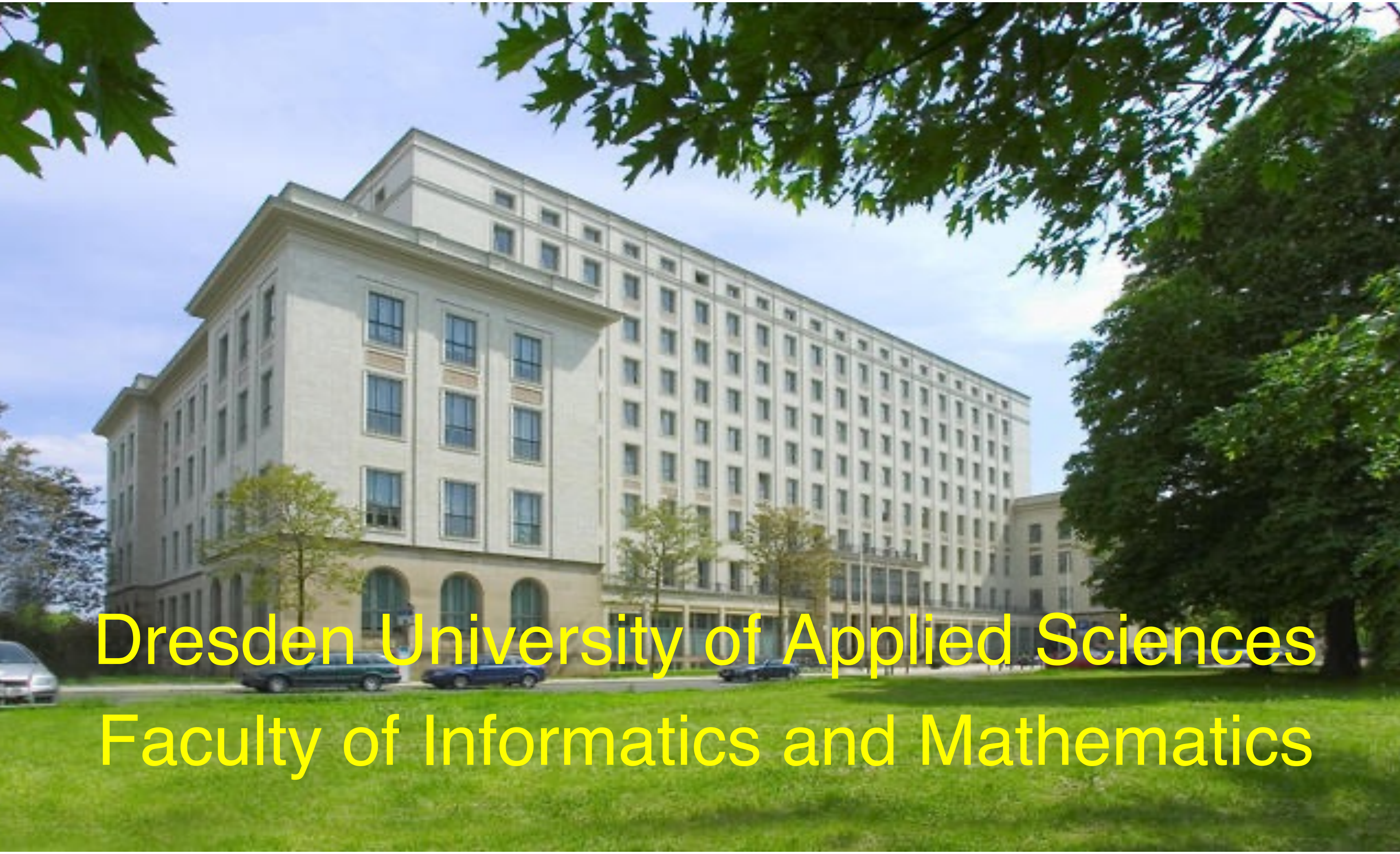
Hochschule für Technik und
Wirtschaft Dresden
University of Applied Sciences



- **Two classic word problems with a similar background are mathematically modeled and a solution is provided.**
- **The first problem, the puzzle of the "Dutch-men‘s Wives", can be found in a women's magazine (The Ladies Diary) as early as 1739 (May (1739)).**
- **A similar problem (“Find Ada‘s Surname”, Workman (1906)) was formulated at the beginning of the last century.**

- The mathematical model leads to a Diophantine quadratic equation, which can be solved using the third binomial formula and a prime factor analysis.
- First, a "stupid trial and error method" (brute force) is used to search for a solution.
- The model is calculated exactly with a pocket calculator.
- The CASIO-calculator/emulator ClassPad II will be used.

- We want to sensitize the pupils to MINT (Mathematics, computer science (Informatics), Natural sciences and Technology) and to link interest with personal experiences.
- We show the pupils that MINT is much more than the cliché of mathematics and computer programming.
- For this purpose, the inhibition threshold for dealing with MINT should be lowered in the school project on the basis of playful, project-oriented didactics and at the same time interest in the topic should be encouraged.

- **Various projects are available.**
- **“In conclusions, we point to the fact that there are several ways to demonstrate interrelations between traditional Maths and IT.”**

Reference:

Hvorecký, Korenova, Barot (2022),

„Combining Brute Force and IT to Solve Difficult Problems“

The first word problem:

the puzzle of the “Dutchmen's Wives”

“There came three Dutchmen of my acquaintance to see me, being lately married.

The men’s names were Hendrick, Claas, and Cornelius; the women’s, Geertruii, Catriin, and Anna.

But I forgot the name of each man’s wife.
They told me they had been at market to buy hogs.

Each person bought as many hogs as they gave shillings for each hog.”

The first word problem:

the puzzle of the "Dutchmen's Wives"

"Hendrick bought 23 hogs more than Catriin, and Claas bought 11 more than Geertruii; likewise, each man laid out 3 guineas more than his wife.

I desire to know the name of each man's wife."

References:
May (1739), Dudeney (1917), Hemme (2019).

Note: the guinea was a British gold coin in circulation from 1663 to 1816, one guinea = 21 shillings.

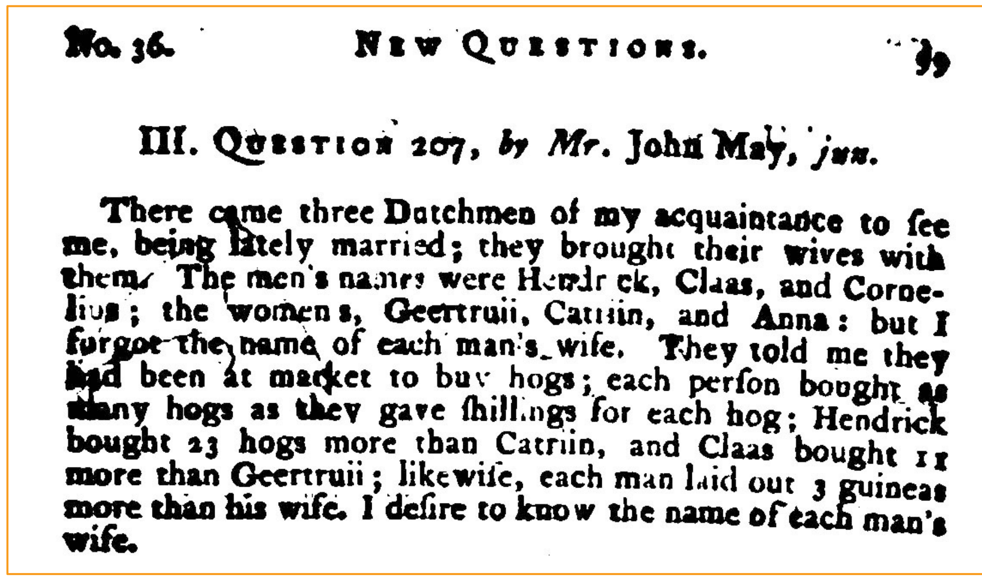
The LADIES Diary: OR, THE Woman's ALMANACK, For the YEAR of our LORD, 1740.

eing the BISSEXTILE, or LEAP-YEAR: ontaining many Delightful and Entertaining Particulars, Peculiarly Adapted for the Uſe and Diverſion of the FAIR-SEX.

eing the Thirty-Seventh ALMANACK ever Publiſh'd of this Kind.

1. HAIL! happy LADIES of the BRITISH Iſle,
On whom the GRACES and the MUSES ſmile,
o nad your lovely Shape, and matchleſs Mein,
The Wonder of the Neighb'ring Nations been;
3. NATURE to make your Triumph more compleat,
To peerleſs CHARMS has added piercing WIT.
4. NO more let SCYTHIA vaunt her FEMALE-HOST,
Nor their SEMIRAMIS th' Aſſyrians boaſt:
WIT join'd to BEAUTY, Fame ſhall now record;
Which lead more Captive than the Conqu'ring Sword.

inted by A. Wilde, for the Company of STATIONERS, 1740



No. 36. NEW QUESTIONS. 99

III. QUESTION 207, by Mr. John May, jun.

There came three Dutchmen of my acquaintance to ſee me, being lately married; they brought their wives with them. The men's names were Hendrick, Claas, and Cornelius; the women's, Geertruii, Catriin, and Anna: but I forgot the name of each man's wife. They told me they had been at market to buy hogs; each perſon bought as many hogs as they gave ſhillings for each hog; Hendrick bought 23 hogs more than Catriin, and Claas bought 11 more than Geertruii; likewiſe, each man laid out 3 guineas more than his wife. I deſire to know the name of each man's wife.

COMBINING BRUTE FORCE AND IT TO SOLVE A CLASSICAL SHOPPING PROBLEM

HTWD Hochschule für Technik und Wirtschaft Dresden University of Applied Sciences

No. 36. NEW QUESTIONS. 99

III. QUESTION 207, *by Mr.* John May, *jun.*

There came three Dutchmen of my acquaintance to ſee me, being lately married; they brought their wives with them. The men's names were Hendrick, Claas, and Cornelius; the women's, Geertruii, Catriin, and Anna: but I forgot the name of each man's wife. They told me they had been at market to buy hogs; each perſon bought as many hogs as they gave ſhillings for each hog; Hendrick bought 23 hogs more than Catriin, and Claas bought 11 more than Geertruii; likewiſe, each man laid out 3 guineas more than his wife. I deſire to know the name of each man's wife.

- **It is about a mathematical problem from 1739 that was published in a women's magazine and is still very popular today and is about married couples and later about mothers with their daughters going shopping.**
- **You find the solution by "trying it out" or, as in the past, by clever thinking, since there was no computing technology or school calculator at that time.**
- **Today "trying out" is easier, at least with a school calculator (here ClassPad II), if you don't have a clever idea for a solution yet.**

Dudeney, H.E. (London, 1917). Amusements in Mathematics , pp. 26–27:

“I wonder how many of my readers are acquainted with the puzzle of the 'Dutchmen‘s Wives‘ – in which you have to determine the names of three men‘s wives, or, rather, which wife belongs to each husband.

Some thirty years ago (i.e. c. 1880) it was “going the rounds" as something quite new, but I recently discovered it in the *Ladies' Diary* for 1739-40, so it was clearly familiar to the fair sex over one hundred and seventy years ago.

How many of our mothers, wives, sisters, daughters, and aunts could solve the puzzle today?

A far greater proportion than then, let us hope.”

The second word problem:

the puzzle “Find Ada‘s Surname”

Henry Ernest Dudeney wrote in 1917

"It was recently submitted to a Sydney evening newspaper that indulges in 'intellect sharpeners' but was rejected with the remark that it is childish and that they only published problems capable of solution!“

Dudeney, H.E. (London, 1917). **Amusements in Mathematics**, p. 27

The second word problem:

29. Five ladies, accompanied each by her daughter, purchased cloth at the same shop. Each of the ten bought as many feet of cloth as she paid farthings per foot. Each mother spent 8s. $5\frac{1}{4}$d. more than her daughter. Mrs. Robinson spent 6s. more than Mrs. Evans, who only spent about a quarter of what Mrs. Jones did, while Mrs. Smith spent most of all. Mrs. Brown bought 21 yds. more than did Bessie, one of the girls, while of the other girls Annie bought 16 yds. more than Mary and spent £3 0s. 8d. more than Emily. The other girl's Christian name was Ada. What was her surname?

From Workman's **Tutorial Arithmetic** (London, 1906), p. 482

https://cs.smu.ca/~dawson/workman/workman3.gif

- **“Five ladies, accompanied by their daughters, bought cloth at the same shop.**
- **Each of the ten paid as many farthing per foot (of cloth) as she bought feet (of cloth), and each mother spent 8 s. 5¼d. more than her daughter.**
- **Mrs Robinson spent 6 s. more than Mrs Evans, who spent about a quarter as much as Mrs Jones.**
- **Mrs. Smith spent the most.**
- **Mrs. Brown bought 21 yards more than Bessie - one of the girls.**
- **Annie bought 16 yards more than Mary and spent £3, 0 s. and 8 d. more than Emily.**
- **The Christian Name of the other was Ada. Now, what was her surname?”**

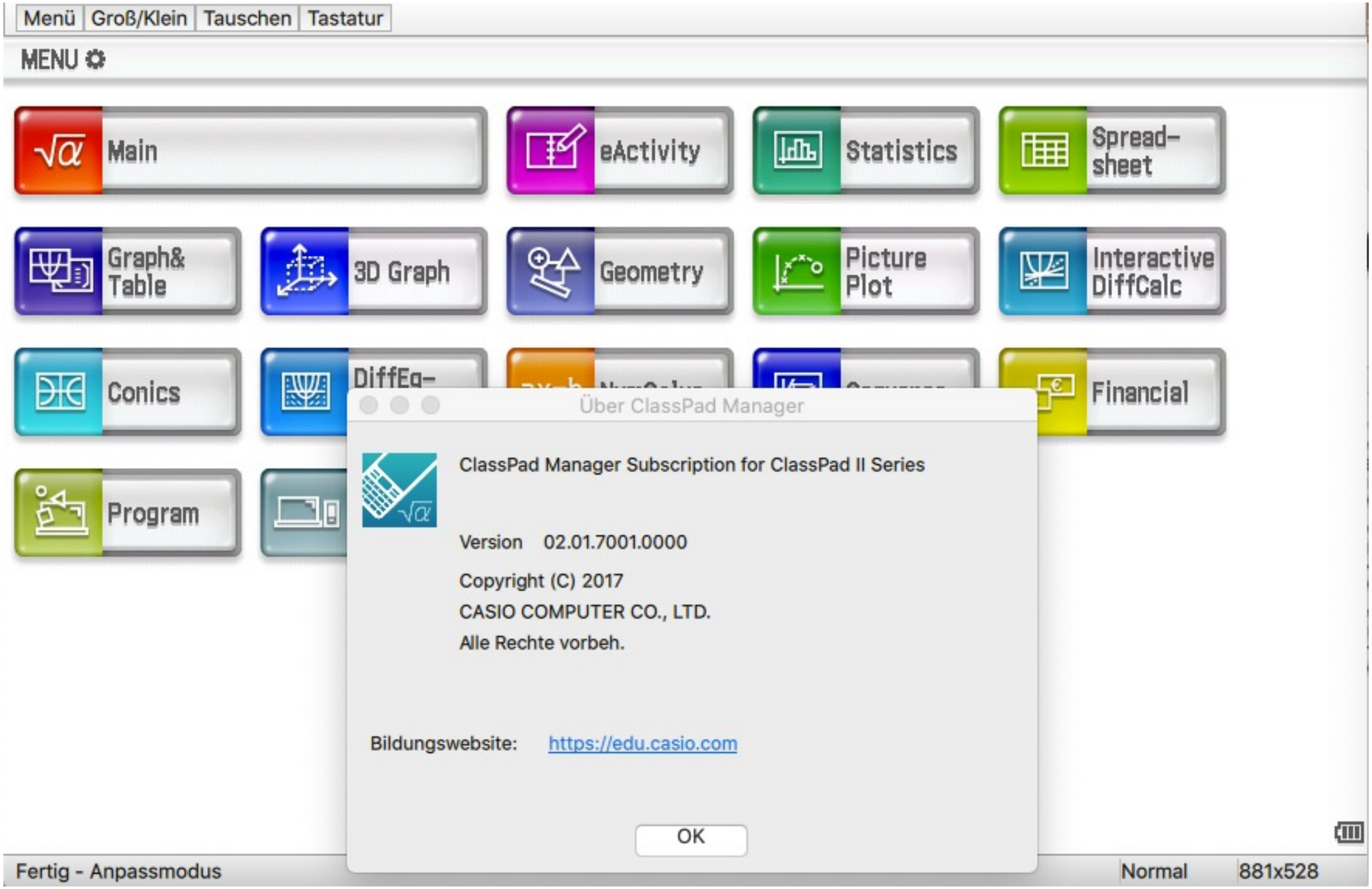
Dudeney, H.E. (London, 1917). **Amusements in Mathematics**, S. 26-27

Workman, W.P. (London, 1918). **The Tutorial Arithmetic: with Answers**, S.482

Beiler, A.H. (NY, 1966). **Recreations in the Theory of Numbers: The Queen of Mathematics Entertains**, S.154

Note: 1£ = 1Pound = 20s. = 240d., 1s. = 1Shilling = 12d., d. = Penny, 1Farthing = ¼ d., 1yard = 3feet

Current software: Status July 2023



Solution to the first word problem:

Analysis of the text and finding a suitable mathematical model:

The names of the men(M): Hendrick, Claas, and Cornelius; the names of the women(W): Geertruii, Catriin and Anna

Each person bought as many hogs as they spent shillings on each hog:

E.g. M=5 hogs for M=5 shillings each gives M^2=25 [s.]

Likewise, each husband paid altogether 3 guineas more than his wife: 3 guineas = 3 * 21 shillings = 63 [s.]

Find a mathematical model!

HTWD Hochschule für Technik und Wirtschaft Dresden University of Applied Sciences

Mathematical model: $M^2 = W^2 + 63$

with **M, W** $\in$ **{1, 2, 3, …}**

You can easily see the pair of numbers: **(M,W) = (8, 1)**

We need three pairs of integer numbers and

consider to the equation $M = (W^2 + 63)^{1/2}$

and solve this equation **using the stupid trial and error method**, successively plugging in W = 1,2,3,... to find integer roots M.

However, we do the trial and error cleverly and use the ClassPad to do so. **Everyone tries it for themselves now!**

We need three pairs of integer numbers and consider

to the equation $M = (W^2 + 63)^{1/2}$

However, we do the trial and error cleverly and use the ClassPad to do so:

The goal is to generate a sequence of numbers "code" with W=1,2,3,...,35, which consists of zeros (0) if the root is not an integer and shows a one (1) only if the root is an integer.

code := seq(… ,W,1,35) = {1,0,0,0, …, 0,0,0}, i.e.

with a small number $\varepsilon := 10^{-20}$ (possibly to correct the sign)

code := seq($f_\varepsilon(W)$, W, 1, 35) = {1,0,0,0, …, 0,0,0}, We **do the trial and error cleverly** and use the ClassPad to do so.

code := seq($f_\varepsilon(W)$, W, 1, 35) = {1,0,0,0, …, 0,0,0},

with a small number $\varepsilon := 10^{-20}$ (possibly to correct the sign)

Here $f(W) = f_\varepsilon(W)$ **is the following function**:

Define $f(W) = (1-\text{signum}(\text{frac}((W^2+63)^{1/2}) - \varepsilon))\ /\ 2$

$\text{frac}((W^2+63)^{1/2})$ returns only the decimal places,

e.g. $(3^2+63)^{1/2} = 8.485281374$, $\text{frac}((3^2+63)^{1/2}) = 0.485281374 > 0$

$(1^2+63)^{1/2} = 8.0000000$, $\text{frac}((1^2+63)^{1/2}) = 0.0000000 = 0$

$\text{frac}((W^2+63)^{1/2}) - \varepsilon > 0$, if $(W^2+63)^{1/2}$ not integer

$\text{frac}((W^2+63)^{1/2}) - \varepsilon < 0$, if $(W^2+63)^{1/2}$ integer code := seq(f(W) , W, 1, 35) = {1,0,0,0, …, 0,0,0},

with a small number $\varepsilon := 10^{-20}$ (possibly to correct the sign)

Here $f(W) = f_\varepsilon(W)$ is the following function:

Define $f(W) = (1-\text{signum}(\,\text{frac}((W^2+63)^{1/2}) - \varepsilon)\,)\,/\,2$

$\text{frac}((W^2+63)^{1/2}) - \varepsilon > 0$, if $(W^2+63)^{1/2}$ not integer

$\text{frac}((W^2+63)^{1/2}) - \varepsilon < 0$, if $(W^2+63)^{1/2}$ integer

The sign function signum(…) returns the sign as a number code +1 or -1, (signum(0) is not defined), e.g.

$\text{signum}(\mathbf{0}.485281374 - 10^{-20}) = +1$

$\text{signum}(\mathbf{0}.000000000 - 10^{-20}) = -1$

Finally, f(W) gives the desired code.

code := seq(f(W) , W, 1, 35) = {1,0,0,0, …, 0,0,0},

with a small number $\varepsilon := 10^{-20}$ (**possibly to correct the sign**)

Here f(W) = $f_\varepsilon(W)$ is the following function:

Define f(W) = (1-signum(frac(**$(W^2+63)^{1/2}$**) - ε)) **/ 2**

The sign function signum(…) returns the sign as a number code +1 or -1, (**signum(0) is not defined**), e.g.

Remark: another definition of f(W) by the help of the heaviside-function is (without ε)

Define f(W) = 2 – 2 * heaviside(frac(**$(W^2+63)^{1/2}$**))

heaviside(x)=H(x)=0, if x<0, H(x)=1/2, if x=0, H(x)=1, if x>0

Finally, $f_\varepsilon(W)$ or f(W) gives the desired code.

HTWD Hochschule für Technik und Wirtschaft Dresden
University of Applied Sciences

Entries in the pocket calculator, e.g. ClassPad II:

$\varepsilon := 10^{-20}$

Define f(W) = (1-signum(frac(**(W²+63)^(1/2)**) - ε)) **/ 2**

$$f(W) = \left(1 - \mathrm{signum}\left(\mathrm{frac}\left((W^2+63)^{1/2}\right) - \varepsilon\right)\right) / 2$$

Finally f(W) gives the desired code:

code := seq(f(W) , W, 1, 35) **=**

{1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0}

It can be seen that there are three integer solutions.

W := seq(W, W, 1, 35) * code =

{1,0,0,0,0,0,0,0,9,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,31,0,0,0,0}

M := $(W^2+63)^{1/2}$ * code =

{8,0,0,0,0,0,0,0,12,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,32,0,0,0,0}

It can be seen that there are three integer solutions.

W := seq(W, W, 1, 35) * code =

{1,0,0,0,0,0,0,0,9,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,31,0,0,0,0}

M := $(W^2+63)^{½}$ * code =

{8,0,0,0,0,0,0,0,12,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,32,0,0,0,0}

So we found the three pairs!

(M, W) = (8,1), (12,9), (32,31)

So we found the three pairs!

(M, W) = (8,1), (12,9), (32,31)

The men's names were Hendrick, Claas, and Cornelius; those of the women, Geertruii, Catriin and Anna.

Hendrick bought 23 more hogs than Catriin, and Claas bought 11 more than Geertruii:

(M, W) = (8, 1), (12=11+1, 9), (32=23+9, 31)

Now the solution can be found quickly!

So we found the three pairs (M, W) = (8, 1), (12, 9), (32, 31)

The men's names were Hendrick, Claas, and Cornelius; those of the women, Geertruii, Catriin and Anna.

Hendrick bought 23 more hogs than Catriin, and Claas bought 11 more than Geertruii:

(M, W) = (8, 1), (12=11+1, 9), (32=23+9, 31)

Claas = 12, Geertruii = 1 and Hendrick = 32, Catriin = 9, thus

(32, 31) = (Hendrick=32, W=31), (12, 9) = (Claas=12, Catriin=9)

and **(M=8, W=1) = (M=8, Geertruii=1),**

Thus it holds **Cornelius = 8** and **Anna = 31**. **Task solved!**

Direct solution of the Diophantine equation

$M^2 = W^2 + 63$, i.e. $M^2 - W^2 = 63$ or

$$(M + W) * (M - W) = 1 * 3 * 3 * 7$$

(3. binomial formula, prime factorization)

The following three systems of equations result:

$M – W = 1$ and $M + W = 3 * 3 * 7 = 63$

$M – W = 3$ and $M + W = 1 * 3 * 7 = 21$

$M – W = 7$ and $M + W = 1 * 3 * 3 = 9$

Which can easily be solved quickly by hand (in your head):

$$(M,W) = (32,31), (M,W) = (12,9), (M,W) = (8,1)$$

Hochschule für Technik und Wirtschaft Dresden
University of Applied Sciences

So we found the three pairs (M, W) = (8, 1), (12, 9), (32, 31)

The men's names were Hendrick, Claas, and Cornelius; those of the women, Geertruii, Catriin and Anna.

Hendrick bought 23 more hogs than Catriin, and Claas bought 11 more than Geertruii:

(M, W) = (8, 1), (12=11+1, 9), (32=23+9, 31)

Claas = 12, Geertruii = 1 and Hendrick = 32, Catriin = 9, thus

(32, 31) = (Hendrick=32, W=31), (12, 9) = (Claas=12, Catriin=9)

and **(M=8, W=1) = (M=8, Geertruii=1),**

Thus it holds **Cornelius = 8** and **Anna = 31**. **Task solved!**

HTWD Hochschule für Technik und Wirtschaft Dresden University of Applied Sciences

Another direct solution:

Write $M^2 = W^2 + 63$ with $M=W+K$, i.e.

$M^2 = (W+K)^2 = W^2 + 2 * W * K + = W^2 + 63$

Thus $K^2 + 2 * W * K - 63 = 0$

and $W = -K/2 + 63/(2K) = -K/2 + 1*3*3*7/(2K)$

Here we check with the ClassPad II:

seq(-K/2 + 63/(2K) ,K,1,9,2) = {31, 9, 3.8, 1, -1}

and we have only tree integer solutions W = {31, 9, 1}

and M = W+K = {31+1, 9+3, 1+7} = {32, 12, 8}

Finally (M,W) = (32,31), (M,W) = (12,9), (M,W) = (8,1)

The solution of the second word problem:

- "Five ladies, accompanied by their daughters, bought cloth at the same shop.
- Each of the ten paid as many farthing per foot (of cloth) as she bought feet (of cloth), and each mother spent 8 s. 5¼d. more than her daughter.
- Mrs Robinson spent 6 s. more than Mrs Evans, who spent about a quarter as much as Mrs Jones.
- Mrs. Smith spent the most.
- Mrs. Brown bought 21 yards more than Bessie - one of the girls.
- Annie bought 16 yards more than Mary and spent £3, 0 s. and 8 d. more than Emily.
- The Christian Name of the other was Ada. Now, what was her surname?"

The solution of the second word problem:

Analysis of the text and finding a suitable mathematical model: The names of the mothers (M):

The names of the mothers (M):

Mrs.Robinson; Mrs.Evans; Mrs.Jones; Mrs.Smith; Mrs.Brown

the names of the daughters(D): **Bessie; Annie; Mary; Emily, Ada**

each mother spent 8 shillings and 5¼ pennies more than her daughter: how much farthing is the difference?

The smallest monetary unit (farthing) is used to calculate with whole numbers:

8s. = 8 shilling = 8*12d. = 96d., d. = penny, 1 farthing = ¼ d.,

Hochschule für Technik und
Wirtschaft Dresden
University of Applied Sciences

The solution of the second word problem:

Analysis of the text and finding a suitable mathematical model: The names of the mothers (M):

The names of the mothers (M):

Mrs.Robinson; Mrs.Evans; Mrs.Jones; Mrs.Smith; Mrs.Brown

the names of the daughters(D): **Bessie; Annie; Mary; Emily, Ada**

each mother spent 8 shillings and 5¼ pennies more than her daughter: 8s.+5¼ d. = (96+5¼)d. = 4* (96+5¼) = 405f.

The smallest monetary unit (farthing) is used to calculate with whole numbers:

8s. = 8 shilling = 8*12d. = 96d., d. = penny, 1 farthing = ¼ d.,

The solution of the second word problem:

Analysis of the text and finding a suitable mathematical model: The names of the mothers (M):

The names of the mothers (M):

Mrs.Robinson; Mrs.Evans; Mrs.Jones; Mrs.Smith; Mrs.Brown

the names of the daughters(D): **Bessie; Annie; Mary; Emily, Ada**

each mother spent 8 shillings and 5¼ pennies (= 405 farthing) more than her daughter:

e.g. T=5 feet of cloth for each T=5 farthing gives T^2=25 [f.]:

Likewise, every mother paid 405 farthing more than her daughter: M^2 = 25 + 405 = 430, M= $430^{1/2}$ is not possible!

Find a mathematical model!

Mathematical model: $M^2 = D^2 + 405$

with **M, D** $\in$ **{1, 2, 3, …}**

It is difficult to recognize a matching pair of numbers.

We need three pairs of integer numbers and

consider to the equation $M = (D^2 + 405)^{1/2}$

and solve this equation **using the stupid trial and error method**, successively plugging in D = 1,2,3,... to find integer roots M.

However, we do the trial and error cleverly and use the ClassPad to do so. **Everyone tries it for themselves now!**

We need five pairs of integer numbers and consider

to the equation $$M = (D^2 + 405)^{1/2}$$

However, we do the trial and error cleverly and use the ClassPad to do so:

The goal is to generate a sequence of numbers "code" with D=1,2,3,...,100, which consists of zeros (0) if the root is not an integer and shows a one (1) only if the root is an integer.

code := seq(… ,D,1,100) = {0,0, …, 0,1,0, …, 0,0,0}, i.e.

with a small number $\varepsilon := 10^{-20}$ (possibly to correct the sign)

code := seq($f_\varepsilon(D)$, W, 1, 100) = {0,0, …, 0,1,0, …, 0,0,0}, We **do the trial and error cleverly** and use the ClassPad to do so.

code := seq($f_\varepsilon(D)$, D, 1, 100) = {1,0,0,0, …, 0,0,0},

with a small number $\varepsilon := 10^{-20}$ (possibly to correct the sign)

Here $f(D) = f_\varepsilon(D)$ **is the following function**:

Define $f(D) = (1 - \text{signum}(\text{ frac}((D^2+405)^{1/2}) - \varepsilon)\ ) / 2$

$\text{frac}((D^2+405)^{1/2})$ returns only the decimal places,

e.g. $(3^2+405)^{1/2} = 8.48528137$, $\text{frac}((3^2+405)^{1/2}) = \mathbf{0}.48528137 > 0$

$(1^2+405)^{1/2} = 8.0000000$, $\text{frac}((1^2+405)^{1/2}) = \mathbf{0}.0000000 = 0$

$\text{frac}((D^2+405)^{1/2}) - \varepsilon > 0$, if $(D^2+405)^{1/2}$ not integer

$\text{frac}((D^2+405)^{1/2}) - \varepsilon < 0$, if $(D^2+405)^{1/2}$ integer

We **do the trial and error cleverly** and use the ClassPad to do so:

code := seq(f(D) , D, 1, 100) = {0,0, …, 0,1,0, …, 0,0,0},

We try to simplify the trial and error by first considering whether only even or odd numbers are possible for M,D:

We have the following model $M^2 = 405 + D^2$, where M, D are positive integers. If D is odd then M is even or if D is even then M is odd.

However D = 2m+1 und M = 2n is not possible:

$(2n)^2 = 405 + (2m+1)^2$ results in $4*n^2 = 4*m^2+4*m+406$ or

$4*(n^2-m^2-m)=406=2*203$ or $2*(n^2-m^2-m)=203$ **Contradiction!**

However, we do the trial and error cleverly and use the ClassPad: instead of increment 1

code := seq(f(D) , d, 1, 100) = {0,0, …, 0,1,0, …, 0,0,0},

It is valid D = 2m and M = 2n+1: now increment 2

With a small number $\varepsilon := 10^{-20}$ (possibly for sign correction)

code1 := seq(f(D) , d, 2, 100, **2**) =

{0,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,
1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} and

code2 := seq(f(D) , d, 102, 210, **2**) =

{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0}

This results in five solutions!

Here $f(D) = f_\varepsilon(D)$ is the following function:

Define $f(D) = (1-\text{signum}(\text{frac}((D^2+405)^{1/2}) - \varepsilon))\ /\ 2$

$\text{frac}((D^2+405)^{1/2}) - \varepsilon > 0$, if $(D^2+405)^{1/2}$ not integer

$\text{frac}((D^2+405)^{1/2}) - \varepsilon < 0$, if $(D^2+405)^{1/2}$ integer

The sign function signum(...) returns the sign as a number code +1 or -1, (signum(0) is not defined), e.g

$\text{signum}(\mathbf{0}.485281374 - 10^{-20}) = +1$

$\text{signum}(\mathbf{0}.000000000 - 10^{-20}) = -1$

Finally f(D) gives the desired code.

It can be seen that there are five integer solutions.

D := seq(D, D, 2, 100, 2) * code1 =

{0,0,6,0,0,0,0,0,18,0,0,0,0,0,0,0,0,0,38,0,0,0,0,0,0,0,0,0,0,0,
0,0,66,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0} and

M := $(D^2+405)^{½}$ * code1 =

{0,0,21,0,0,0,0,0,27,0,0,0,0,0,0,0,0,0,43,0,0,0,0,0,0,0,0,0,0,0,
0,0,69,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}

So we already have the four pairs

(M, D) = (21,6), (27,18), (43,38), (69,66)

found!

Furthermore

D := seq(D, D, 102, 210, 2) * code2 =
{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,203,0,0,0,0}

and

M := $(D^2+405)^{1/2}$ * code2 =
{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,202,0,0,0,0}

We have found the fifth pair:

(M, D) = (203,202)

We have found the five pairs:

(M, D) = (21,6), (27,18), (43,38), (69,66), (203,202)

The names of the mothers (M):

Mrs.Robinson; Mrs.Evans; Mrs.Jones; Mrs.Smith; Mrs.Brown

the names of the daughters(D): **Bessie; Annie; Mary; Emily, Ada**

Mrs. Robinson spent 6 shillings more than Mrs. Evans, who spent about a quarter as much as Mrs. Jones.

It holds: 6s. = 6*12d. = 6*12*4f. = 288f.,

4*441f. = 1764f. near 1849

M^2 = {21,27,43,69,203}^{2} = {441,729=441+288,1849,4761,41209}

Now the solution can be found quickly!

The five pairs are

(M, D) = (21,6), (27,18), (43,38), (69,66=18(Mary)+48), (203,202)

= (Mrs.Evans,6),(Mrs.Robinson,18),(Mrs.Jones,38),(69,66),(203,202)

Mrs. Smith spent the most: (Mrs.Smith = 203, 202)

Thus (Mrs.Brown = 69 = 63 + 6(Bessie), 66).

Now the daughters can be assigned:

Mrs. Brown(=69 feet) bought 21 **yards(=63 feet)** more than Bessie - one of the girls. Thus Bessie **= 6 feet** and the first pair is

(21,6) = (Mrs.Evans = 21, Bessie Evans = 6)

Annie bought 16 **yards (=48 feet)** more than Mary
and spent **£**3, 0 **shillings** and 8 **pence (=2912f.)** more than Emily.

3£+0s.+8d. = 3*20*12d.+8d. = 728d. = 4*728f. = 2912f.

Annie bought 16 **yards (=48 feet)** more than Mary
and spent **£**3, 0 **shillings** and 8 **pence (=2912f.)** more than Emily.

3£+0s.+8d. = 3*20*12d.+8d. = 728d. = 4*728f. = 2912f.

D^2 = {6, 18(Mary), 38, 66=18+48(Annie), 202}^{2} =

{36, 324, 1444(Emily), 4356=1444+2912(Annie), 40804},

because 2912 = 4356-1444, i.e.

(M=69, D=66) = **(Mrs. Brown, Annie Brown)**,

(M=27, D=18) = **(Mrs. Robinson, Mary Robinson),**

(M=43, D=38) = **(Mrs. Jones, Emily Jones)**.

Finally stays (M=203, D=202) = **(Mrs. Smith, Ada Smith)**.

Task solved!

Direct solution of the Diophantine equation

$M^2 = D^2 + 405$, i.e. $M^2 - D^2 = 405$ or

$$(M + D) * (M - D) = 1 * 3^4 * 5$$

(3. binomial formula, prime factorization)

The following three systems of equations result:

$M – D = 1$ and $M + D = 3^4 * 5 = 405$

$M – D = 3$ and $M + D = 1 * 3^3 * 5 = 135$

$M – D = 5$ and $M + D = 1 * 3^4 = 81$

$M – D = 3^2 = 9$ and $M + D = 1 * 3^2 * 5 = 45$

$M – D = 3*5 = 15$ and $M + D = 1 * 3^3 = 27$

which can easily be solved quickly by hand (in your head):

(M, D) = (21, 6), (27, 18), (43, 38), (69, 66), (203, 202)

Thus the pairs (M,D), see above,

(M=21,D=6) = (Mrs.Evans, Bessie Evans)

(M=27, D=18) = **(Mrs. Robinson, Mary Robinson),**

(M=43, D=38) = **(Mrs. Jones, Emily Jones)**.

(M=69, D=66) = **(Mrs. Brown, Annie Brown)**,

(M=203, D=202) = **(Mrs. Smith, Ada Smith)**.

Task solved!

HTWD Hochschule für Technik und Wirtschaft Dresden University of Applied Sciences

Another direct solution:

Write $M^2 = D^2 + 405$ with $M = D+K$, i.e.

$M^2 = (D+K)^2 = D^2 + 2 * D * K + K^2 = D^2 + 405$

Thus $K^2 + 2 * D * K - 405 = 0$

and $D = -K/2 + 405/(2K) = - K / 2 + 1 * 3^4 * 5 / (2K)$

Here we check with the ClassPad II:

fRound(seq(-K/2 + 405/(2K) ,K,1,21,2), 2) =

{202, 66, 38, 25.43, 18, 12.91, 9.08, 6, 3.41, 1.16, -0.86}

we have five integer solutions D = {202, 66, 38, 18, 6} and

M = D+K = {202+1, 66+3, 38+5, 18+9, 6+15} = {203, 69, 43, 27, 21}

Finally (M, D) = (203,202), (69,66), (43,38), (27,18), (21,6)

References:

Dudeney, H.E. (1917, 1958). ***Amusements in Mathematics*.** Nelson London, pp. 26-27 http://djm.cc/library/Amusements_in_Mathematics_Dudeney_edited02.pdf

Hemme, H.(2019). ***Die Holländer kaufen Schweine***, Spektrum der Wissenschaft, https://www.spektrum.de/raetsel/die-hollaender-kaufen-schweine/1579982

Hvorecký, J., Korenova, L., Barot, T. (2022). ***Combining Brute Force and IT to Solve Difficult Problems.*** Electronic Proceedings of the 27th Asian Technology Conference in Mathematics (ATCM) 2022, Dec. 9-12, Prague, https://atcm.mathandtech.org/EP2022/regular/21959.pdf

May, J. (1739). ***Three Dutchmen***, The Ladies' Diary: Puzzle https://www.cantorsparadise.com/three-dutchmen-f9f08ac19d73

Workman, W.P. (1906, 1918). ***The Tutorial Arithmetic: with Answers***. Univ. Tutorial Press London, p. 482 https://cs.smu.ca/~dawson/workman/workman3.gif


Thank you very much for your attention!