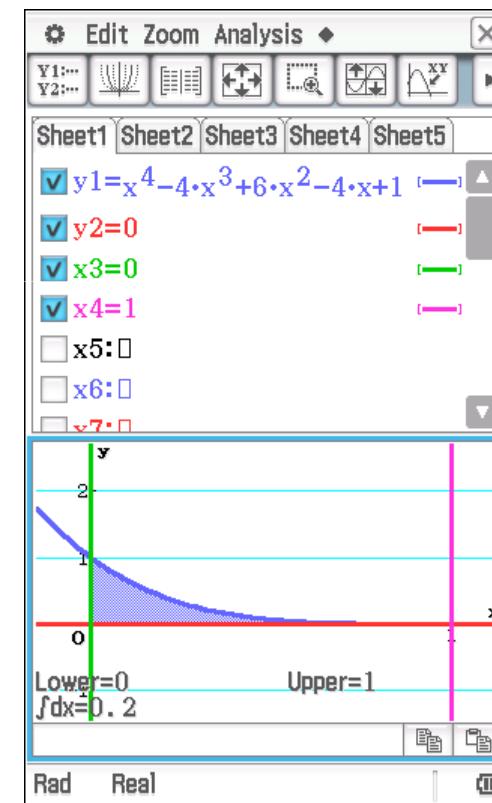
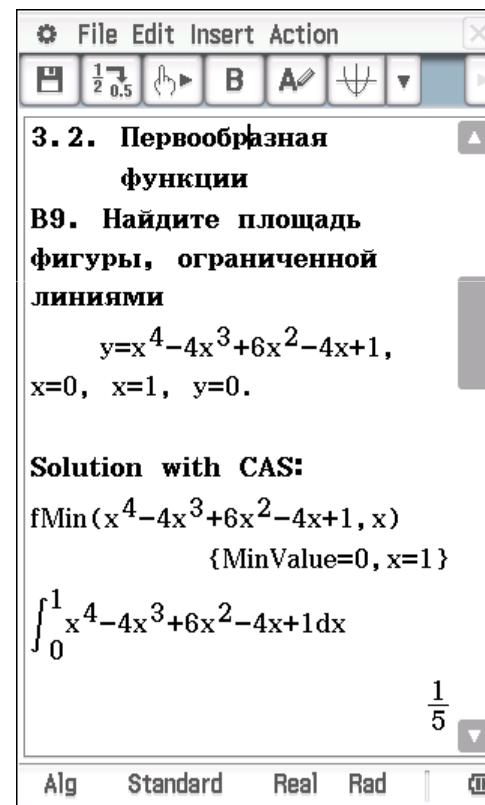
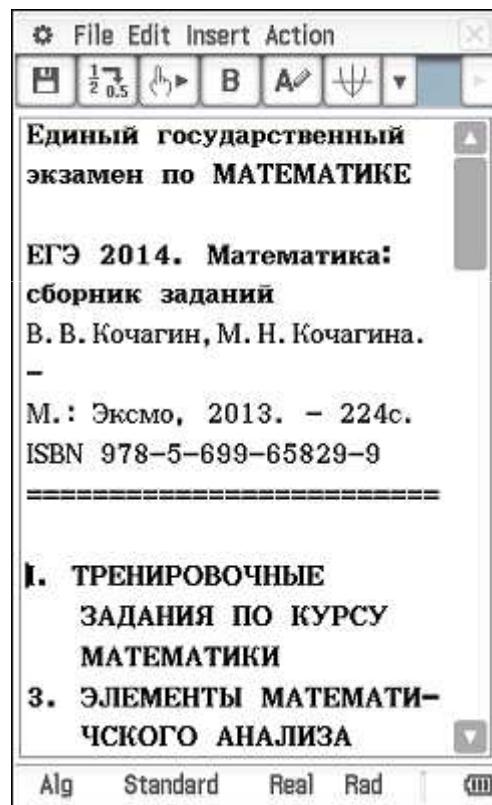
3.1. Производная функции



ответ: $4.472136 \leq h(x) \leq 10$

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

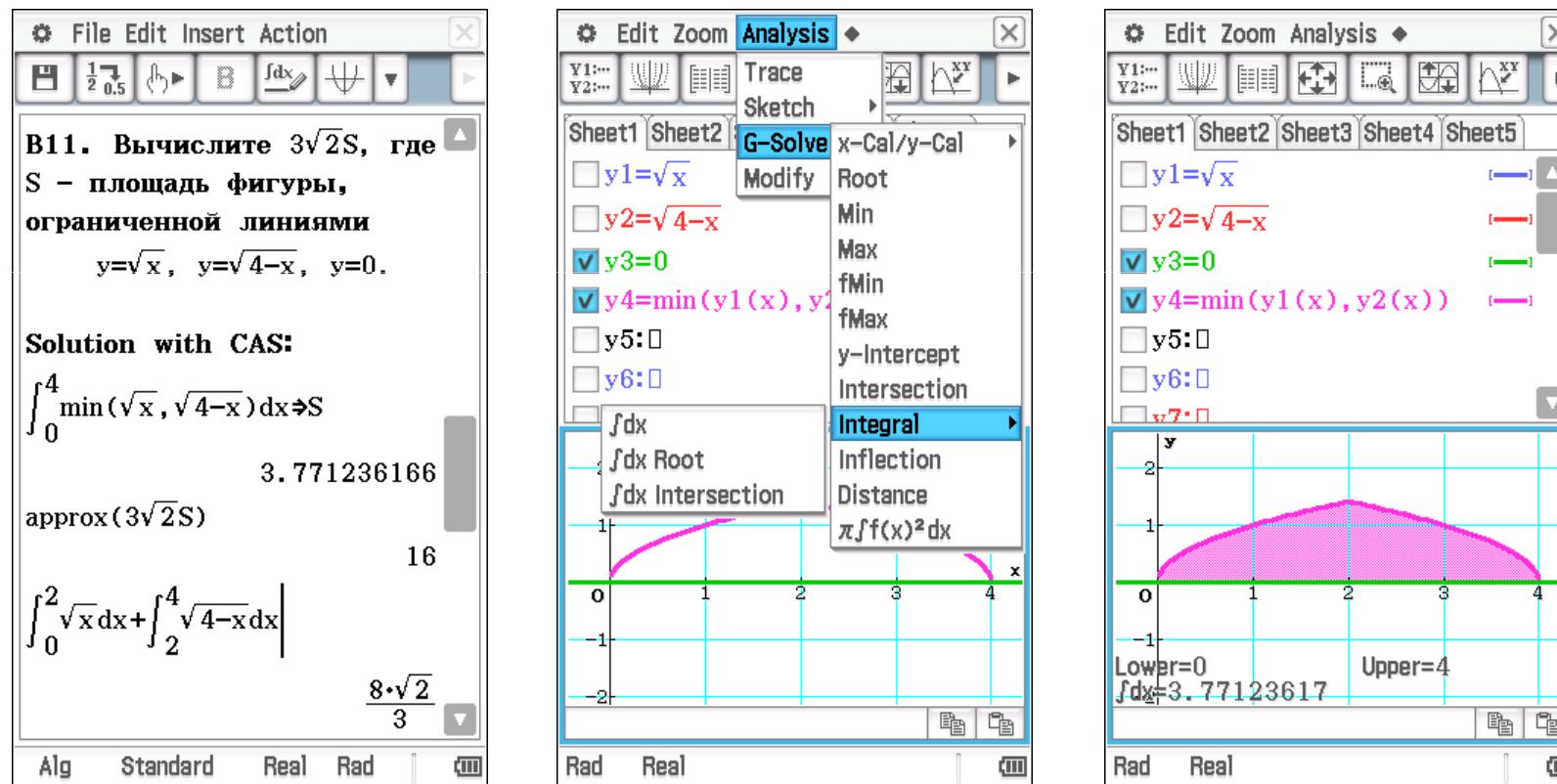
3.2. Первообразная функции



ответ: 0.2

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

3.2. Первообразная функции



ответ: 16

eActivity: 3. ЭЛЕМЕНТЫ МАТЕМАТИЧЕСКОГО АНАЛИЗА

3.2. Первообразная функции

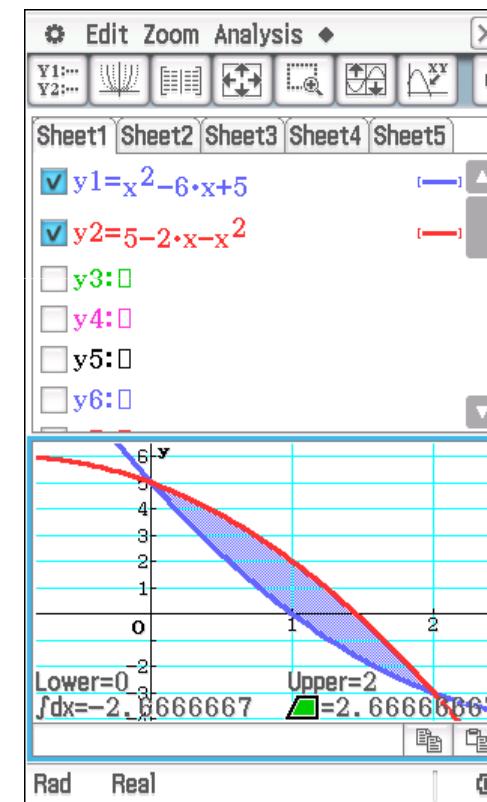
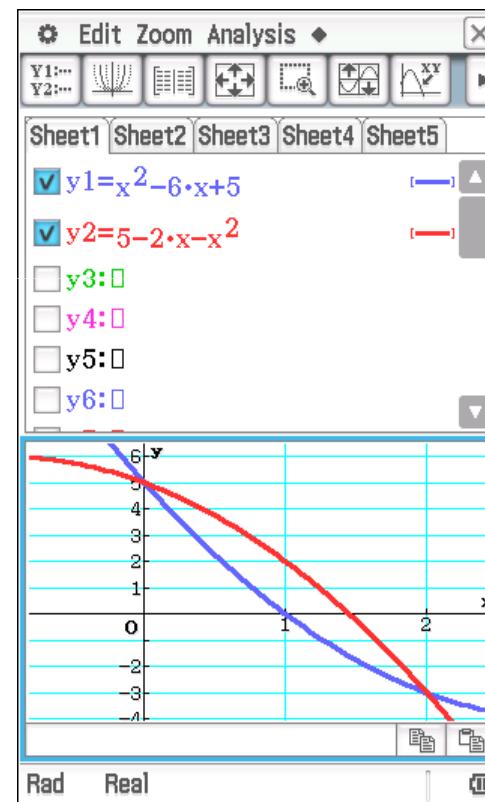
C20. Найдите площадь фигуры, ограниченной линиями
 $y=x^2-6x+5$ и $y=5-2x-x^2$.

Solution with CAS:

```
solve(x^2-6x+5=5-2x-x^2, x)
{x=0, x=2}
 $\int_0^2 (5-2x-x^2)-(x^2-6x+5) dx$ 
 $\frac{8}{3}$ 
```

ответ: $2+\frac{2}{3}$

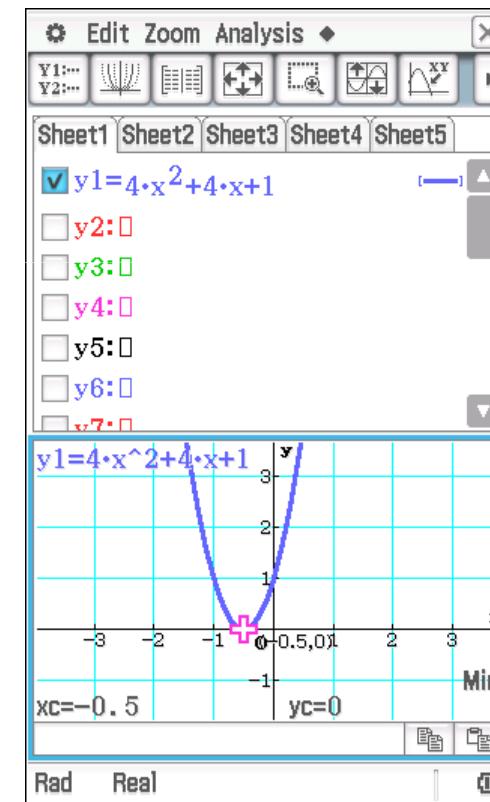
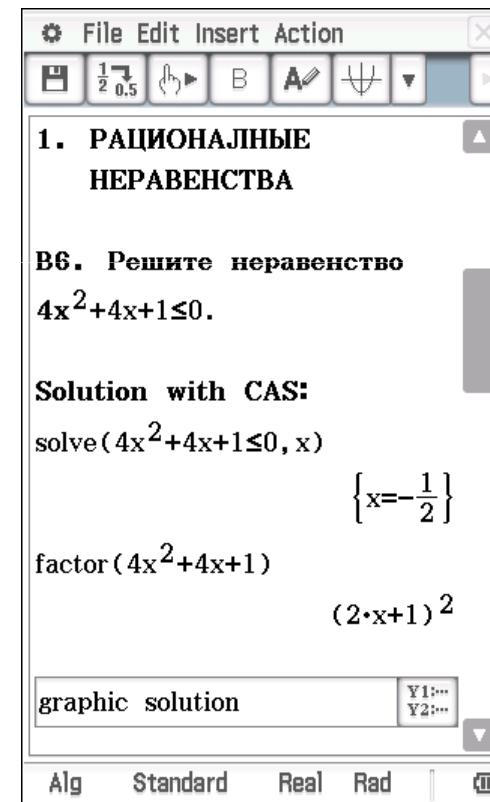
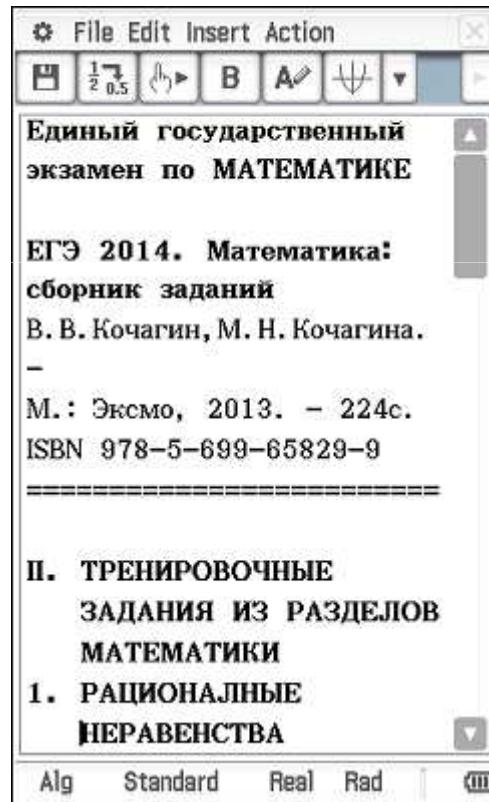
graphical: Integral intersect



ответ: $2+\frac{2}{3}$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА



eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА

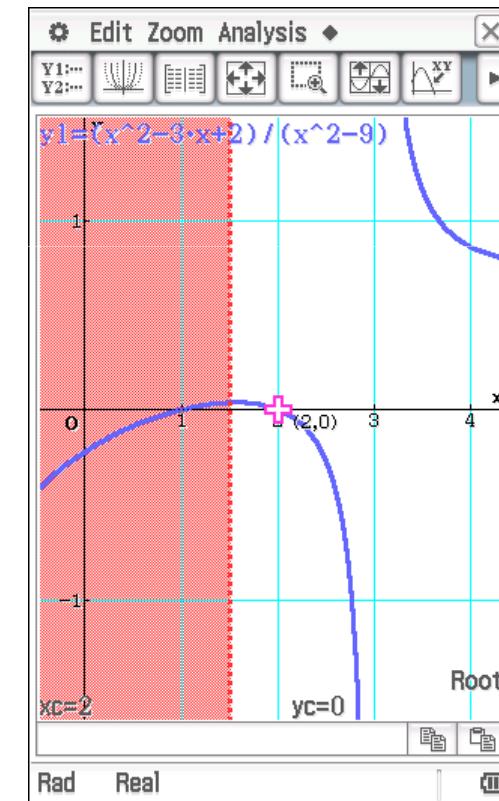
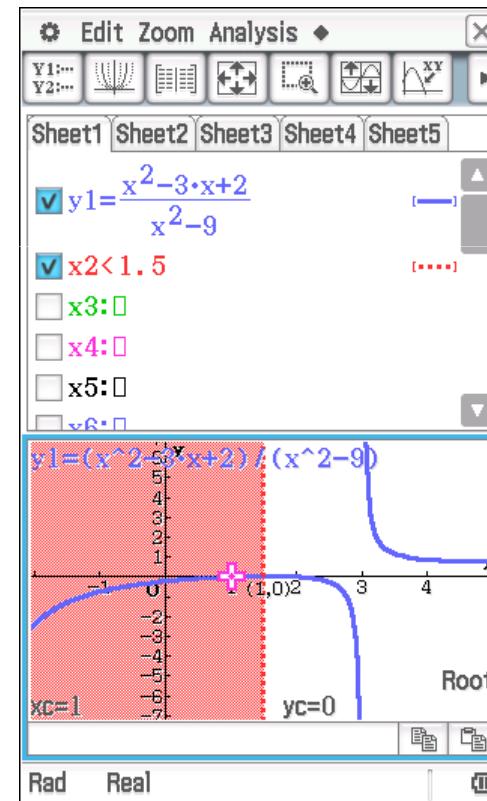
B38. Найдите корень уравнения $\frac{x^2-3x+2}{x^2-9}=0$, удовлетворяющий неравенству $-(5-2x) > -(6.5-3x)$.

Solution with CAS:

```
solve(-(5-2x) > -(6.5-3x), x)
{x < 3/2}
```

```
solve((x^2-3x+2=0, x)
{x=1, x=2}
```

```
solve((x^2-3x+2=0 | x < 3/2, x)
{x=1}
```



ответ: 1

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА

B39. Среди решений уравнения $\frac{2x-2}{x+3} + \frac{x+3}{x-3} = 5$ найдите те, которые не удовлетворяют неравенству $-x^2 - 7x + 8 < 0$.

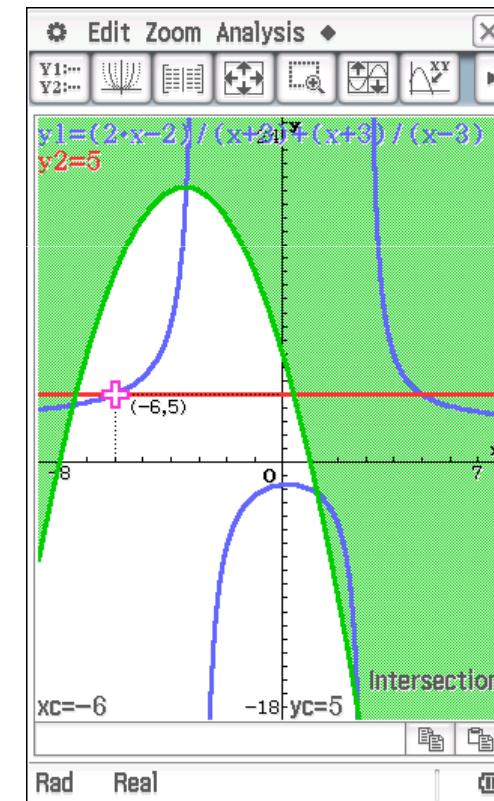
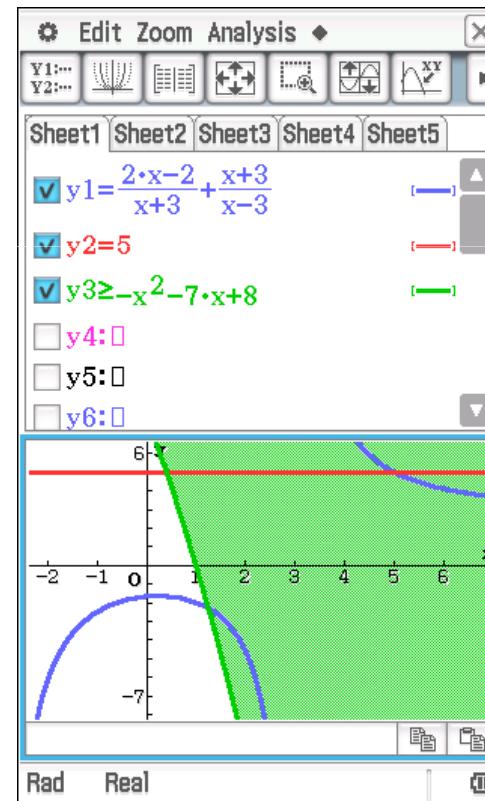
Solution with CAS:

```
solve((2x-2)/(x+3) + (x+3)/(x-3) = 5, x)
{x=-6, x=5}

judge(-x^2 - 7x + 8 < 0 | x=-6)
FALSE

judge(-x^2 - 7x + 8 < 0 | x=5)
TRUE
```

ответ: $x = -6$



ответ: -6

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА

C60. Решите неравенство

$$\frac{\sqrt{20+x-x^2}}{2x-3} \leq \frac{\sqrt{20+x-x^2}}{x-6}.$$

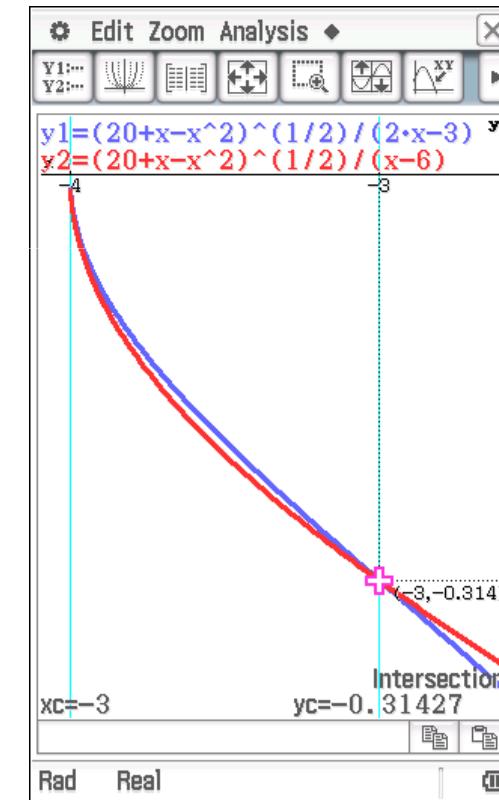
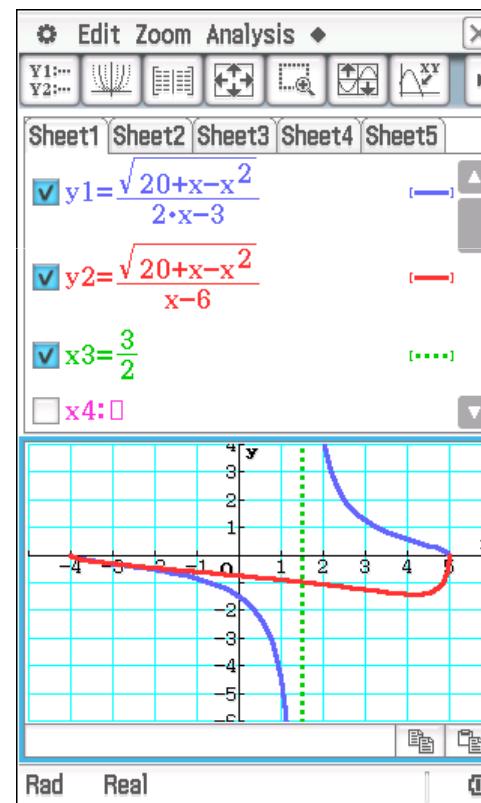
Solution with CAS:

```
solve(\frac{\sqrt{20+x-x^2}}{2x-3} \leq \frac{\sqrt{20+x-x^2}}{x-6}, x)
{x=-4, x=5, -3 ≤ x < 3/2}
```

```
solve(\sqrt{20+x-x^2}=0, x)
{x=-4, x=5}
```

```
solve(\frac{1}{2x-3} \leq \frac{1}{x-6}, x)
{-3 ≤ x < 3/2, 6 < x}
```

ответ: $x=-4, x=5, -3 \leq x < \frac{3}{2}$



ответ: $x=-4, x=5, -3 \leq x < \frac{3}{2}$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

1. РАЦИОНАЛНЫЕ НЕРАВЕНСТВА

C62. Решите неравенство

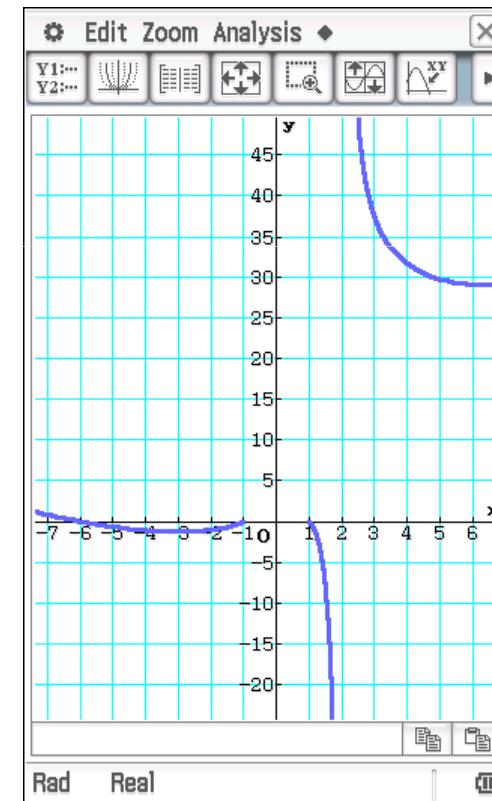
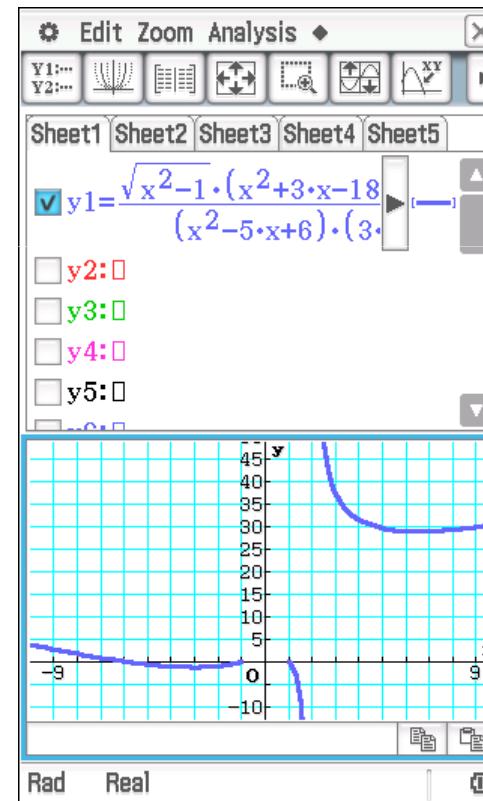
$$\sqrt{x^2-1} \cdot \frac{(x^2+3x-18)(4x^2-4x+1)}{(x^2-5x+6)(3x^2-8x+14)} \leq 0.$$

Solution with CAS:

```
solve(√(x^2-1) · (x^2+3x-18) · (4x^2-4x+1) / ((x^2-5x+6) · (3x^2-8x+14)), x)
```

ответ: $-6 \leq x \leq -1, 1 \leq x < 2$

graphical solution



ответ: $-6 \leq x \leq -1, 1 \leq x < 2$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

B45. Найдите наименьшее значение функции $y=\log_{\frac{1}{3}}(81-x^2)$.

Solution with CAS:

$$fMin(\log_{\frac{1}{3}}(81-x^2), x)$$

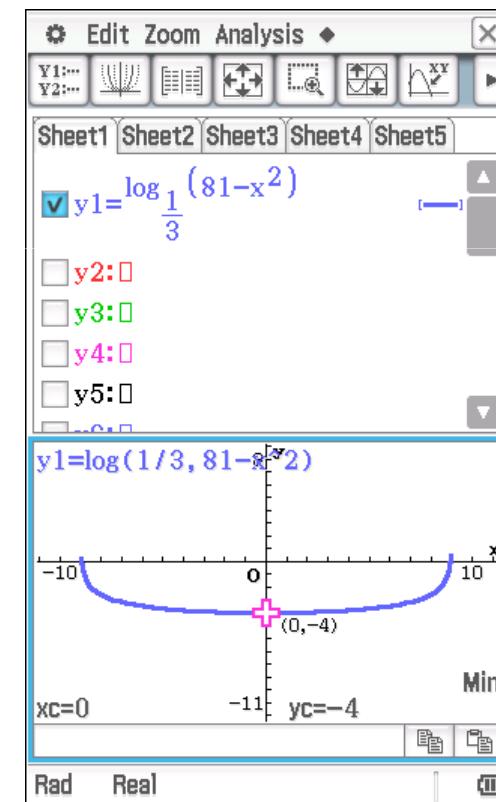
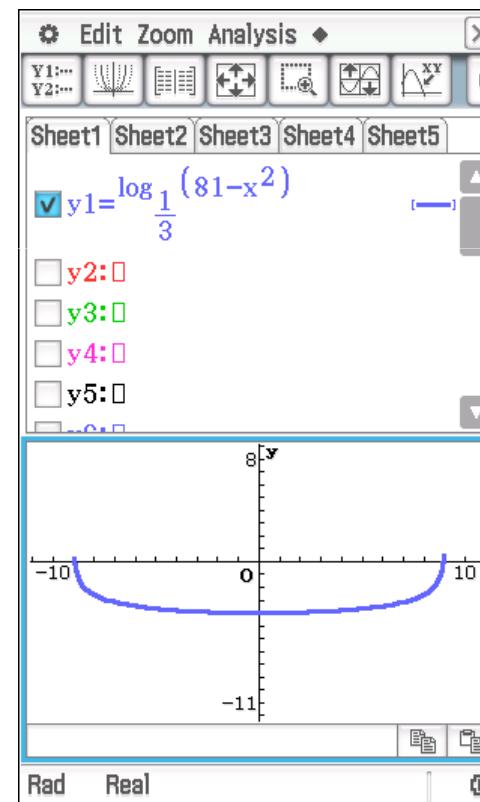
{MinValue=-4, x=0}

$$\log_{\frac{1}{3}}(81-x^2)$$

$$\frac{-\ln(-x^2+81)}{\ln(3)}$$

ответ: $y=-4$ for $x=0$

Alg Standard Real Rad



ответ: $y=-4$ for $x=0$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

C81. Найдите множество значений функции $y=\log_3(x-|x|+3)$.

Solution with CAS:

$$fMin(\log_3(x-|x|+3), x)$$

$$\left\{ \text{MinValue}=-\infty, x=-\frac{3}{2} \right\}$$

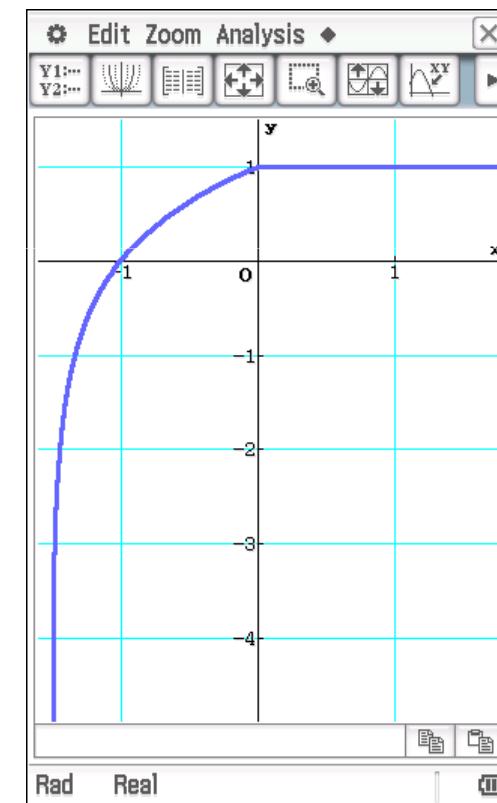
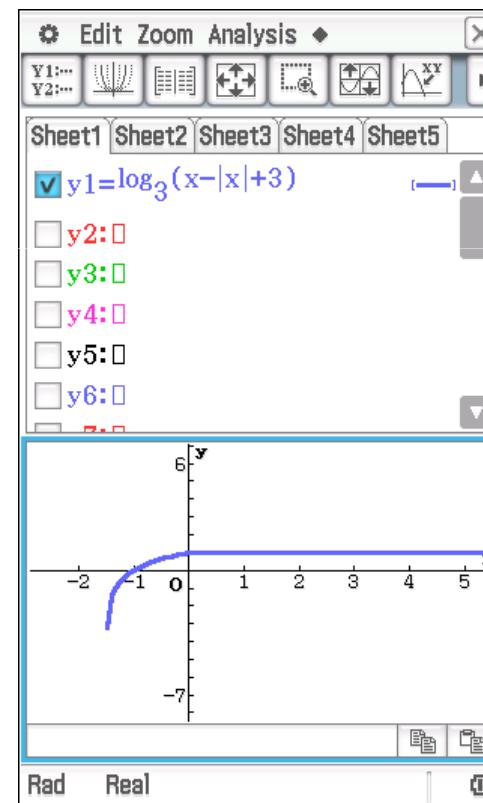
$$fMax(\log_3(x-|x|+3), x)$$

$$\left\{ \text{MaxValue}=1, x=1, x=\infty \right\}$$

ответ: $-\infty < x \leq 1$

graphical solution

Alg Standard Real Rad



ответ: $-\infty < x \leq 1$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

C82. Найдите множество значений функции

$$y = \left(\frac{1}{2}\right)^{\frac{12}{3-\sin(x)}}.$$

Solution with CAS:

$$fMin\left(\left(\frac{1}{2}\right)^{\frac{12}{3-\sin(x)}}, x\right)$$

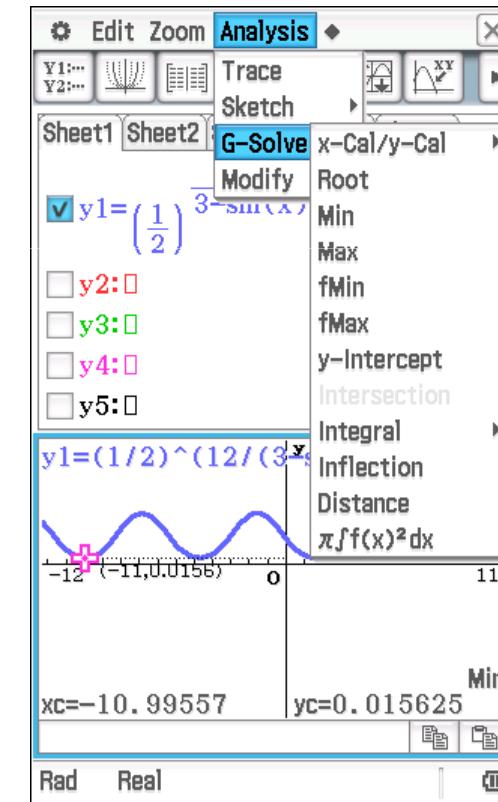
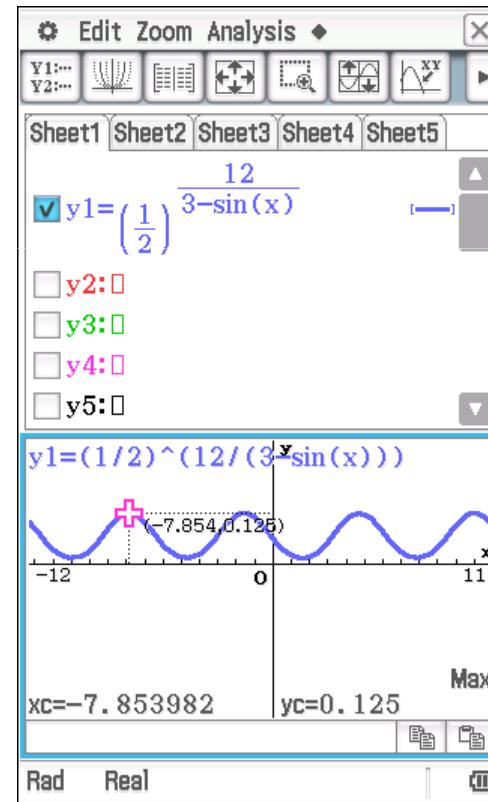
$$\left\{ \text{MinValue} = \frac{1}{64}, x = 2\pi \cdot \text{constn}(1) \right.$$

$$fMax\left(\left(\frac{1}{2}\right)^{\frac{12}{3-\sin(x)}}, x\right)$$

$$\left\{ \text{MaxValue} = \frac{1}{8}, x = 2\pi \cdot \text{constn}(1) \right.$$

graphical solution

Alg Standard Real Rad



ответ: $1/64 \leq x \leq 1/8$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

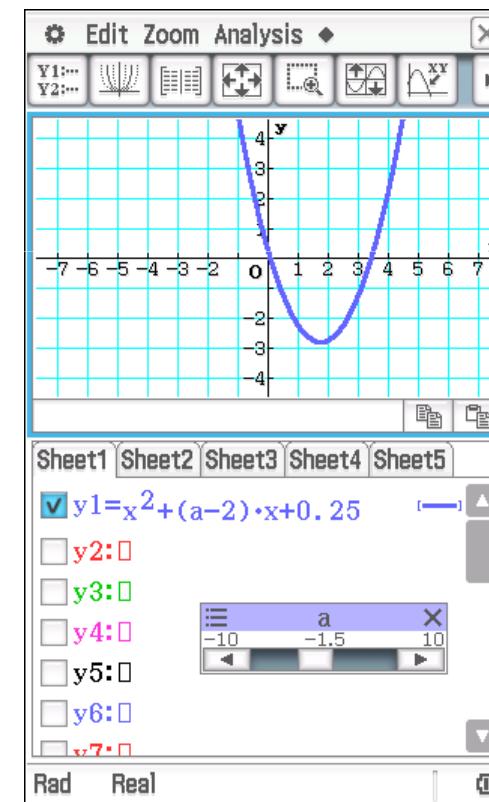
2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

C83. При каких значениях а функция $y=x^2+(a-2)x+\frac{1}{4}$ не принимает отрицательных значений?

Solution with CAS:

```
Define y(x)=x^2+(a-2)x+0.25
done
 $\frac{d}{dx}(y(x))=0$ 
 $2 \cdot x + a - 2 = 0$ 
solve(ans, x)
 $\left\{ x = \frac{-a}{2} + 1 \right\}$ 
 $y(x) | x = \frac{-a}{2} + 1$ 
 $\left( \frac{a}{2} - 1 \right)^2 - (a-2) \cdot \left( \frac{a}{2} - 1 \right) + \frac{1}{4}$ 
simplify(ans)
 $\frac{-(a-1) \cdot (a-3)}{4}$ 
solve(ans≥0, a)
 $\{1 \leq a \leq 3\}$ 
dynamic graphical solution
```

```
dx
 $2 \cdot x + a - 2 = 0$ 
solve(ans, x)
 $\left\{ x = \frac{-a}{2} + 1 \right\}$ 
 $y(x) | x = \frac{-a}{2} + 1$ 
 $\left( \frac{a}{2} - 1 \right)^2 - (a-2) \cdot \left( \frac{a}{2} - 1 \right) + \frac{1}{4}$ 
simplify(ans)
 $\frac{-(a-1) \cdot (a-3)}{4}$ 
solve(ans≥0, a)
 $\{1 \leq a \leq 3\}$ 
dynamic graphical solution
```



ответ: $1 \leq a \leq 3$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

2. ИССЛЕДОВАНИЕ ФУНКЦИЙ ЭЛЕМЕНТАРНЫМИ МЕТОДАМИ

C86. При каком значении а область определения функции

$$y = \sqrt[6]{-x^2 + 4x + a} + \sqrt{x - 3}$$

состоит из одной точки?

Solution with CAS:

$$\frac{d}{dx} (-x^2 + 4x + a) = 0$$

$$-2x + 4 = 0$$

$$\text{solve}(ans, x)$$

$$\{x=2\}$$

$$\frac{d^2}{dx^2} (-x^2 + 4x + a) < 0$$

$$-2 < 0$$

$$\text{Define } y1(x) = \sqrt[6]{-x^2 + 4x + a} + \sqrt{x - 3}$$

$$\text{done}$$

$$\text{solve}(-x^2 + 4x + a = 0 | x=3, a)$$

$$\{a=-3\}$$

ответ: $a = 3$

dynamic graphical solution

Alg Standard Real Rad

$\frac{d}{dx} (-x^2 + 4x + a) = 0$

$$-2x + 4 = 0$$

$$\text{solve}(ans, x)$$

$$\{x=2\}$$

$\frac{d^2}{dx^2} (-x^2 + 4x + a) < 0$

$$-2 < 0$$

Define $y1(x) = \sqrt[6]{-x^2 + 4x + a} + \sqrt{x - 3}$

done

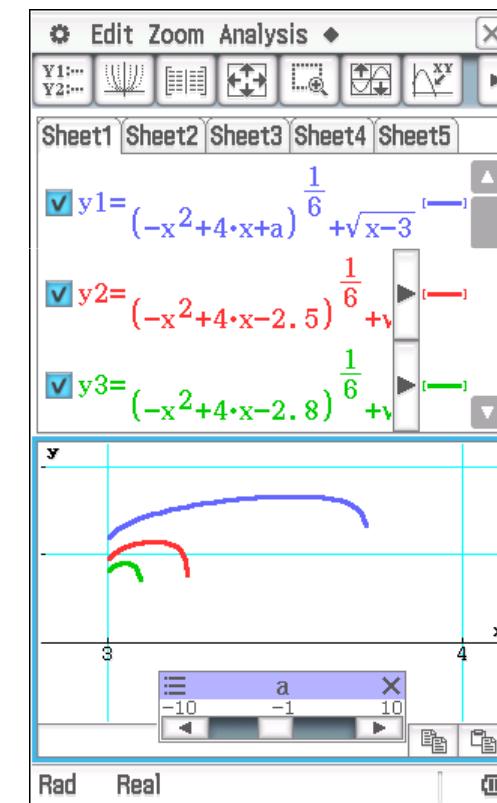
$\text{solve}(-x^2 + 4x + a = 0 | x=3, a)$

$$\{a=-3\}$$

ответ: $a = 3$

dynamic graphical solution

Alg Standard Real Rad



ответ: $a = -3$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

3. ТЕКСТОВЫЕ ЗАДАЧИ

File Edit Insert Action

1 2 0.5 B A $\frac{d}{dx}$

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II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

3. ТЕКСТОВЫЕ ЗАДАЧИ

Alg Standard Real Rad

File Edit Insert Action

1 2 0.5 B A $\frac{d}{dx}$

B32. Сумма двух чисел равна 1100. Найдите наибольшее из них, если 6% одного из них равны 5% другово.

Solution with CAS:

$$\begin{cases} x+y=1100 \\ 0.06x=0.05y \end{cases} \quad |_{x, y}$$

{x=500, y=600}

solve({x+y=1100, 0.06x=0.05y})

{x=500, y=600}

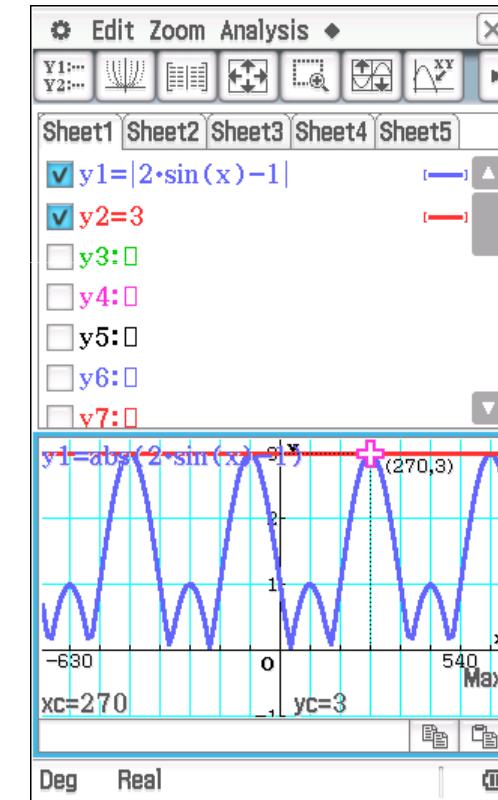
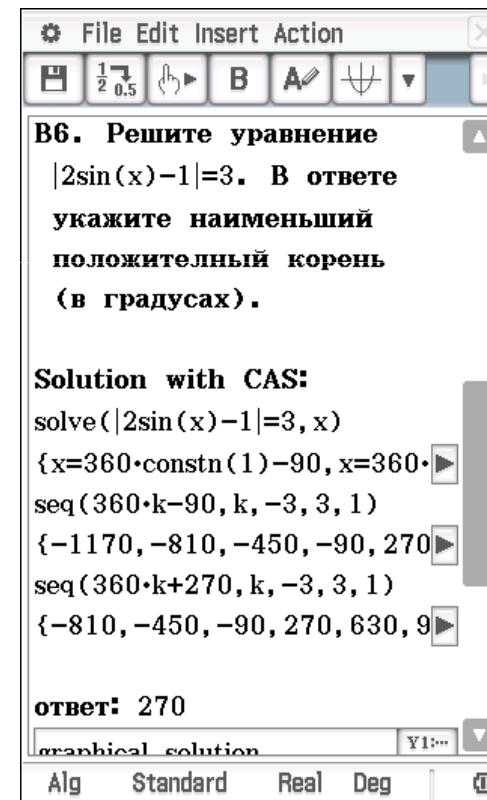
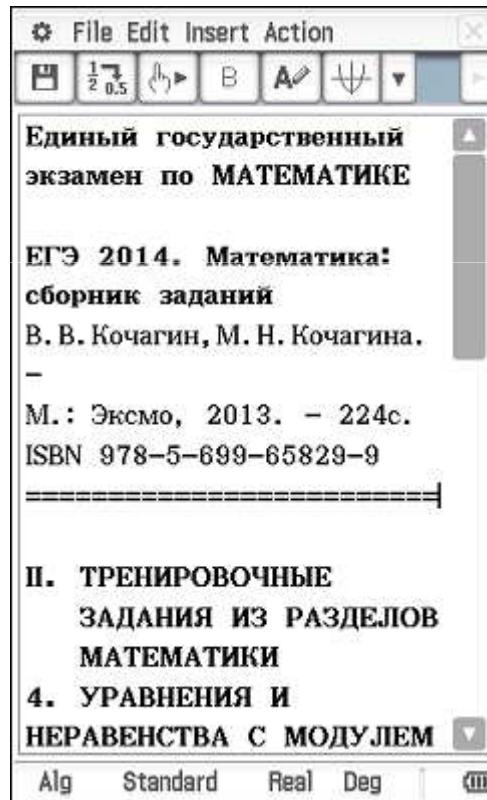
ответ: 600

Alg Standard Real Rad

ответ: 600

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ



ответ: 270°

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ

B25. Укажите середину промежутка, являющегося решением неравенства

$$\left(\frac{1}{2}\right)^{|4x+1|} \geq \frac{1}{8}.$$

Solution with CAS:

```
solve((1/2)^|4x+1| >= 1/8, x)
{-1 <= x <= -1/2}
```

$$\frac{-1+0.5}{2} = -\frac{1}{4}$$

ответ: -0.25

Sheet1

$y_1 = \left(\frac{1}{2}\right)^{|4x+1|}$

$y_2 \geq \frac{1}{8}$

$y_3 :=$

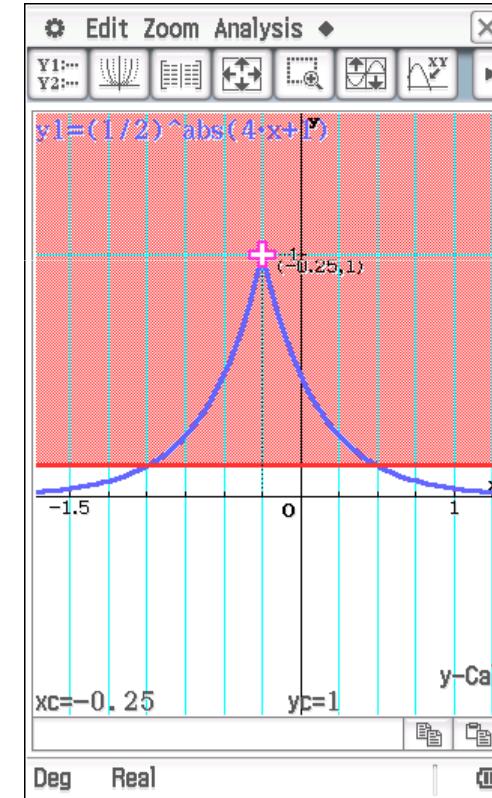
$y_4 :=$

$y_5 :=$

$y_1 = (1/2)^{\text{abs}(4 \cdot x + 1)}$

$x_c = -0.25$

$y_c = 1$



ответ: -0.25

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ

C71. Решите неравенство

$$|x+1| + |x+2| + |x-1| + |x-2| < 8x - 32.$$

Solution with CAS:

```
solve(|x+1| + |x+2| + |x-1| + |x-2| < 8x)
```

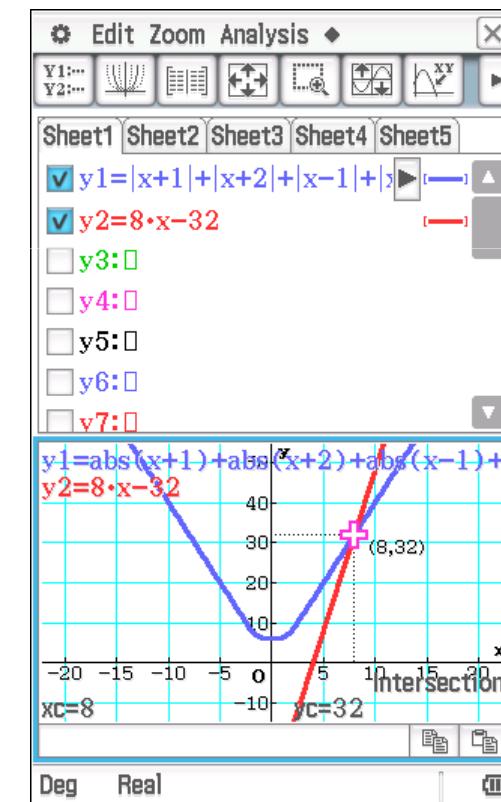
graphical solution

critical points: $x=-2, x=-1, x=1, x=2$

```
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
-4*x < 8*x - 32
solve(ans | x < -2, x)
```

No Solution

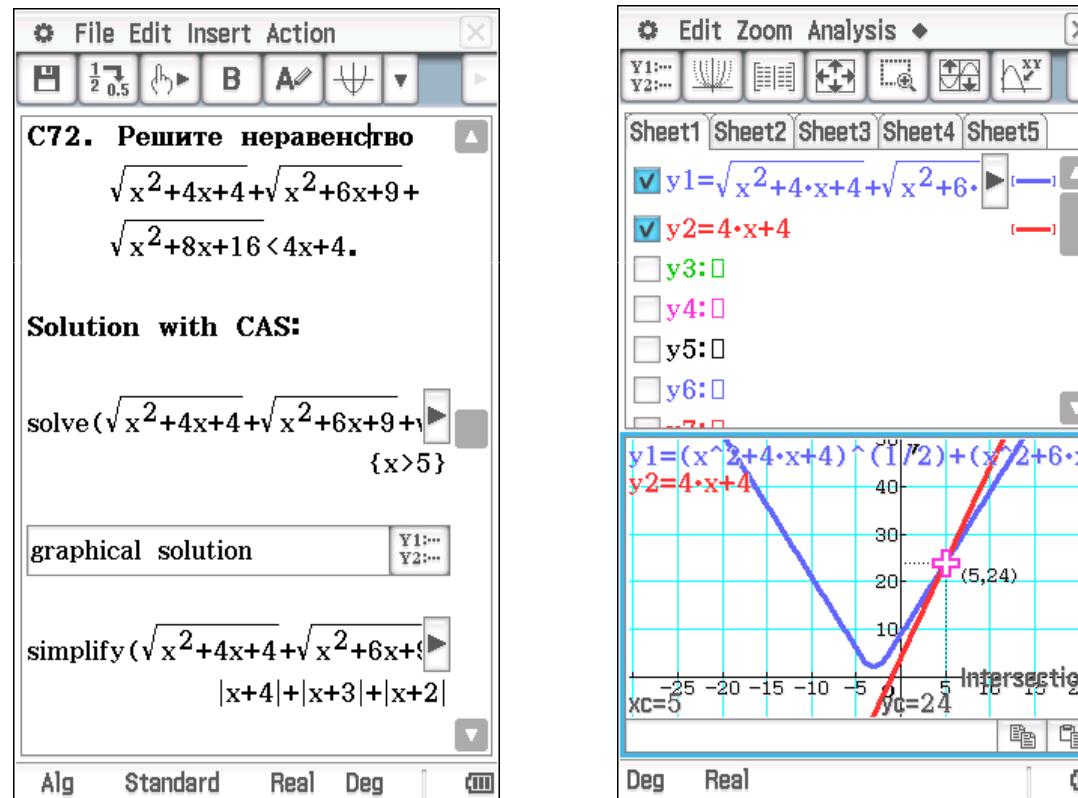
```
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
-2*x + 4 < 8*x - 32
solve(ans | x < -1 and x ≥ -2, x)
No Solution
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
6 < 8*x - 32
solve(ans | x < 1 and x ≥ -1, x)
No Solution
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
2*x + 4 < 8*x - 32
solve(ans | x < 2 and x ≥ 1, x)
No Solution
simplify(|x+1| + |x+2| + |x-1| + |x-2|)
4*x < 8*x - 32
solve(ans | x ≥ 2, x)
```



ответ: $x > 8$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ



ответ: $x > 5$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ

The figure consists of three side-by-side screenshots of the ClassPad calculator interface, illustrating the step-by-step solution of the inequality $|\log_3(x^2-16) + x - 5| \geq |\log_3(x^2-16)| + |x-5|$.

Screenshot 1: Shows the input of the inequality and a note about the triangle inequality $|a+b| \leq |a| + |b|$, leading to the conclusion that the inequality is only possible if $\log_3(x^2-16) + x - 5 = |\log_3(x^2-16)| + |x-5|$.

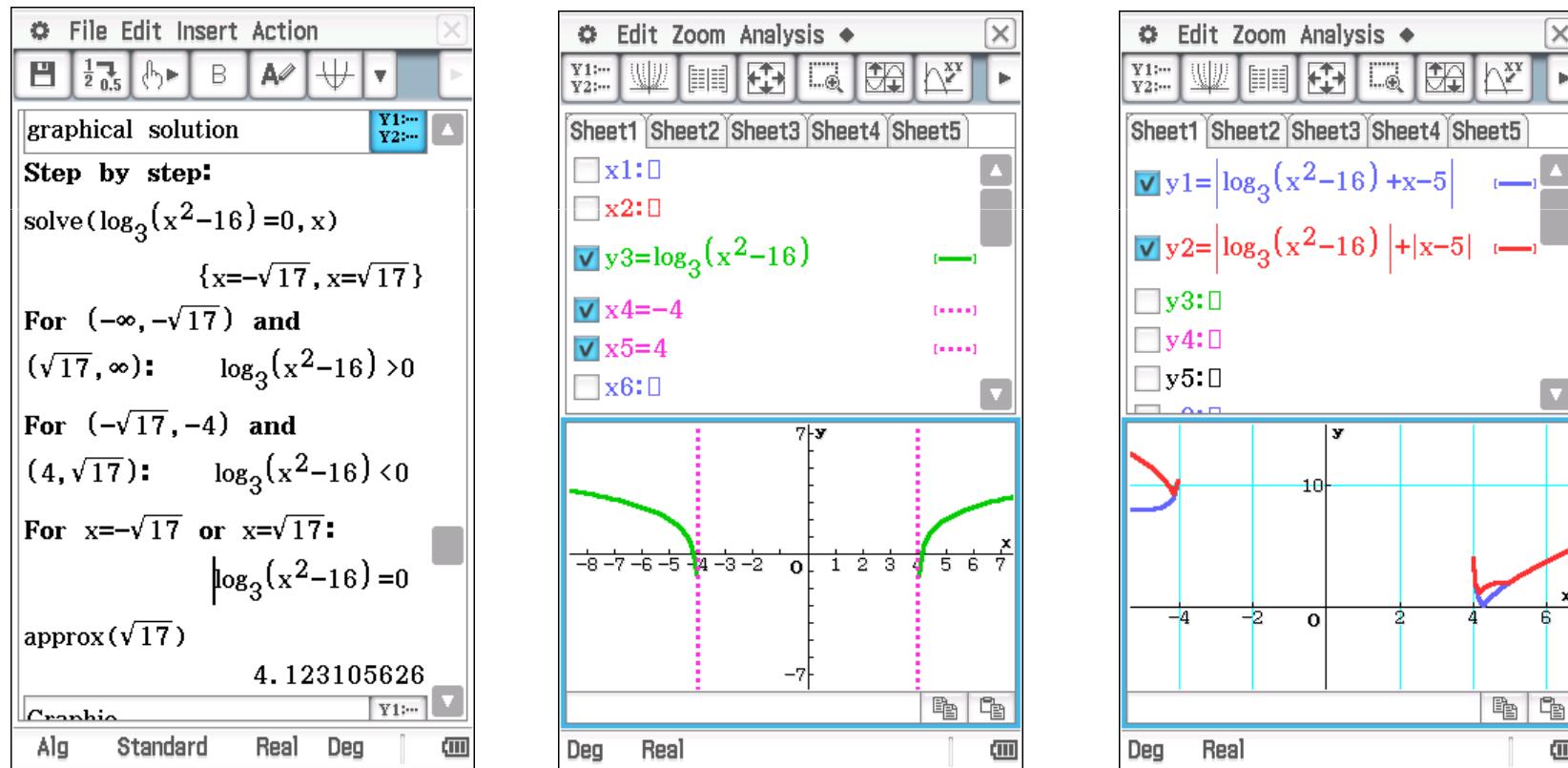
Screenshot 2: Shows the use of the solve function to find solutions. It displays the warning "CAS: WARNING! More solutions may exist" and lists two numerical solutions: $x=6$ and $x=6.006246748$. It also shows the command to solve for $(4, \sqrt{17})$ and the numerical solve command.

Screenshot 3: Shows the final result of the solve command, displaying the solution $x=4.1231056$ and the message "Left-Right=0". It also shows the equation again and the numerical values $x=4.12310562561766$, $\text{Lower}=4$, and $\text{Upper}=4.12310562561766$.

ответ: $x \in [-\sqrt{17}, -4] \cup (4, \sqrt{17}] \cup [5, \infty)$

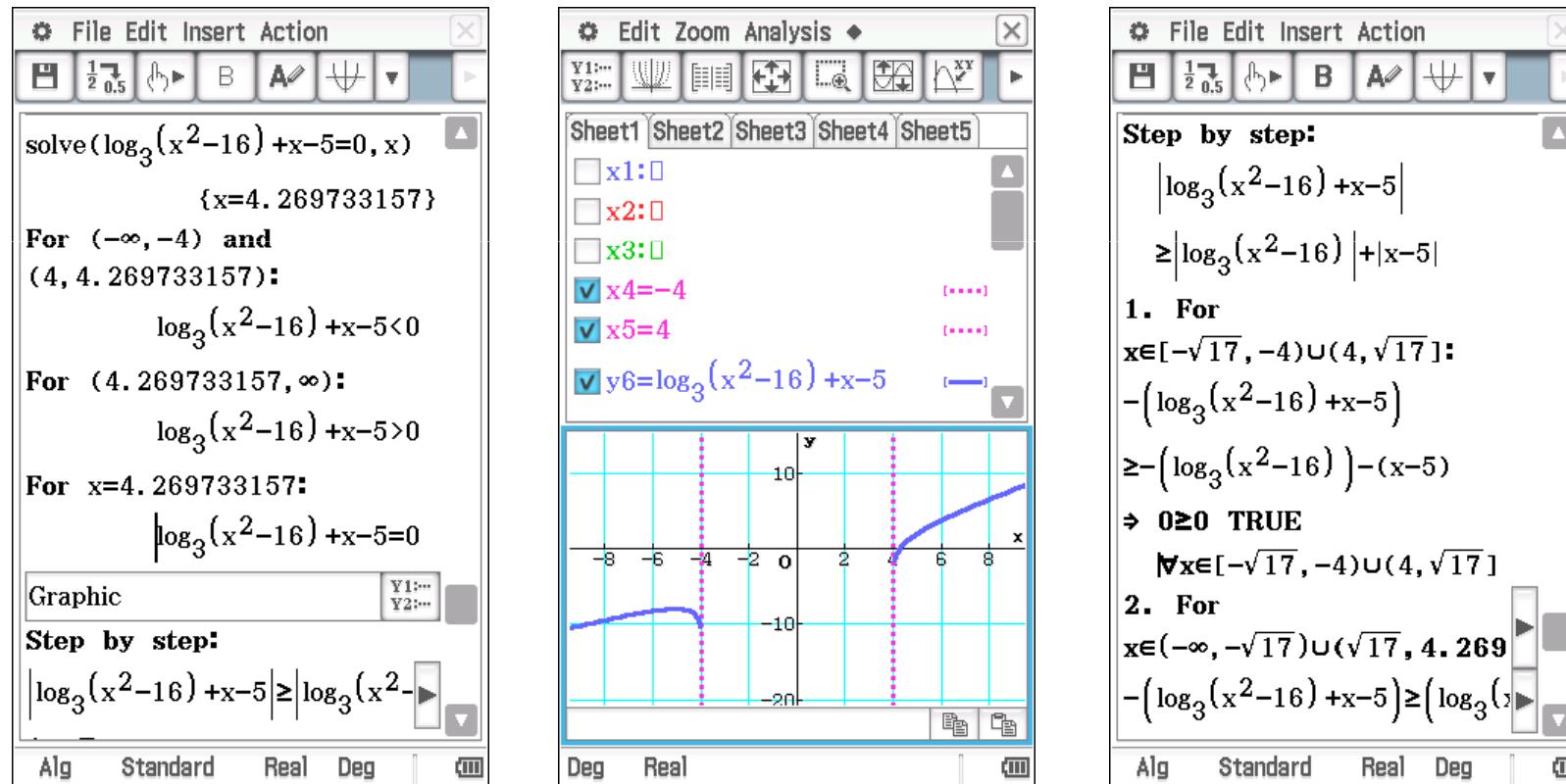
eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ



eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

4. УРАВНЕНИЯ И НЕРАВЕНСТВА С МОДУЛЕМ



ответ: $x \in [-\sqrt{17}, -4] \cup (4, \sqrt{17}] \cup [5, \infty)$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

The screenshot shows a ClassPad calculator interface with a menu bar (File, Edit, Insert, Action) and toolbars for text, equations, and graphs. The main window displays a step-by-step solution for the inequality $|\log_3(x^2-16)| + x - 5 \geq |\log_3(x^2-16)| + |x-5|$.

The steps are as follows:

- For $x \in [-\sqrt{17}, -4) \cup (4, \sqrt{17}]$:
 $-(\log_3(x^2-16)) + x - 5 \geq -(\log_3(x^2-16)) - (x-5)$
 $\Rightarrow 0 \geq 0$ TRUE
 $\forall x \in [-\sqrt{17}, -4) \cup (4, \sqrt{17}]$
- For $x \in (-\infty, -\sqrt{17}) \cup (\sqrt{17}, 4.269733157]$:
 $-(\log_3(x^2-16)) + x - 5 \geq (\log_3(x^2-16)) - (x-5)$
 $\Rightarrow -2\log_3(x^2-16) \geq 0$
 $\text{solve}(-2\log_3(x^2-16) \geq 0, x)$
 $\Rightarrow \text{no solution}$
- For $x \in (4.269733157, 5)$:
 $(\log_3(x^2-16)) + x - 5 \geq (\log_3(x^2-16)) - (x-5)$
 $\Rightarrow 2x - 10 \geq 0 \Rightarrow x \geq 5$ no solution
- For $x \in [5, \infty)$:
 $(\log_3(x^2-16)) + x - 5 \geq (\log_3(x^2-16)) + (x-5)$
 $\Rightarrow 0 \geq 0$ TRUE $\forall x \in [5, \infty)$

Ответ: $x \in [-\sqrt{17}, -4) \cup (4, \sqrt{17}] \cup [5, \infty)$

At the bottom, there are buttons for Alg, Standard, Real, and Deg.

ответ: $x \in [-\sqrt{17}, -4) \cup (4, \sqrt{17}] \cup [5, \infty)$

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

File Edit Insert Action

1 2 0.5 ↗ ↘ B A ↗ ↘

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II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

Alg Standard Real Deg

File Edit Insert Action

1 2 0.5 ↗ ↘ B A ↗ ↘

1. В 11 классе 30 человек.
18 человек изучают английский язык,
16 – немецкий,
9 – оба языка.

Сколько человек изучают:
а) только английский язык,
б) только немецкий язык,
в) не изучают ни одного языка?

Solution with CAS:

Alg Standard Real Deg

File Edit Insert Action

1 2 0.5 ↗ ↘ B A ↗ ↘

```
seq(x, x, 1, 30, 1)⇒Ω
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}▶
seq(x, x, 1, 18, 1)⇒A
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}▶
seq(x, x, 10, 25, 1)⇒B
{10, 11, 12, 13, 14, 15, 16, 17}▶
[[dim(Ω), dim(A), dim(B)]]
[30 18 16]
```

Set theory: Intersection A∩B
A∩B

No computation of A∩B –
only in text mode possibel.
Students wrote a programm
for set theory.
(see another vcf-file)

Alg Standard Real Deg

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

The image shows two side-by-side screenshots of the ClassPad software interface.

Left Screenshot:

- Top menu: File, Edit, Insert, Action.
- Toolbar: Includes icons for fraction, decimal, percentage, and various mathematical operations.
- Text area: "4-fields-table"

| | | |
|-----------|------------------|------------------------|
| | B | \bar{B} |
| A | $A \cap B$ | $A \cap \bar{B}$ |
| \bar{A} | $\bar{A} \cap B$ | $\bar{A} \cap \bar{B}$ |
| Ω | | |
- Text area: "given:"
- Table:

| | | |
|-----------|----|-----------|
| | B | \bar{B} |
| A | 9 | 18 |
| \bar{A} | 16 | 30 |
- Text area: "fill the table:"
- Table:

| | | | |
|-----------|---|-----------|----|
| | B | \bar{B} | |
| A | 9 | 9 | 18 |
| \bar{A} | 7 | 16 | 30 |
- Bottom menu: Alg, Standard, Real, Deg, Unit.

Right Screenshot:

- Top menu: File, Edit, View, Draw.
- Toolbar: Includes icons for text, pen, triangle, square, circle, and other drawing tools.
- Text area: "Venn-diagram"

| | |
|------------------|--|
| A 9 9 18 | |
| \bar{A} 7 5 | |
| 16 30 | |
| B \bar{B} | |
| A 9 9 18 | |
| \bar{A} 7 5 12 | |
| 16 14 30 | |
- Venn diagram: A rectangle labeled Ω contains two overlapping circles labeled A and B. Circle A has value 9, circle B has value 7, and their intersection has value 9.

ответ: а)9 б)7 в)5

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

File Edit Insert Action

31. Три стрелка независимо друг от друга стреляют по мишени.

Вероятность попадания первого стрелка равна 0,7; второго – 0,6; третьего – 0,4.

Какова вероятность:
 а) хотя бы одного попадания,
 б) трех попаданий, если каждый сделал по одному выстрелу.

Alg Standard Real Rad

File Edit Insert Action

Solution: $P(A)=0.7$,
 $P(B)=0.6$, $P(C)=0.4$

a)
 $P(A \cup B \cup C) = 1 - P(\bar{A} \cap \bar{B} \cap \bar{C}) =$
 $1 - 0.3 \cdot 0.4 \cdot 0.6$

approx(ans) $\frac{116}{125}$
 0.928

b)
 $P(A \cap B \cap C) = P(A)P(B)P(C) =$
 $0.7 \cdot 0.6 \cdot 0.4$

approx(ans) $\frac{21}{125}$
 0.168

Alg Standard Real Rad

File Edit View Draw

$P(A \cap B \cap C) = P(A)P(B)P(C) =$
 $0.7 \cdot 0.6 \cdot 0.4$

$\frac{21}{125}$
 approx(ans) 0.168

Venn-diagram

ответ: а)0.928 б)0.168

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ – Venn-diagram with 4 sets $A, B, C, D \subset \Omega$

The figure consists of three side-by-side screenshots of the ClassPad software interface, showing the process of creating and analyzing a Venn diagram with four sets.

Left Window: Displays the input code for generating four sets A, B, C, and D from the universal set Ω (100 elements). The code uses the seq command to generate sequences for each set and then converts them to sets using the expToStr command.

```

Venn-diagram with 4 sets
A, B, C, D ⊂ Ω
seq(x, x, 0, 100, 1) ⇒ Ω
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...}
seq(x, x, 0, 100, 2) ⇒ A
{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, ...}
seq(x, x, 0, 100, 3) ⇒ B
{0, 3, 6, 9, 12, 15, 18, 21, 24, ...}
seq(x, x, 0, 100, 5) ⇒ C
{0, 5, 10, 15, 20, 25, 30, 35, 40, ...}
seq(x, x, 0, 100, 7) ⇒ D
{0, 7, 14, 21, 28, 35, 42, 49, 56, ...}

```

Middle Window: Shows the execution of commands to convert sets A, B, C, and D into strings and then pass them to the StrOVenn command. The output shows the sets A, B, C, and D as strings.

```

seq(x, x, 0, 100, 7) ⇒ D
{49, 56, 63, 70, 77, 84, 91, 98}
expToStr Ω, Ω
done
expToStr A, A
done
expToStr B, B
done
expToStr C, C
done
expToStr D, D
done
StrOVenn(Ω, A, B, C, D, 4, 2, 1)
done
stop

```

Right Window: Displays the resulting Venn diagram with four overlapping circles labeled A, B, C, and D. The counts of elements in the various regions are shown: A only (23), B only (12), C only (6), D only (22), A ∩ B (11), A ∩ C (3), A ∩ D (4), B ∩ C (3), B ∩ D (2), C ∩ D (1), and A ∩ B ∩ C ∩ D (1).

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ – Venn-diagram with 4 sets $A, B, C, D \subset \Omega$

• Edit
a=...
b=...

A \ (B \cup C \cup D), B \ (A \cup C \cup D), C \ (A \cup B \cup D), D \ (A \cup B \cup C), $\Omega \setminus$ (A \cup B \cup C \cup D)
{2, 4, 8, 16, 22, 26, 32, 34, 38, 44, 46, 52, 58, 62, 64, 68, 74, 76, 82, 86, 88, 92, 94}
{3, 9, 27, 33, 39, 51, 57, 69, 81, 87, 93, 99}
{5, 25, 55, 65, 85, 95}
{7, 49, 77, 91}
{1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97}
AnB, AnC, AnD, BnC, BnD, CnD, AnBnC, AnBnD, AnCnD, BnCnD, AnBnCnD
{0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96}
{0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100}
{0, 14, 28, 42, 56, 70, 84, 98}
{0, 15, 30, 45, 60, 75, 90}
{0, 21, 42, 63, 84}
{0, 35, 70}
{0, 30, 60, 90}
{0, 42, 84}
{0, 70}
{0}
{0}
(AnB) \ (C \cup D), (AnC) \ (B \cup D), (AnD) \ (B \cup C), (BnC) \ (A \cup D), (BnD) \ (A \cup C), (CnD) \ (A \cup B)
{6, 12, 18, 24, 36, 48, 54, 66, 72, 78, 96}
{10, 20, 40, 50, 80, 100}
{14, 28, 56, 98}
{15, 45, 75}
{21, 63}
{35}
(AnBnC) \ D, (AnBnD) \ C, (AnCnD) \ B, (BnCnD) \ A
{30, 60, 90}
{42, 84}
{70}
Ø

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ – Compute $\Omega \setminus (A \cup B \cup C \cup D)$ with 4 sets $A, B, C, D \subset \Omega$

```

File Edit Insert Action
[File] [Edit] [Insert] [Action]
[Stop] [1/2] [0.5] [Back] [Forward] [B] [A] [Graph] [Down]

StrOVenn(Ω, A, B, C, D, 4, 2, 1)
done

stop
Compute Ω \ (A ∪ B ∪ C ∪ D):
Menge(A, "∪", B)
done

Menge(Result, "∪", C)
done

Menge(Result, "∪", D)
done

Menge(Ω, "-", Result)
done

Result
" {1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97} "
Ω \ (A ∪ B ∪ C ∪ D) contains 1 and all prime numbers (except 2, 3, 5, 7)

Alg Standard Real Deg

```

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

File Edit Insert Action

35. Рассмотрите ряд чисел:
40, 90, 50, 20, 20, 50.

Найдите:

- размах,
- среднее арифметическое,
- медиану,
- моду

Solution with CAS:

```
{40, 90, 50, 20, 20, 50}⇒list
{40, 90, 50, 20, 20, 50}
dim(list)
6
```

Alg Standard Real Rad

File Edit Insert Action

размах (range)
max(list) – min(list)

среднее арифметическое (mean)
mean(list)

sortA(list)
{20, 20, 40, 50, 50, 90}

медиан (median)
median(list)

мода (mode)
mode(list)

OneVariable list

Alg Standard Real Rad

File Edit Insert Action

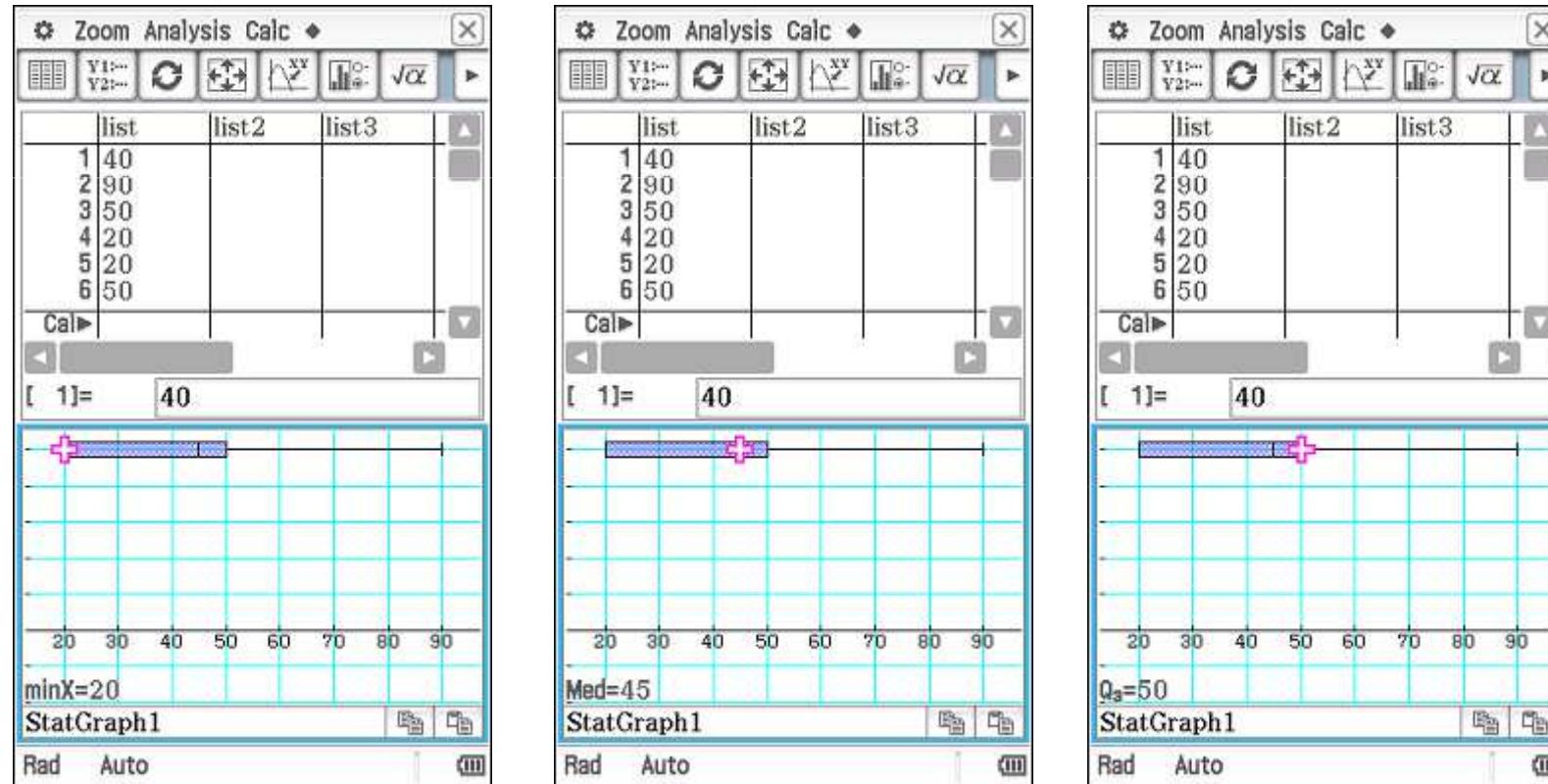
OneVariable

$\bar{x} = 45$
 $\Sigma x = 270$
 $\Sigma x^2 = 15500$
 $\sigma_x = 23.629078$
 $s_x = 25.884358$
 $n = 6$
 $\min X = 20$
 $Q_1 = 20$
 $Med = 45$
 $Q_3 = 50$
 $\max X = 90$
 $Mode = 20$
 $Mode = 50$
 $ModeN = 2$
 $ModeF = 2$

Alg Standard Real Rad

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ (Boxplot)



eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

File Edit Insert Action

40. В таблице записаны результаты ежедневного измерения температуры воздуха (в градусах Цельсия) на метеостанции в полдень в первую декаде февраля.

| Число месяца | 1 | 2 |
|-----------------|----|----|
| Температура, °C | -8 | -6 |

Найдите среднюю температуру в полдень в эту декаду.

Alg Standard Real Rad

File Edit Insert Action

40. В таблице записаны результаты ежедневного измерения температуры воздуха (в градусах Цельсия) на метеостанции в полдень в первую декаде февраля.

| | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|----|---|----|----|----|---|---|----|
| — | -2 | 0 | -2 | -2 | -1 | 1 | 0 | 0 |

Найдите среднюю температуру в полдень в эту декаду.

Alg Standard Real Rad

File Edit Insert Action

Solution with CAS:

```
trn([-8 -6 -2 0 -2 -2 -1])
```

$$\begin{bmatrix} -8 \\ -6 \\ -2 \\ 0 \\ -2 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

```
matToList(ans, 1) → list
{-8, -6, -2, 0, -2, -2, -1, 1, 0}
```

```
mean(ans)
```

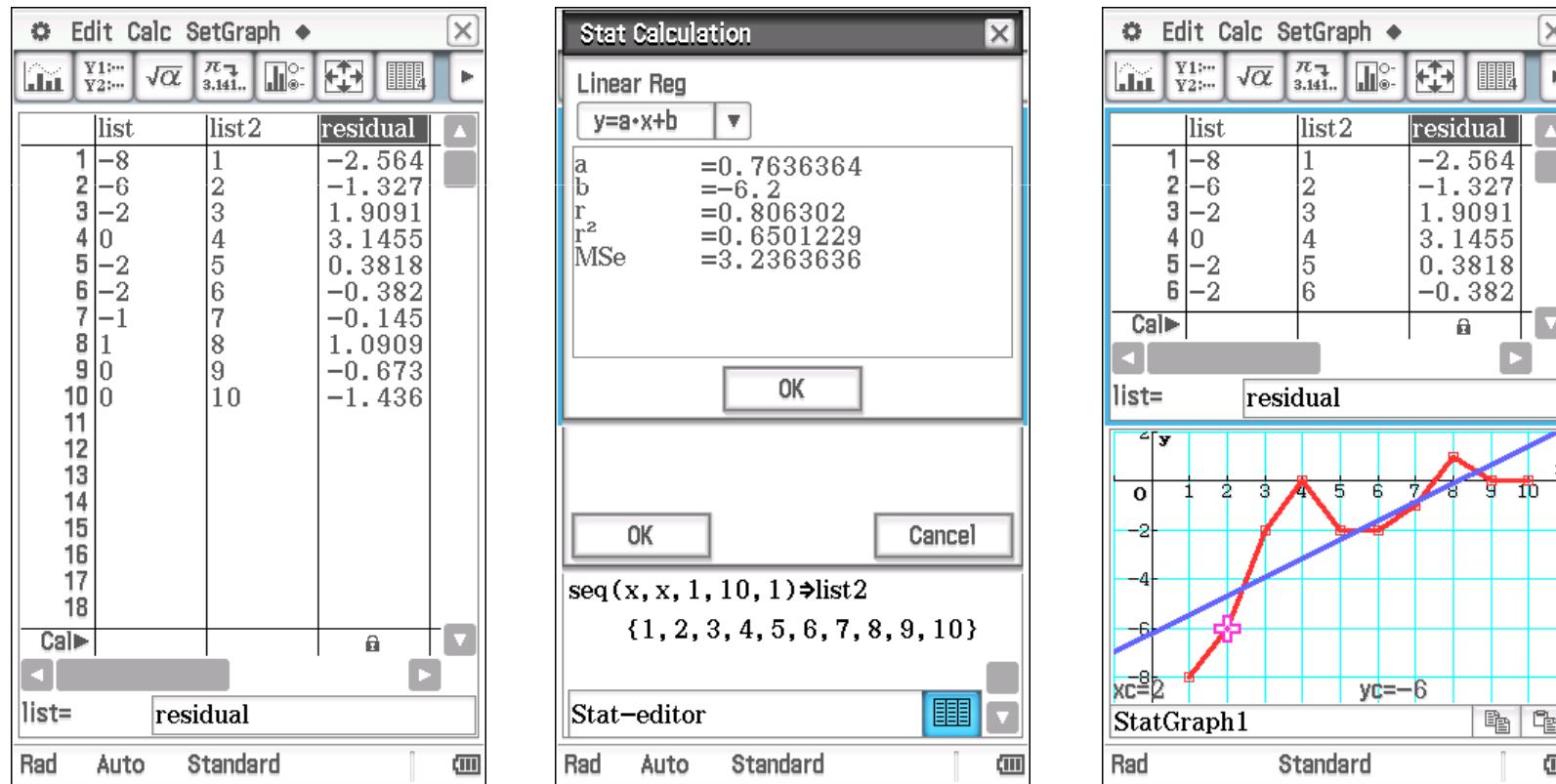
-2

Alg Standard Real Rad

ответ: -2°

eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ



eActivity: II. ТРЕНИРОВОЧНЫЕ ЗАДАНИЯ ИЗ РАЗДЕЛОВ МАТЕМАТИКИ

5. ЭЛЕМЕНТЫ КОМБИНАТОРИКИ, ТЕОРИИ ВЕРОЯТНОСТЕЙ И СТАТИСТИКИ

Method of least squares:
Define $y(x) = a \cdot x + b$
done
Define $F(a, b) = \sum_{k=1}^{10} ((y(k) - list[k])^2)$
done
 $\frac{d}{da}(F(a, b)) = 0 \Rightarrow Equ1$
 $770 \cdot a + 110 \cdot b + 94 = 0$
 $\frac{d}{db}(F(a, b)) = 0 \Rightarrow Equ2$
 $110 \cdot a + 20 \cdot b + 40 = 0$
 $\begin{cases} Equ1 \\ Equ2 \end{cases} \Big|_{a, b}$
 $\left\{ a = \frac{42}{55}, b = -\frac{31}{5} \right\}$

Method of least squares:
Define $y(x) = a \cdot x + b$
done
 $\frac{d}{da}(F(a, b)) = 0 \Rightarrow Equ1$
 $770 \cdot a + 110 \cdot b + 94 = 0$
 $\frac{d}{db}(F(a, b)) = 0 \Rightarrow Equ2$
 $110 \cdot a + 20 \cdot b + 40 = 0$
 $\begin{cases} Equ1 \\ Equ2 \end{cases} \Big|_{a, b}$
 $\left\{ a = \frac{42}{55}, b = -\frac{31}{5} \right\}$

Define $F(a, b) = \sum_{k=1}^{10} ((y(k) - list[k])^2)$
done
 $\frac{d}{da}(F(a, b)) = 0 \Rightarrow Equ1$
 $770 \cdot a + 110 \cdot b + 94 = 0$
 $\frac{d}{db}(F(a, b)) = 0 \Rightarrow Equ2$
 $110 \cdot a + 20 \cdot b + 40 = 0$
 $\begin{cases} Equ1 \\ Equ2 \end{cases} \Big|_{a, b}$
 $\left\{ a = \frac{42}{55}, b = -\frac{31}{5} \right\}$
approx(ans)
 $\{a = 0.7636363636, b = -6.2\}$

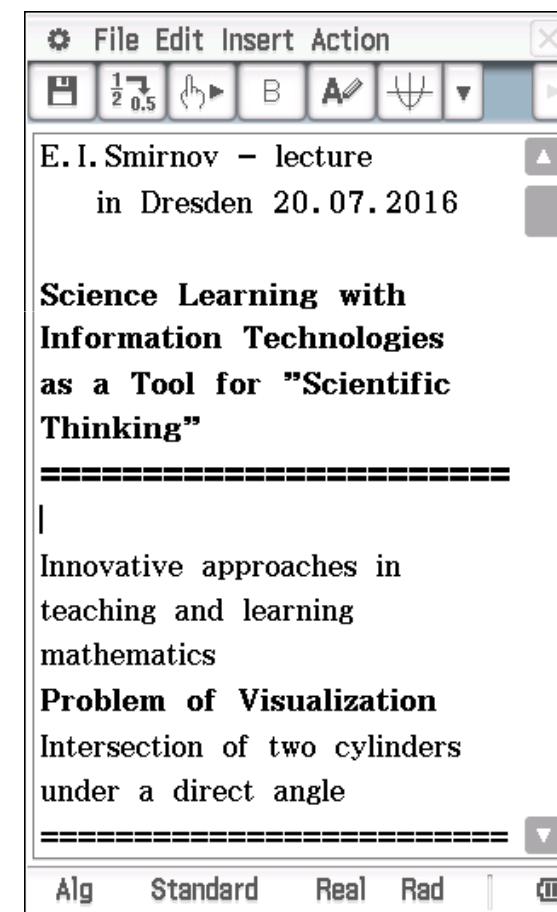
eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

**Innovative approaches in teaching
and learning mathematics**

Problem of Visualization

**Intersection of two cylinders
under a direct angle**



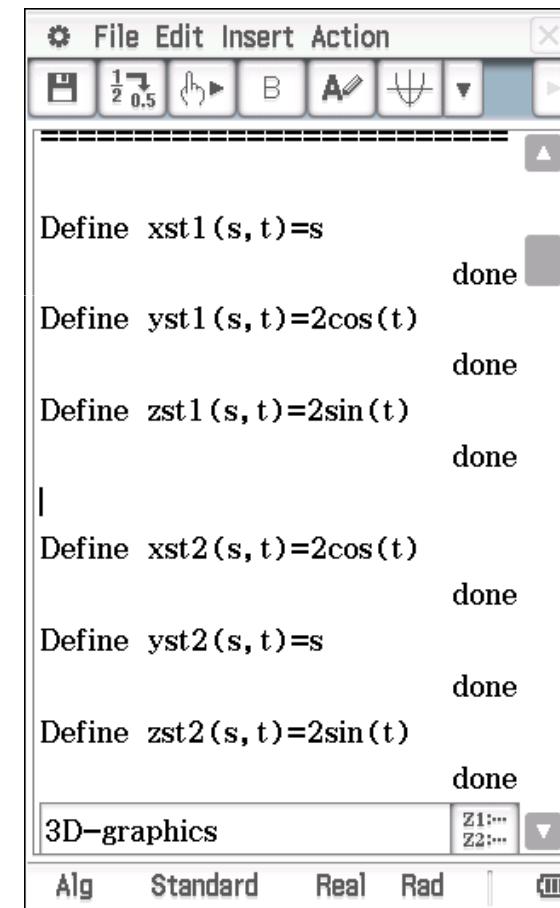
eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

**Innovative approaches in teaching
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Problem of Visualization

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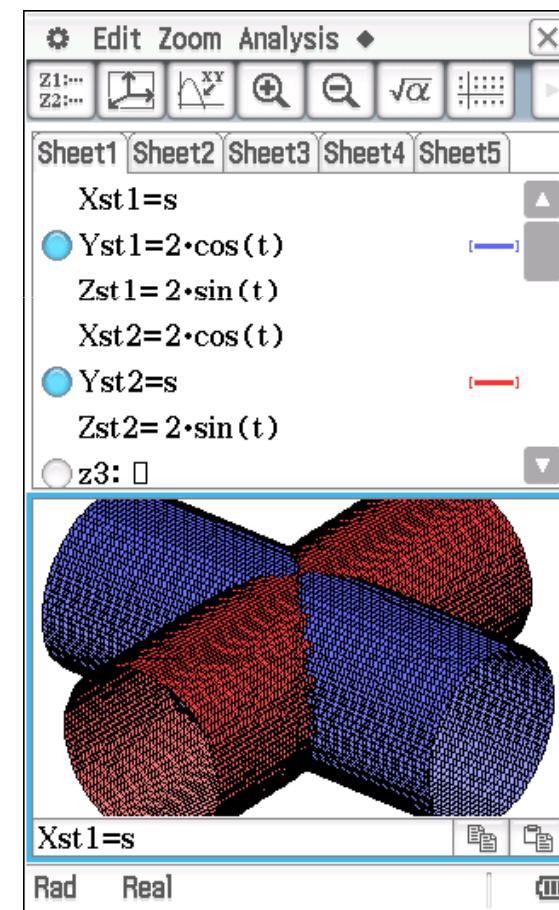
eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

**Innovative approaches in teaching
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File Edit Insert Action

Rotation matrix (z-axis):

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ with } \gamma = \frac{\pi}{6}$$

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2\cos(t) \\ s \\ 2\sin(t) \end{bmatrix} \mid \gamma = \frac{\pi}{6}$$

$$\begin{bmatrix} \frac{-s + \sqrt{3} \cdot \cos(t)}{2} \\ \frac{\sqrt{3} \cdot s + \cos(t)}{2} \\ 2 \cdot \sin(t) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 \\ 2s \\ 0 \end{bmatrix} \mid \gamma = \frac{\pi}{6}$$

$$\begin{bmatrix} -s \\ \frac{s}{\sqrt{3}} \\ 0 \end{bmatrix}$$

Alg Standard Real Rad

File Edit Insert Action

Rotation matrix (z-axis):

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ with } \gamma = \frac{\pi}{6}$$

$$\begin{bmatrix} -\sin(\gamma) & 0 \\ \cos(\gamma) & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} 2\cos(t) \\ s \\ 2\sin(t) \end{bmatrix} \mid \gamma = \frac{\pi}{6}$$

$$\begin{bmatrix} \frac{-s + \sqrt{3} \cdot \cos(t)}{2} \\ \frac{\sqrt{3} \cdot s + \cos(t)}{2} \\ 2 \cdot \sin(t) \end{bmatrix}$$

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 0 \\ 2s \\ 0 \end{bmatrix} \mid \gamma = \frac{\pi}{6}$$

$$\begin{bmatrix} -s \\ \frac{s}{\sqrt{3}} \\ 0 \end{bmatrix}$$

Alg Standard Real Rad

File Edit Insert Action

Define $xst3(s, t) = \frac{-s + \sqrt{3} \cdot \cos(t)}{2}$ done

Define $yst3(s, t) = \frac{\sqrt{3} \cdot s + \cos(t)}{2}$ done

Define $zst3(s, t) = 2 \cdot \sin(t)$ done

Define $xst4(s, t) = -s$ done

Define $yst4(s, t) = \sqrt{3} \cdot s$ done

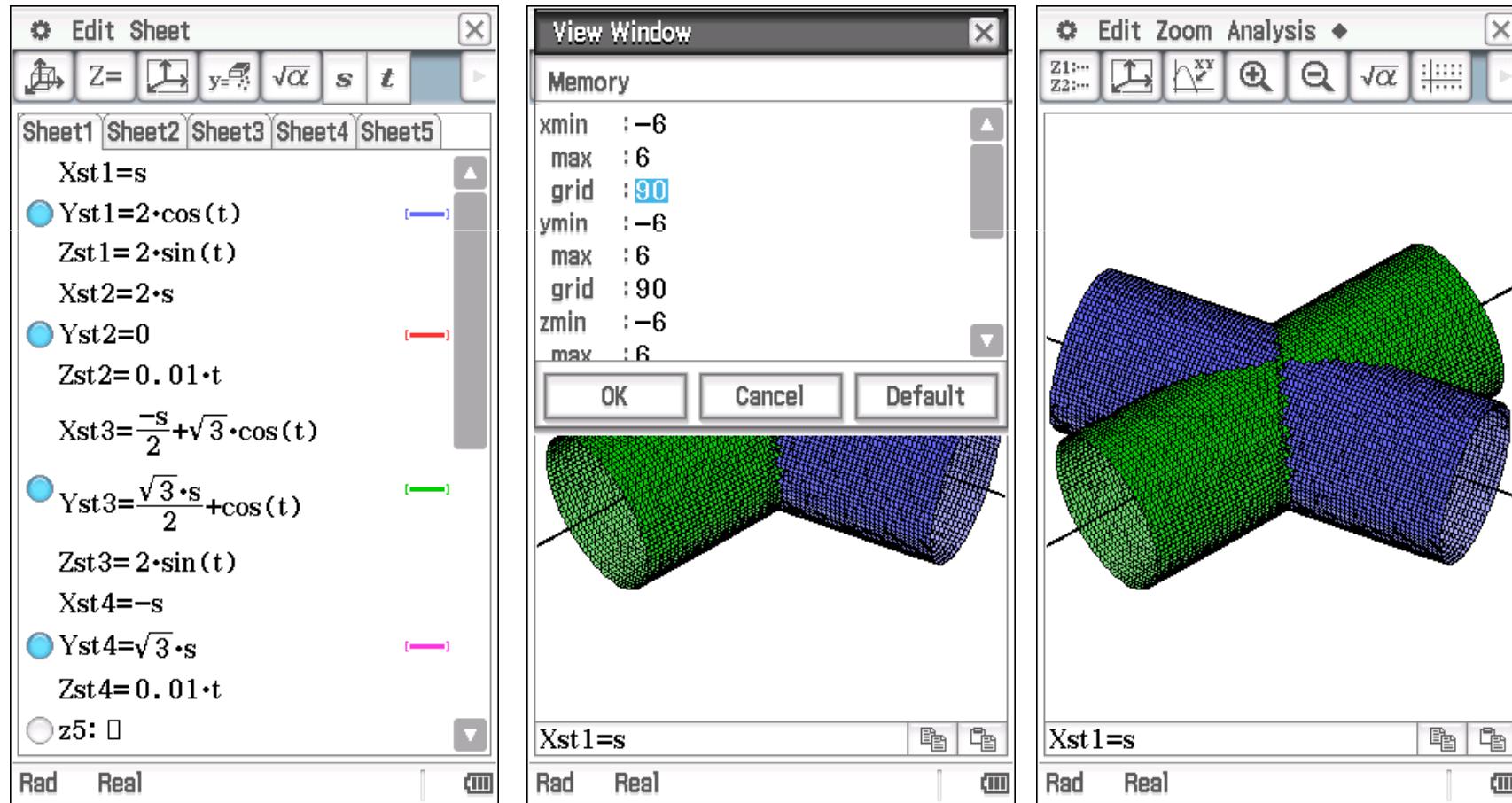
Define $zst4(s, t) = 0.01t$ done

3D-graphics with rotation Z1... Z2...

Alg Standard Real Rad

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File Edit Insert Action

Recognition problem

To find the volume of intersection of the two cylinders
 $r=0$ (no rotation)

Define $xst1(s, t) = s$
done

Define $yst1(s, t) = 2\cos(t)$
done

Define $zst1(s, t) = 2\sin(t)$
done

Define $xst2(s, t) = 2\cos(t)$
done

Define $yst2(s, t) = s$
done

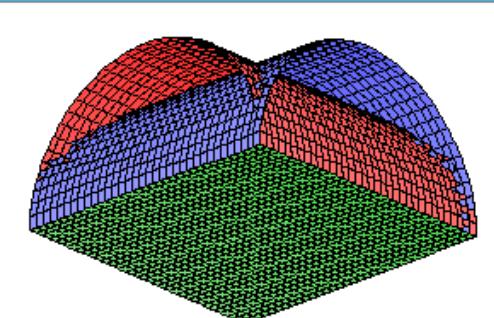
Define $zst2(s, t) = 2\sin(t)$
done

Alg Standard Real Rad

Edit Zoom Analysis

Sheet1 Sheet2 Sheet3 Sheet4 Sheet5

Xst1=s
Yst1=2·cos(t)
Zst1=2·sin(t)
Xst2=2·cos(t)
Yst2=s
Zst2=2·sin(t)
z3=0



Xst1=s

Rad Real

File Edit Insert Action

Define $z3(x, y) = 0$
done

3D-graphics without rota

radius R=2:

$V := 8 * \int_0^2 \int_{-x}^x \int_0^{\sqrt{4-x^2}} 1 dz dy dx$

$\frac{128}{3}$

$V := 8 * \int_0^R \int_{-x}^x \int_0^{\sqrt{R^2-x^2}} 1 dz dy dx$

$\frac{16 \cdot R^3}{3}$

Alg Standard Real Rad

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

The image displays three windows from the ClassPad software:

- Left Window (Calculator View):**

Rotation matrix (z-axis):

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ with angle } \gamma$$

$$\begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2\cos(t) \\ s \\ 2\sin(t) \end{bmatrix}$$

$$\begin{bmatrix} -s\cdot\sin(\gamma)+2\cdot\cos(t)\cdot\cos(\gamma) \\ s\cdot\cos(\gamma)+2\cdot\cos(t)\cdot\sin(\gamma) \\ 2\cdot\sin(t) \end{bmatrix}$$

ans | $\gamma = -\frac{\pi}{6}$

$$\begin{bmatrix} \frac{s}{2}+\sqrt{3}\cdot\cos(t) \\ \frac{\sqrt{3}\cdot s}{2}-\cos(t) \end{bmatrix}$$
- Middle Window (Edit Sheet):**

Sheet1 Sheet2 Sheet3 Sheet4 Sheet5

 - Xst1=s
 - Yst1=2·cos(t)
 - Zst1=2·sin(t)
 - Xst2=2·s
 - Yst2=0
 - Zst2=0.01·t
 - Xst3= $\frac{s}{2}+\sqrt{3}\cdot\cos(t)$
 - Yst3= $\frac{\sqrt{3}\cdot s}{2}-\cos(t)$
 - Zst3=2·sin(t)
 - Xst4=s
 - Yst4= $\sqrt{3}\cdot s$
 - Zst4=0.01·t
 - z5: □
- Right Window (View Window):**

Memory

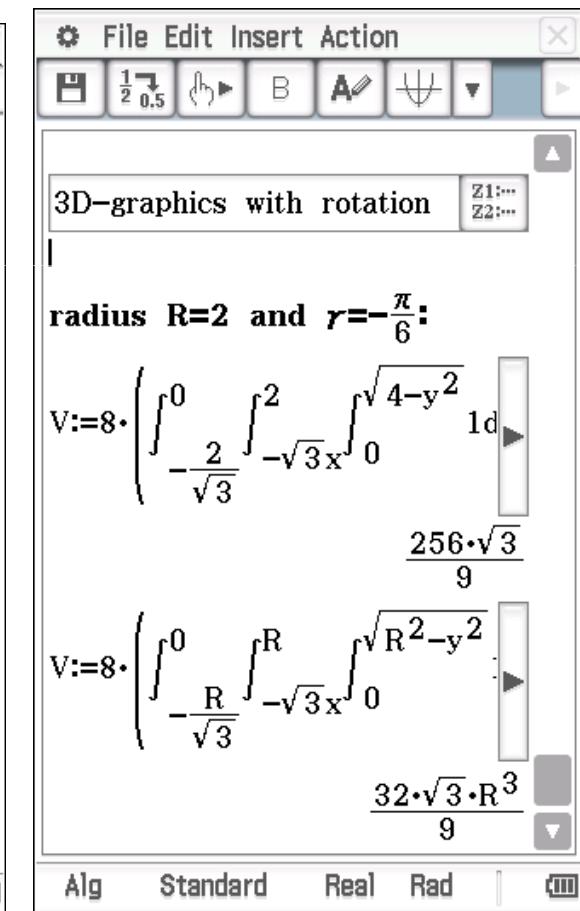
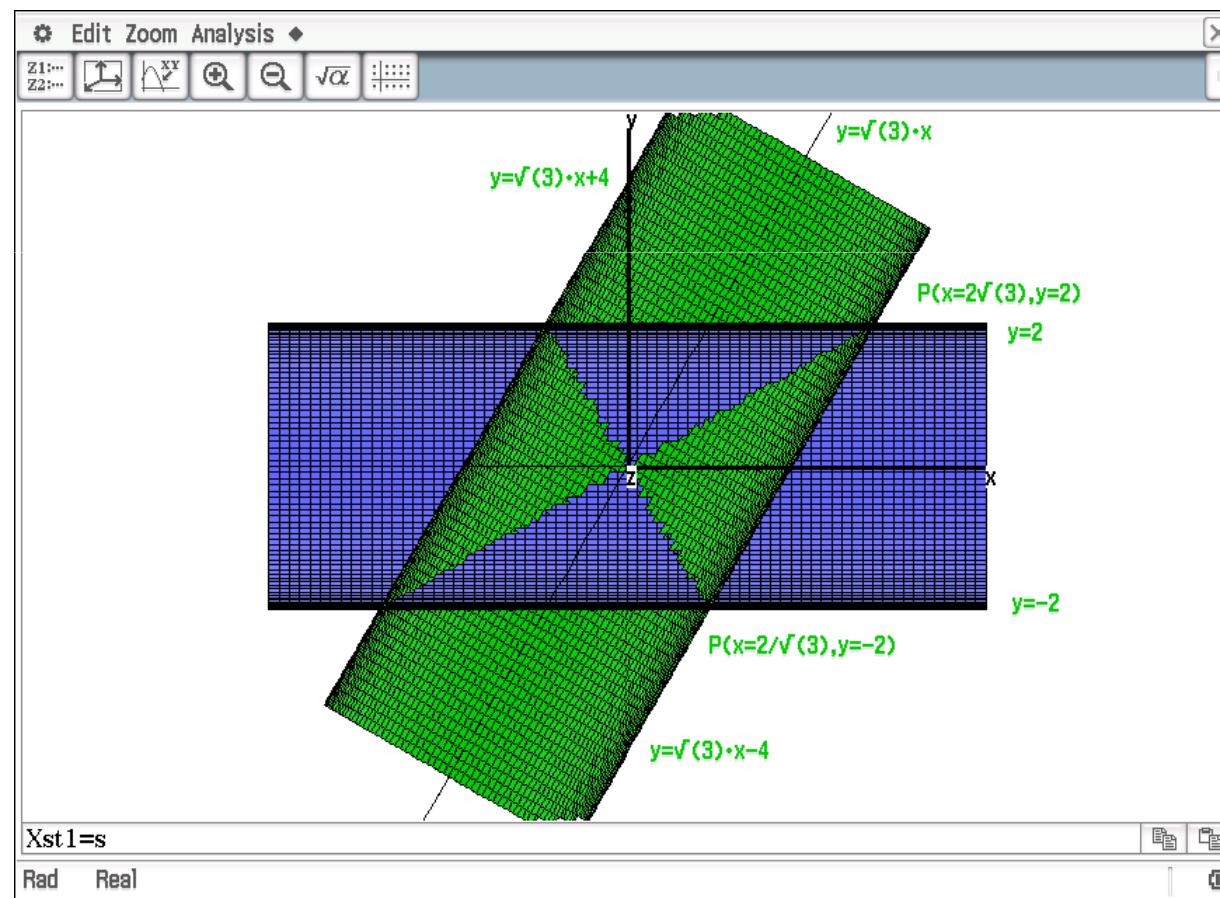
 - zmin : -5
 - max : 5
 - angle θ : -90
 - angle ϕ : 0
 - smin : -5
 - max : 5
 - tmin : 0
 - max : π

OK Cancel Default

A 3D plot showing a surface with a grid, colored in green and blue.

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"



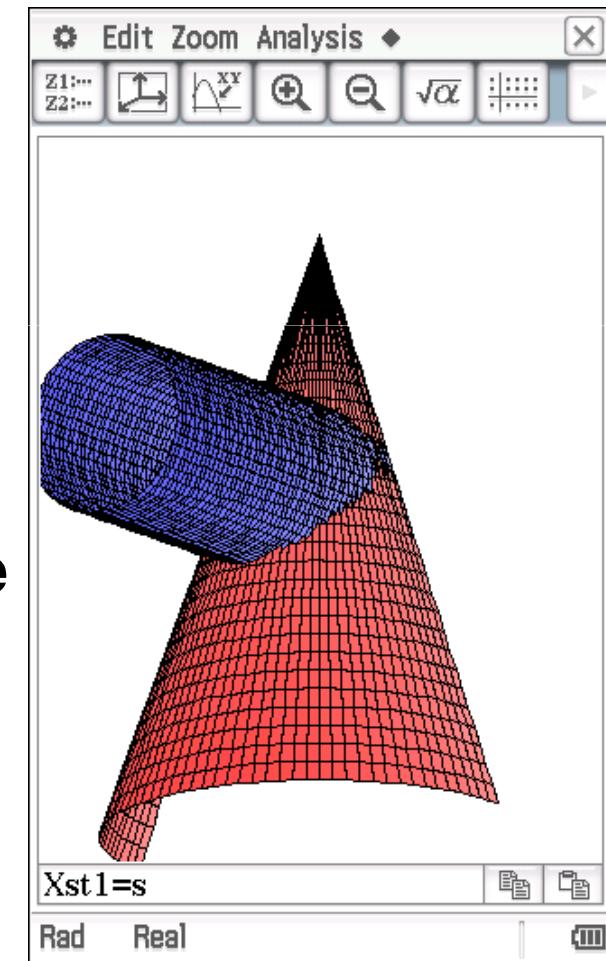
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**Science Learning with Information Technologies
as a Tool for "Scientific Thinking"**

**Innovative approaches in teaching
and learning mathematics**

Problem of Visualization

Intersection of a cylinder and a cone



eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

E. I. Smirnov – lecture
in Dresden 20.07.2016

Science Learning with
Information Technologies
as a Tool for "Scientific
Thinking"

Innovative approaches in
teaching and learning
mathematics

Problem of Visualization
Intersection of a cylinder and a
cone

Alg Standard Real Rad

solve($\sqrt{1-x^2} = -c*x+3, x$)

$$\left\{ x = \frac{3 \cdot c - \sqrt{c^2 - 8}}{c^2 + 1}, x = \frac{3 \cdot c + \sqrt{c^2 - 8}}{c^2 + 1} \right.$$

solve($c^2 - 8, c$)

$$\{ c = -2\sqrt{2}, c = 2\sqrt{2} \}$$

Define xst1(s, t) = s
done

Define yst1(s, t) = cos(t)
done

Define zst1(s, t) = sin(t)
done

Define xst2(s, t) = s * cos(t/2)
done

Define vst2(s, t) = s * sin(t/2)

3D-graphics Z1:... Z2:...

R=1 (cylinder)

V := $2 \cdot \int_{-1}^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_0^{\sqrt{\left(\frac{z-3}{2\sqrt{2}}\right)^2 - y^2}} 1 dx dz dy$

$$5.762207211$$

Alg Standard Real Rad

$$V := 2 \cdot \int_{-1}^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_0^{\sqrt{\left(\frac{z-3}{2\sqrt{2}}\right)^2 - y^2}} 1 dx dz dy$$

$$5.762207211$$

$$\int_0^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_0^{\sqrt{\left(\frac{z-3}{2\sqrt{2}}\right)^2 - y^2}} 1 dx dz dy$$

$$\sqrt{-2 \cdot (8 \cdot y^2 - z^2 + 6 \cdot z - 9)}$$

Alg Standard Real Rad

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

The left window shows the 'Edit Sheet' interface with the following content:

- Sheet1 (selected)
- Sheet2
- Sheet3
- Sheet4
- Sheet5

Definitions:

- $Yst1 = \cos(t)$
- $Zst1 = \sin(t)$
- $Xst2 = s \cdot \cos\left(\frac{t}{2}\right)$
- $Yst2 = s \cdot \sin\left(\frac{t}{2}\right)$
- $Zst2 = 3 - 2 \cdot \sqrt{2} \cdot |s|$

At the bottom, there is a 3D plot of a cylinder.

The right window shows the 'File Edit Insert Action' interface with the following content:

R=1 (cylinder)

$$V := 2 \cdot \int_{-1}^1 \left(\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \left(\int_0^{\sqrt{\left(\frac{z-3}{2\sqrt{2}}\right)^2 - y^2}} 1 dx \right) dy \right) dz$$

Result: 5.762207211

$$\int_0^{\sqrt{\left(\frac{z-3}{2\sqrt{2}}\right)^2 - y^2}} 1 dx$$

Result: $\frac{\sqrt{-2 \cdot (8 \cdot y^2 - z^2 + 6 \cdot z - 9)}}{4}$

$$\int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \frac{\sqrt{-2 \cdot (8 \cdot y^2 - z^2 + 6 \cdot z - 9)}}{4} dy$$

$$\frac{z^2 \cdot \sin^{-1}\left(\frac{2 \cdot \sqrt{2} \cdot \sqrt{-z^2 + 1}}{\sqrt{z^2 - 6 \cdot z + 9}}\right)}{8} - \frac{3 \cdot z \cdot \sin^{-1}\left(\frac{2 \cdot \sqrt{2} \cdot \sqrt{-z^2 + 1}}{\sqrt{z^2 - 6 \cdot z + 9}}\right)}{4} + \frac{|3 \cdot z - 1| \cdot \sqrt{-2 \cdot (z^2 - 1)}}{4} + \frac{9 \cdot \sin^{-1}\left(\frac{2 \cdot \sqrt{2} \cdot \sqrt{-z^2 + 1}}{\sqrt{z^2 - 6 \cdot z + 9}}\right)}{8}$$

Result: 5.762207211

Mode buttons at the bottom: Alg, Standard, Real, Rad

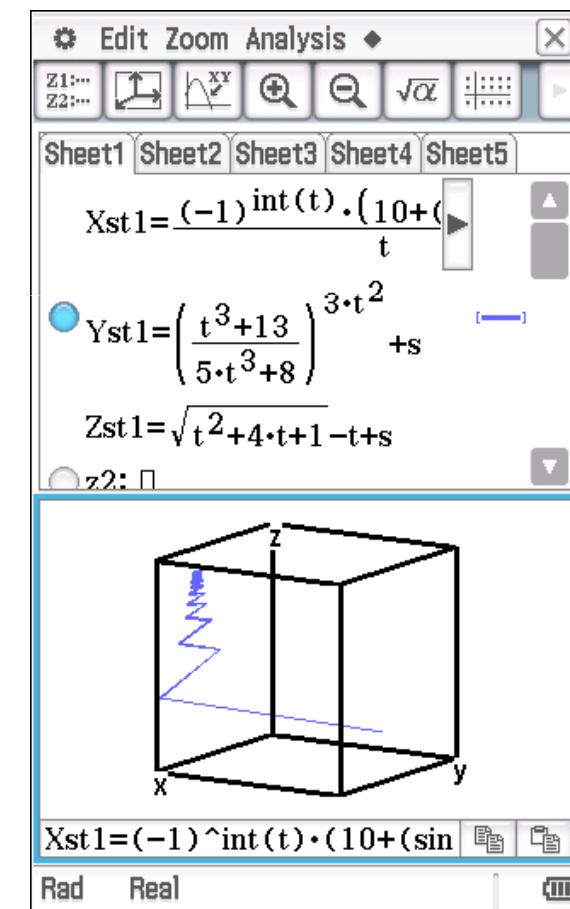
eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

Innovative approaches in teaching and learning mathematics

Problem of subject's Essence

To find the limit of a sequence M_n



eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

E. I. Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

Innovative approaches in teaching and learning mathematics

Problem of subject's Essence

To find the limit of a sequence M_n

$$\lim_{n \rightarrow \infty} (M_n) = M^*$$

$$M_n = \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix}, \quad M^* = \begin{bmatrix} x^* \\ y^* \\ z^* \end{bmatrix}$$

Define $x(n) = (-1)^n \cdot \frac{10 + (\sin(n))^2}{n}$

$x(n)$

$$\frac{(-1)^n \cdot ((\sin(n))^2 + 10)}{n}$$

$\lim_{n \rightarrow \infty} (x(n))$
Undefined

$\lim_{n \rightarrow \infty} (|x(n)|)$
0

=====

Define $y(n) = \left(\frac{n^3 + 13}{5n^3 + 8} \right)^{3n^2}$

done

$\lim_{n \rightarrow \infty} (y(n))$
0

=====

Define $z(n) = \sqrt{n^2 + 4n + 1} - n$

eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

File Edit Insert Action

lim (z(n))
n→∞ 2

=====

x*=0, y*=0, z*=2.

Compute the norm of

$\left[\begin{matrix} x(n) \\ y(n) \\ z(n) \end{matrix} \right] - \left[\begin{matrix} 0 \\ 0 \\ 2 \end{matrix} \right]$:

norm $\left(\left[\begin{matrix} x(n) \\ y(n) \\ z(n) \end{matrix} \right] - \left[\begin{matrix} 0 \\ 0 \\ 2 \end{matrix} \right] \right)$

$\sqrt{(\sin(n))^4 + n^2 \cdot \left(\left(\frac{n^3 + 13}{5 \cdot n^3 + 8} \right)^2 \right)}$

Alg Standard Real Rad

File Edit Insert Action

Define M(n)= $\begin{bmatrix} x(n) \\ y(n) \\ z(n) \end{bmatrix}$

done

M(n)

$\begin{bmatrix} (-1)^n \cdot ((\sin(n))^2 + 10) \\ n \\ \left(\frac{n^3 + 13}{5 \cdot n^3 + 8} \right)^3 \cdot n^2 \\ -n + \sqrt{n^2 + 4 \cdot n + 1} \end{bmatrix}$

$M^+ := \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

Alg Standard Real Rad

File Edit Insert Action

$\sqrt{(\sin(n))^4 + n^2 \cdot \left(\left(\frac{n^3 + 13}{5 \cdot n^3 + 8} \right)^2 \right)}$

=====

approx(seq(M(n), n, 1, 3, 1))

$\begin{bmatrix} 0.5 \\ -3.339971619 \\ 1.152250099e-15 \end{bmatrix}, \begin{bmatrix} 1.69041576 \end{bmatrix} \}$

approx(M(10))

$\begin{bmatrix} 1.029595897 \\ 6.074877816e-209 \\ 1.874342087 \end{bmatrix}$

approx(M(100))

$\begin{bmatrix} 0.1025640616 \\ 0 \\ 1.985293057 \end{bmatrix}$

Alg Standard Real Rad

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Science Learning with Information Technologies as a Tool for "Scientific Thinking"

The image displays three side-by-side screenshots of the ClassPad software interface, showing its use for scientific calculations and 3D plotting.

Screenshot 1 (Left):

- Shows several approximations of matrices M(n) for n=1, 3, 10, 100, and 10^8 .
- For $M(1)$, the result is a 2x2 matrix: $\begin{bmatrix} -10.70807342 & 5.41341 \\ 1.248975876 & 4.91741 \end{bmatrix}$.
- For $M(10)$, the result is a 3x3 matrix: $\begin{bmatrix} 1.029595897 & & \\ 6.074877816 \times 10^{-209} & & \\ 1.874342087 & & \end{bmatrix}$.
- For $M(100)$, the result is a 3x3 matrix: $\begin{bmatrix} 0.1025640616 & & \\ 0 & & \\ 1.985293057 & & \end{bmatrix}$.
- For $M(10^8)$, the result is a 3x3 matrix: $\begin{bmatrix} 1.086795142 \times 10^{-7} & & \\ 0 & & \\ 2 & & \end{bmatrix}$.

Screenshot 2 (Middle):

- Shows the definition of three functions:

 - $xst1(s, t) = (-1)^{\text{int}(t)}$
 - $yst1(s, t) = \left(\frac{t^3 + 13}{5t^3 + 8}\right)^{3t}$
 - $zst1(s, t) = \sqrt{t^2 + 4t + 1} - t + s$

- Shows a 3D plot of a wavy surface defined by these functions within a cube.
- Shows coordinate values: $zc = 1.4503897$, $xc = -10.70717$, $sc = 9 \times 10^{-4}$, $yc = 1.2498759$, $tc = 1$.

Screenshot 3 (Right):

- Shows the same 3D plot of the wavy surface within a cube.
- Shows coordinate values: $zc = 1.4503897$, $xc = -10.70717$, $sc = 9 \times 10^{-4}$, $yc = 1.2498759$, $tc = 1$.
- Shows the function definition: $Xst1 = (-1)^{\text{int}(t)} \cdot (10 + (\sin$

eActivity: E.I.Smirnov – lecture in Dresden 20.07.2016

Science Learning with Information Technologies as a Tool for "Scientific Thinking"

File Edit Insert Action

3D-graphic z1...
z2...

=====

Computing the limits step by step:

x(n):

$$\left| (-1)^n \cdot \frac{10 + (\sin(n))^2}{n} \right| = \frac{10 + (\sin(n))^2}{n} < \frac{10+1}{n} = \frac{11}{n} \rightarrow 0, \quad n \rightarrow \infty$$

Alg Standard Real Rad

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y(n):

$$\left(\frac{n^3 + 13}{5n^3 + 8} \right)^{3n^2} \rightarrow \left(\frac{1}{5} \right)^\infty = 0, \quad n \rightarrow \infty$$

approx(seq($\frac{n^3 + 13}{5n^3 + 8}$, n, 1, 10, 1))

{1.076923077, 0.4375, 0.27}

$$\frac{n^3 + 13}{5n^3 + 8} < 1, \text{ if } n > 1.$$

$$\frac{n^3 + 13}{5n^3 + 8} = \frac{1}{5} \cdot \frac{n^3 + 13}{n^3 + 8/5}$$

$$= \frac{1}{5} \cdot \left(\frac{n^3 + 1.6}{n^3 + 1.6} + \frac{11.4}{n^3 + 1.6} \right)$$

$$= \frac{1}{5} \cdot \left(1 + \frac{11.4}{n^3 + 1.6} \right)$$

Alg Standard Real Rad

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$$= \frac{1}{5} \cdot \left(\frac{n^3 + 1.6}{n^3 + 1.6} + \frac{11.4}{n^3 + 1.6} \right)$$

$$= \frac{1}{5} \cdot \left(1 + \frac{11.4}{n^3 + 1.6} \right)$$

z(n):

$$\sqrt{n^2 + 4n + 1} - n = \frac{4n + 1}{\sqrt{n^2 + 4n + 1} + n}$$

$$= \frac{4 + 1/n}{\frac{1}{n} \sqrt{n^2 + 4n + 1} + n/n}$$

$$= \frac{4 + 1/n}{\sqrt{1 + 4/n + 1/n^2} + 1} \rightarrow \frac{4}{1+1} = 2,$$

for $n \rightarrow \infty$

Alg Standard Real Rad

Levenberg-Marquardt Algorithm using ClassPad



Большое спасибо за ваше внимание!