Solution of Linear Equation Systems with Parameters

cp.
http://www.informatik.htw-dresden.de/~paditz/Paditz_Beitrag_CEJ_2006.pdf

3x+2y+t*z=0⇒equ1

$$3\cdot x+2\cdot y+t\cdot z=0$$

0x+1y-4z=-1⇒equ2

$$y-4\cdot z=-1$$

1x+3y+0z=1⇒equ3

$$x+3\cdot y=1$$

-1x+0y+2z=1⇒equ4

$$-x+2\cdot z=1$$

$$\left\{\begin{matrix}\text{equ1}\\ \text{equ2}\\ \text{equ3}\\ \text{equ4}\end{matrix}\right|_{x,y,z,u}$$

$$\left\{x=\frac{4\cdot t+8}{t-28},y=\frac{-(t+12)}{t-28},z=\frac{-10}{t-28},u=u\right\}$$

What happens with t=28?

$$\left\{\begin{matrix}\text{equ4}\\ \text{equ2}\\ \text{equ1}\\ \text{equ3}\end{matrix}\right|_{x,y,z,u}$$

$$\left\{x=\frac{-(t+4)}{t+14},y=\frac{-(t-6)}{t+14},z=\frac{5}{t+14},u=u\right\}$$

What happens with t=-14?

$$\left.\begin{cases} equ4 \\ equ2 \\ equ3 \\ equ1 \end{cases}\right|_{x,y,z,u}$$

$$\left\{x=-\frac{2}{7}, y=\frac{3}{7}, z=\frac{5}{14}, u=u\right\}$$

Here it seems for all t we get an unique solution (and parameter t disappears?)?

$$\begin{bmatrix} 3 & 2 & t & 0 \\ 0 & 1 & -4 & -1 \\ 1 & 3 & 0 & 1 \\ -1 & 0 & 2 & 1 \end{bmatrix} \Rightarrow mat$$

$$\begin{bmatrix} 3 & 2 & t & 0 \\ 0 & 1 & -4 & -1 \\ 1 & 3 & 0 & 1 \\ -1 & 0 & 2 & 1 \end{bmatrix}$$

ref(mat)

$$\begin{bmatrix} 1 & \frac{2}{3} & \frac{t}{3} & 0 \\ 0 & 1 & \frac{-t}{7} & \frac{3}{7} \\ 0 & 0 & 1 & \frac{5}{3\cdot t+14} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here it seems we have to study the special case t=-3/14?

rref(mat)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here it seems the parameter t is without meaning (t disappears?)?

But in case t=0:

ref(mat|t=0)

$$\begin{bmatrix} 1 & \frac{2}{3} & 0 & 0 \\ 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{5}{14} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rref(mat|t=0)

$$\begin{bmatrix} 1 & 0 & 0 & -\frac{2}{7} \\ 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{5}{14} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank(mat)

4

rank(mat|t=0)

3

The rank-function can't compute the rank in dependence on the parameter t!
If we use the "ref" or "rref" or "rank" functions the CP should give a message "no solution with parameters"

another exercise:

$$\begin{bmatrix} j & 2 & t & 3+2j \\ 0 & 1 & 2j & 1+j \\ s & 0 & 4 & -1 \end{bmatrix} \Rightarrow mat$$

$$\begin{bmatrix} j & 2 & t & 3+2\cdot j \\ 0 & 1 & 2\cdot j & 1+j \\ s & 0 & 4 & -1 \end{bmatrix}$$

rank(mat)

3

rank(mat|s=-j and t=-4+4j)

2

Again, the rank is depending on the parameters s and t

Now using the new created functions LinEqSys and AVRank in Main-menu

"Read at first the eActivity LinEqSys_AVRank"

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$$\begin{bmatrix} 3 & 2 & t & 0 \\ 0 & 1 & -4 & 1 \\ 1 & 3 & 0 & -1 \\ -1 & 0 & 2 & -1 \end{bmatrix} \Rightarrow matST$$

$$\begin{bmatrix} 3 & 2 & t & 0 \\ 0 & 1 & -4 & 1 \\ 1 & 3 & 0 & -1 \\ -1 & 0 & 2 & -1 \end{bmatrix}$$

LinEqSys(matST,1,1)

done

matnew⇒matT1

$$\begin{bmatrix} -\frac{2}{3} & \frac{-t}{3} & 0 \\ 1 & -4 & 1 \\ \frac{7}{3} & \frac{-t}{3} & -1 \\ \frac{2}{3} & \frac{t}{3}+2 & -1 \end{bmatrix}$$

LinEqSys(matT1,2,1)

done

matnew⇒matT2

$$\begin{bmatrix} \frac{-(t+8)}{3} & \frac{2}{3} \\ 4 & -1 \\ \frac{-(t-28)}{3} & -\frac{10}{3} \\ \frac{t+14}{3} & -\frac{5}{3} \end{bmatrix}$$

"If t≠28 than we have a third exchange-step"

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LinEqSys(matT2,3,1)

done

matnew⇒matET

$$\begin{bmatrix} \frac{4\cdot(t+2)}{t-28} \\ \frac{-40}{t-28}-1 \\ \frac{-10}{t-28} \\ \frac{-5\cdot t}{t-28} \end{bmatrix}$$

"matET is the solution, if the last element -5t/(t-2▸

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matET|t=0

$$\begin{bmatrix} -\frac{2}{7} \\ \frac{3}{7} \\ \frac{5}{14} \\ 0 \end{bmatrix}$$

"Thus x=-2/7, y=3/7, z=5/14"

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"Remark: rank of matST is 3 for t=0 and 4 otherwis▸

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AVRank(matST,1,1)

done

matnew⇒matT1

$$\begin{bmatrix} 1 & -4 & 1 \\ \frac{7}{3} & \frac{-t}{3} & -1 \\ \frac{2}{3} & \frac{t}{3}+2 & -1 \end{bmatrix}$$

AVRank(matT1,1,1)

done

```
matnew⇒matT2
```

$$\begin{bmatrix} \frac{-(t-28)}{3} & -\frac{10}{3} \\ \frac{t+14}{3} & -\frac{5}{3} \end{bmatrix}$$

```
"If t≠28 we compute:"
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"If t≠28 we compute:"

```
AVRank(matT2,1,1)
```

done

```
matnew⇒matT3
```

$$\begin{bmatrix} \frac{-5\cdot t}{t-28} \end{bmatrix}$$

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"If t≠-14 we compute:"
```

"If t≠-14 we compute:"

```
AVRank(matT2,2,1)
```

done

```
matnew⇒matT3
```

$$\begin{bmatrix} \frac{-5\cdot t}{t+14} \end{bmatrix}$$

```
"Thus we have 3 steps, i.e. rank equals 3 and for▸
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□

```
[i 2 t  -3-2i]
[0 1 2i -1-i ]→matST
[s 0 4  1    ]

                                  [i 2 t   -3-2·i]
                                  [0 1 2·i -1-i  ]
                                  [s 0 4   1     ]
AVRank(matST,2,2)
                                             done
matnew→matT1
                                    [i t-4·i -1]
                                    [s 4      1]
AVRank(matT1,1,1)
                                             done
matnew→matT2
                              [4·s+s·t·i+4 -s·i+1]
"If 4·s+s·t·i+4≠0 we have 3 exchange steps"
          "If 4·s+s·t·i+4≠0 we have 3 exchange steps"
"If -s·i+1≠0 we have 3 exchange steps too"
           "If -s·i+1≠0 we have 3 exchange steps too"
solve({4·s+s·t·i+4=0,-s·i+1=0},{s,t})
                                  {s=-i,t=-4+4·i}
"Thus we have for s=-i,t=-4+4·i: rank(A)=rank(A,-▸
    "Thus we have for s=-i,t=-4+4·i: rank(A)=rank(A,-▸
"now consider 4·s+s·t·i+4≠0 and s=-i"
                "now consider 4·s+s·t·i+4≠0 and s=-i"
4·s+s·t·i+4|s=-i
                                         t+4-4·i
"i.e. t≠-4+4·i"
                                    "i.e. t≠-4+4·i"
"Thus we have for s=-i and t≠-4+4·i: rank(A)=ranl▸
    "Thus we have for s=-i,t≠-4+4·i: rank(A)=rank(A,-▸
"now consider 4·s+s·t·i+4=0 and s≠-i"
                "now consider 4·s+s·t·i+4=0 and s≠-i"
"i.e. rank(A)=2<rank(A,-b)=3"
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"i.e. rank(A)=2<rank(A,-b)=3"
"finally if 4·s+s·t·i+4≠0 and -s·i+1≠0: rank(A)=rar▸
```
"finally if 4·s+s·t·i+4≠0 and -s·i+1≠0: rank(A)=rar▸

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"Now we consider the LinEqSys"
```
"Now we consider the LinEqSys"

```
LinEqSys(matST,2,2)
                                                done
matnew→matT1
```
$$\begin{bmatrix} i & t-4\cdot i & -1 \\ 0 & -2\cdot i & 1+i \\ s & 4 & 1 \end{bmatrix}$$

```
LinEqSys(matT1,1,1)
                                                done
matnew→matT2
```
$$\begin{bmatrix} t\cdot i+4 & -i \\ -2\cdot i & 1+i \\ 4\cdot s+s\cdot t\cdot i+4 & -s\cdot i+1 \end{bmatrix}$$

```
"If 4·s+s·t·i+4≠0 we have an unique solution"
```
"If 4·s+s·t·i+4≠0 we have an unique solution"

```
LinEqSys(matT2,3,1)
                                                done
matnew→matET
```
$$\begin{bmatrix} \dfrac{-(t\cdot i+4+4\cdot i)}{4\cdot s+s\cdot t\cdot i+4} \\ \dfrac{-((1-i)\cdot s\cdot t+(-6-4\cdot i)\cdot s-4-6\cdot i)}{4\cdot s+s\cdot t\cdot i+4} \\ \dfrac{(s+i)\cdot i}{4\cdot s+s\cdot t\cdot i+4} \end{bmatrix}$$

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"have a look on det(A):"
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"have a look on det(A):"

$$\det\left(\begin{bmatrix} i & 2 & t \\ 0 & 1 & 2i \\ s & 0 & 4 \end{bmatrix}\right)$$

$-s\cdot t+4\cdot s\cdot i+4\cdot i$

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factorOut(ans,i)
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$(4\cdot s+s\cdot t\cdot i+4)\cdot i$

"In case det(A)=0 and s=−i, i.e. t=−4+4·i, we ha▸

"In case det(A)=0 and s=−i, i.e. t=−4+4·i, we ha▸

```
matT2|s=-i and t=-4+4·i
```

$$\begin{bmatrix} -4\cdot i & -i \\ -2\cdot i & 1+i \\ 0 & 0 \end{bmatrix}$$

"solution: x=−4ic−i, y=−2ic+1+i, z=c, c∈C"

"solution: x=−4ic−i, y=−2ic+1+i, z=c, c∈C"

□