HOCHSCHULE FÜR TECHNIK UND WIRTSCHAFT DRESDEN
UNIVERSITY OF APPLIED SCIENCES

IMPROVEMENT OF STUDENTS' UNDERSTANDING OF ALGEBRA OF SETS AND VENN-DIAGRAMS

Preface:

In **elementary algebra** students consider **expressions** with variables and study general rules with **operators** (introduced in the arithmetic with numbers), e.g. $3x^2-2x \cdot y+c$, and solve **equations**, e.g. $3x^2-2x \cdot y+c=0$.
Here $3x^2$, $2x \cdot y$ and c are several **terms** and **–** , **+** and $\cdot$ are **operators** between the terms.

From the same point of view now we consider the **algebra of sets**. Again we have terms, expressions and equations with sets A, B, C,... and the operators \ , ∪ and ∩.

IMPROVEMENT OF STUDENTS' UNDERSTANDING OF ALGEBRA OF SETS AND VENN-DIAGRAMS

The basics of set theory consists in sets, elements, lists, set-builder notation, subsets, equal sets, the empty set, union, intersection, difference, symmetric difference and Cartesian product and power sets.

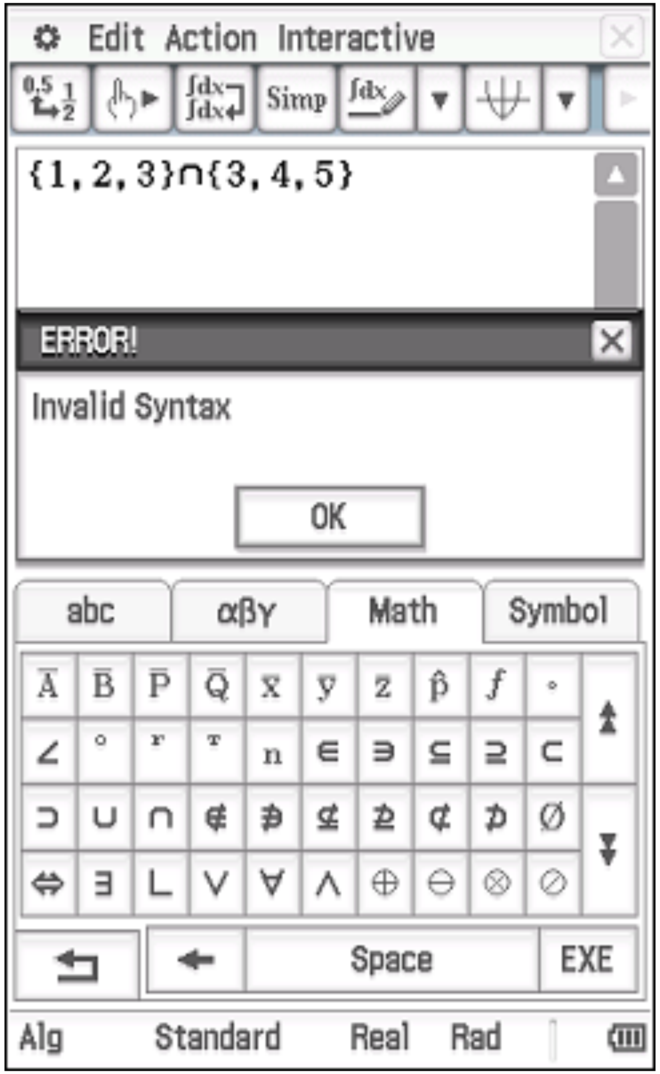
Sets are one of the most fundamental concepts in mathematics but we can't calculate with set operations and set relations on any calculators, e.g. on the ClassPad.
The graphic calculators (sometimes with CAS) were introduced in many German schools.

On the other hand we can write in the text mode with special symbols of the set theory in the ClassPad, e.g. $\in$, $\notin$, $\cup$, $\cap$, \ , $\subset$, $\subseteq$, $\neq$, ….

The problem with calculators:

We can write in the text mode with special symbols of the set theory

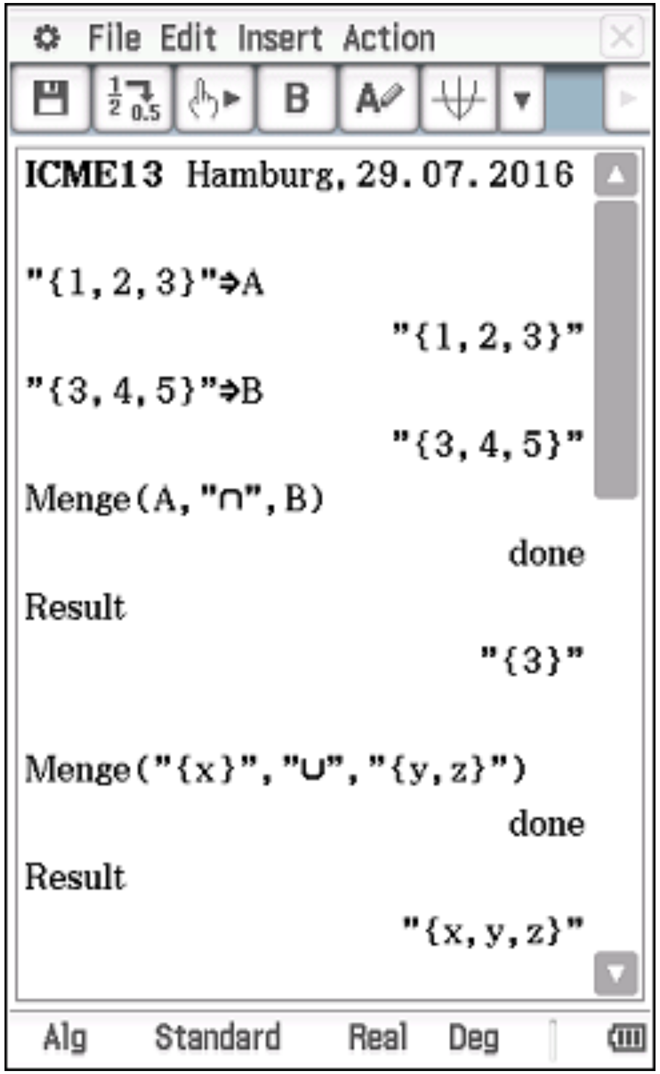
but we can't calculate with set operations and set relations.



1. The first solution:

Students wrote a single Basic-program **Menge** for operations and relations in the set theory

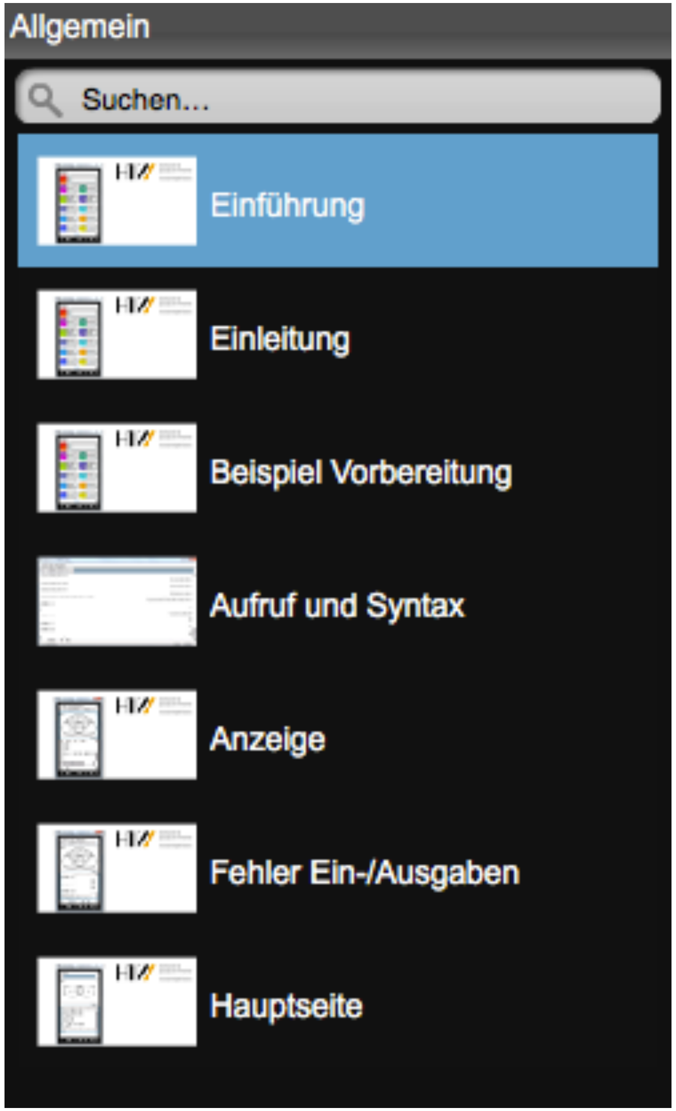
See the manual:
http://www.informatik.htw-dresden.de/~paditz/Bedienungsanleitung_Menge_Version_0_9_13.pdf



The solution by P.Koehler:

Paul wrote a single Basic-program **StrOVenn** for operations in the set theory and published videotutorials and vcp-files.

See the web-page:
http://koehler.blackscripts.de/sub/classpad_menge/



2. More possible solutions:

Other students of informatics tried to introduce the set theory in the operating system of the calculator in the following manner:

They give several ways of solution:

1. Create a so called **AddIn for ClassPad330** to calculate with sets of real numbers or finite sets of words and
2. Create a **Basic-program for ClassPad400** to calculate with finite sets of numbers or words.
3. For the visualizing we get **Venn-diagrams** for a basic set Ω and up to four subsets A, B, C, D of Ω.
4. An important application consists in the **basics of probability theory** if the sets are random events.

Examples of Sets:

In the mathematics these days essentially **everything is a set**.

Some **knowledge of the set theory** is a necessary part of the background everyone needs for the further study of math.

We want to review here elementary-school set theory and the algebra of sets. **Finally we will use the calculator to compute and draw Venn-diagrams.**

A set is a collection of things (called its members or elements), e.g. the set of the **prime numbers** less 10: **A={2,3,5,7}** or the **set B of all real solutions of the polynomial equation** $x^4-17x^3+101x^2-247x+210=0$, i.e. B=A. B and A were defined in different ways.

Let be $\varnothing = \{x \mid x \neq x\}$ the **empty set**. We can form the set $\{\varnothing\}$ and $\{\{\varnothing\}\}$ and finally $\{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}\}$, a three element set.

Two familiar operations on sets are the **union** and **intersection**.

For example $\{x,y\}\cup\{z\}=\{x,y,z\}$ or $\{2,3,5,7\}\cap\{1,2,3,4\}=\{2,3\}$.

Any set A will have one or more subsets.

In fact if A has n elements then A has 2^n subsets.

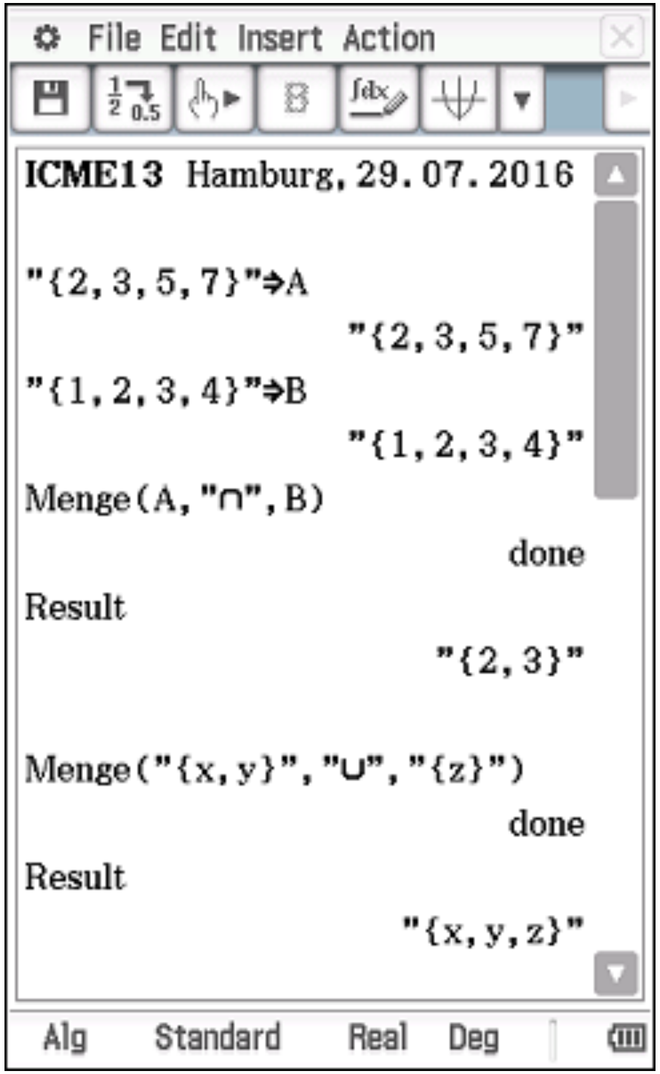
We can gather all of the subsets of A into one collection called the **power set** of A.

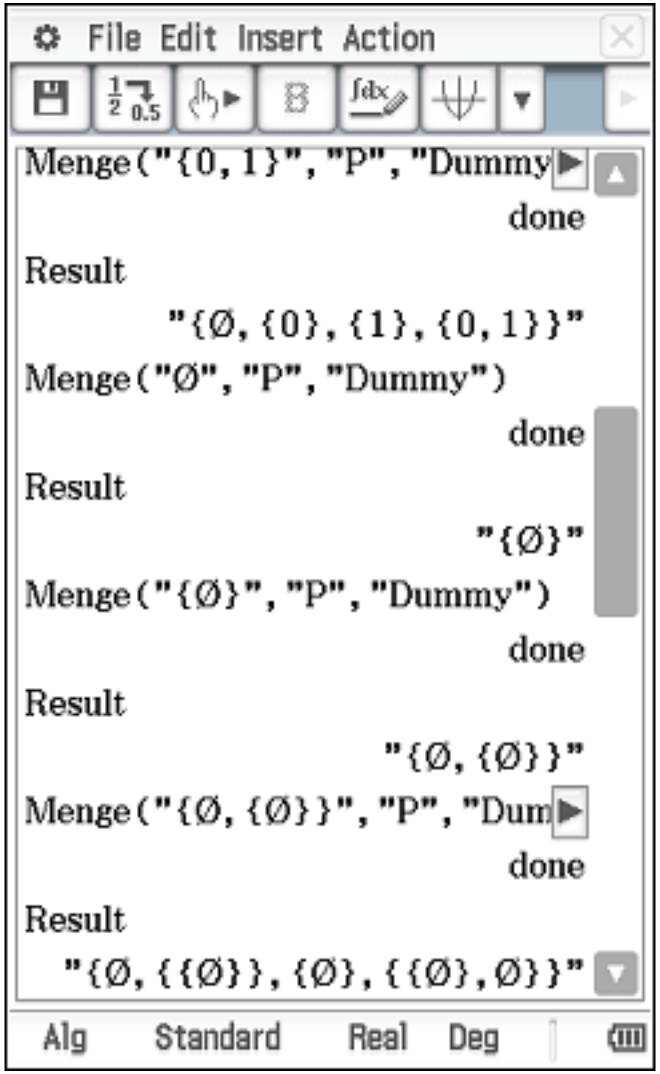
For example
the **power set of {0,1}** is $\{\varnothing,\{0\},\{1\},\{0,1\}\}$ and
the **power set of** $\varnothing$ is $\{\varnothing\}$ and
the **power set of** $\{\varnothing\}$ is $\{\varnothing,\{\varnothing\}\}$.

Understanding of Sets and Venn-Diagrams

{2,3,5,7∩{1,2,3,4} ={2,3} or {x,y}∪{z}={x,y,z}.

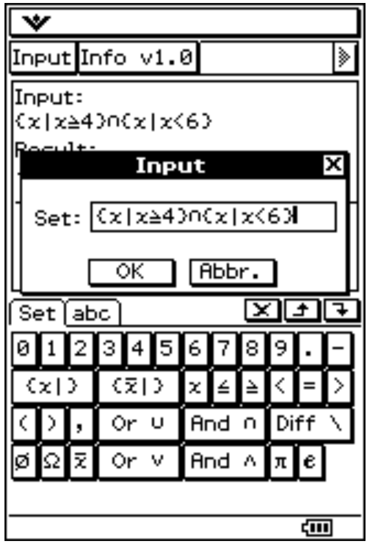
the **power set of {0,1}** is {∅,{0},{1},{0,1}} and
the **power set of** ∅ is {∅} and
the **power set of {∅}** is {∅,{∅}}.

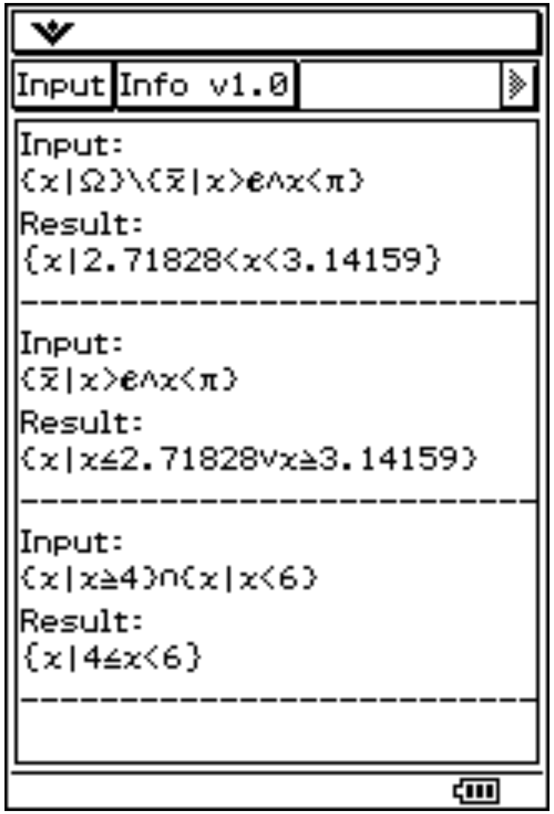




3. The AddIn „REAL SETS“ for ClassPad330:

The students wrote a program in C++ and used then the CASIO-SDK (software development kid) to compile the source program into a ClassPad AddIn.

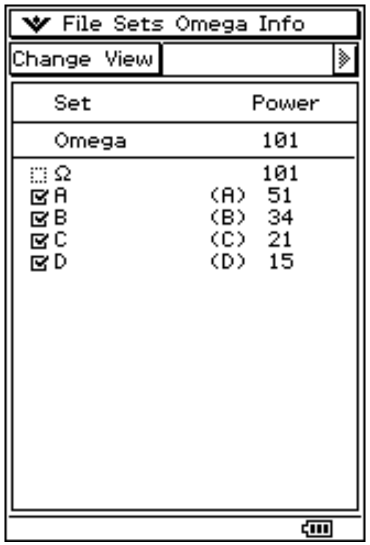


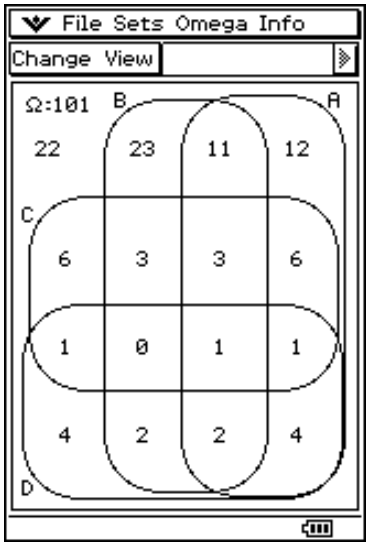


4. The AddIn „VENN4SETS“ for ClassPad330:

Other students wrote a program in C++ and used then the CASIO-SDK (software development kid) to compile the source program into a ClassPad AddIn.

Let be $\Omega=\{0,1,2,3,\dots,99,100\}$,
$A=\{0,2,4,6,\dots,98,100\}$, $B=\{0,3,6,9,\dots,96,99\}$,
$C=\{0,5,10,15,\dots,95,100\}$, $D=\{0,7,14,21,\dots,91,98\}$





5. THE PROGRAM "STROVENN" FOR CP400

The program **StrOVenn** works with sets, which are **Str**ing-variables. The **O**utput is a **Venn**-diagram. Therefore the program is named **StrOVenn**. The syntax of the input is

StrOVenn(Ω,A,B,C,D,4,2,1) or

StrOVenn(Ω,A,B,C,dummy,3,2,1) or

StrOVenn(Ω,A,B,dummy,dummy,2,2,1).

The last parameter 1 describes the type of data: numeric or alpha-numeric sets. Later the parameter 0 in the last position should be for the single type: numeric sets. Finally the fixed parameter 2 stands for a color Venn-diagram (ClassPad400) or 1 for a black/white diagram (ClassPad330).

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The tool – ClassPad 400

Again let be

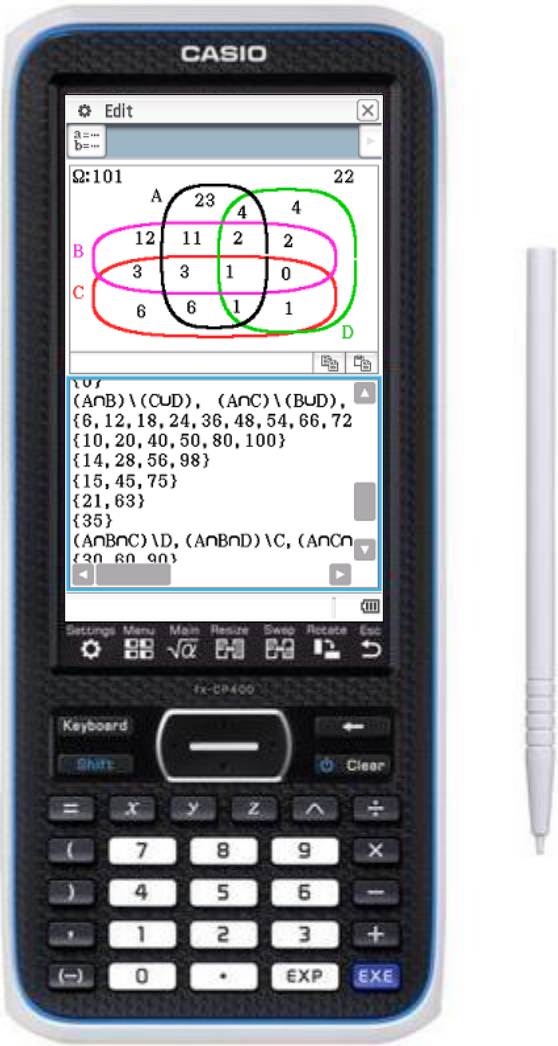
Ω=“{0,1,2,3,4,5,…,99,100}“

A=“{0,2,4,6,8,10,…,98,100}“

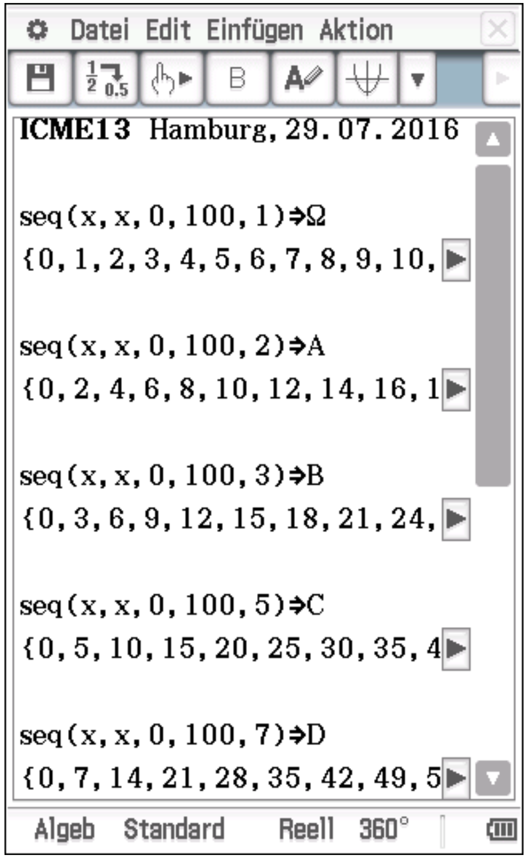
B=“{0,3,6,9,12,14,…,96,99}“

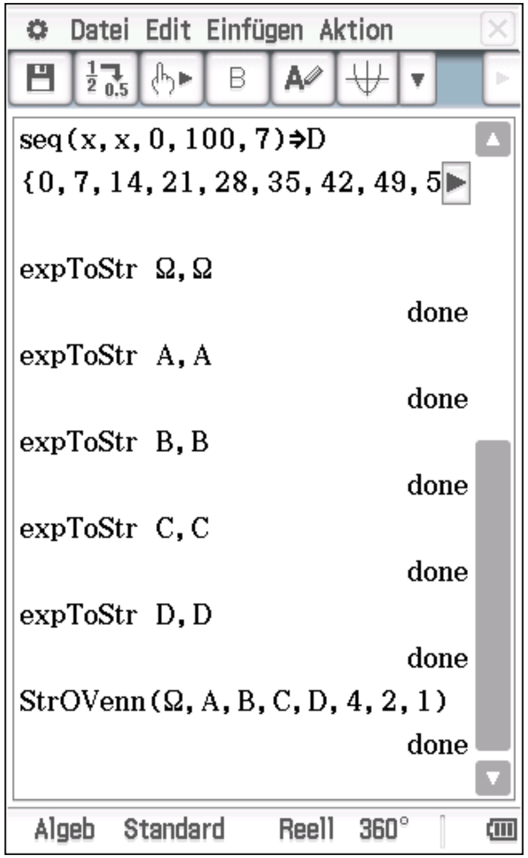
C=“{0,5,10,15,20,…,95,100}“

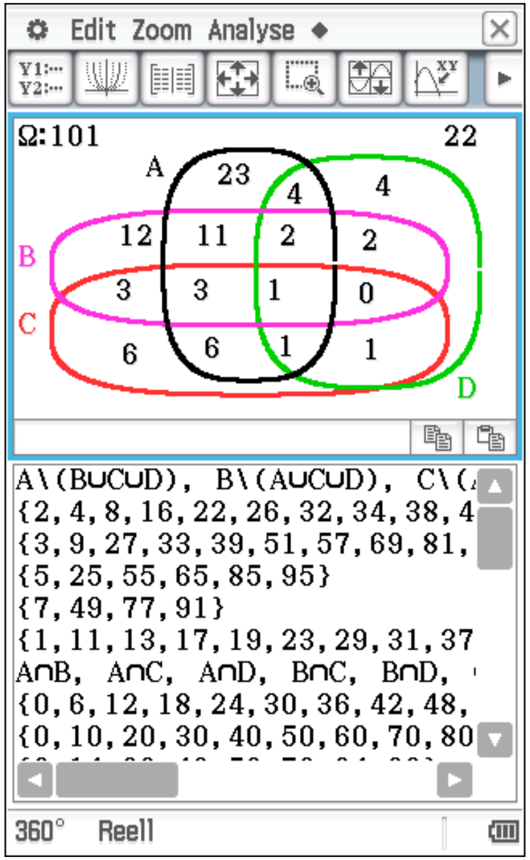
D=“{0,7,14,21,28,…,91,98}“



With **StrOVenn(Ω,A,B,C,D,4,2,1)** we get the results:







Understanding of Sets and Venn-Diagrams

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With the PC-ClassPad Manager we get the full screen:

Edit

```
A\(B∪C∪D), B\(A∪C∪D), C\(A∪B∪D), D\(A∪B∪C), Ω\(A∪B∪C∪D)
{2,4,8,16,22,26,32,34,38,44,46,52,58,62,64,68,74,76,82,86,88,92,94}
{3,9,27,33,39,51,57,69,81,87,93,99}
{5,25,55,65,85,95}
{7,49,77,91}
{1,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97}
A∩B, A∩C, A∩D, B∩C, B∩D, C∩D, A∩B∩C, A∩B∩D, A∩C∩D, B∩C∩D, A∩B∩C∩D
{0,6,12,18,24,30,36,42,48,54,60,66,72,78,84,90,96}
{0,10,20,30,40,50,60,70,80,90,100}
{0,14,28,42,56,70,84,98}
{0,15,30,45,60,75,90}
{0,21,42,63,84}
{0,35,70}
{0,30,60,90}
{0,42,84}
{0,70}
{0}
{0}
(A∩B)\(C∪D), (A∩C)\(B∪D), (A∩D)\(B∪C), (B∩C)\(A∪D), (B∩D)\(A∪C), (C∩D)\(A∪B)
{6,12,18,24,36,48,54,66,72,78,96}
{10,20,40,50,80,100}
{14,28,56,98}
{15,45,75}
{21,63}
{35}
(A∩B∩C)\D, (A∩B∩D)\C, (A∩C∩D)\B, (B∩C∩D)\A
{30,60,90}
{42,84}
{70}
Ø
```

A student discovered all prime numbers between 0 and 100 in the set

$$\Omega \setminus (A \cup B \cup C \cup D) = \{1,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97\}$$

where 1 is no prime number, but 2,3,5,7 (not in this set).

Probability theory: Let $P(\omega)=1/n$, for all $\omega \in \Omega$ and $n=|\Omega|=101$

What is the probability $\mathbf{P((A\cap B)\setminus(C\cup D))}$ of the difference $(A\cap B)\setminus(C\cup D)$?

What is the conditionel probability $\mathbf{P((A\cap B)|(C\cup D))}$ with the condition $(C\cup D)$?

By the help of the Venn-diagram it is easy to get the solution.

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Probability theory:

What is the probability **P((A∩B)\(C∪D))** of the difference

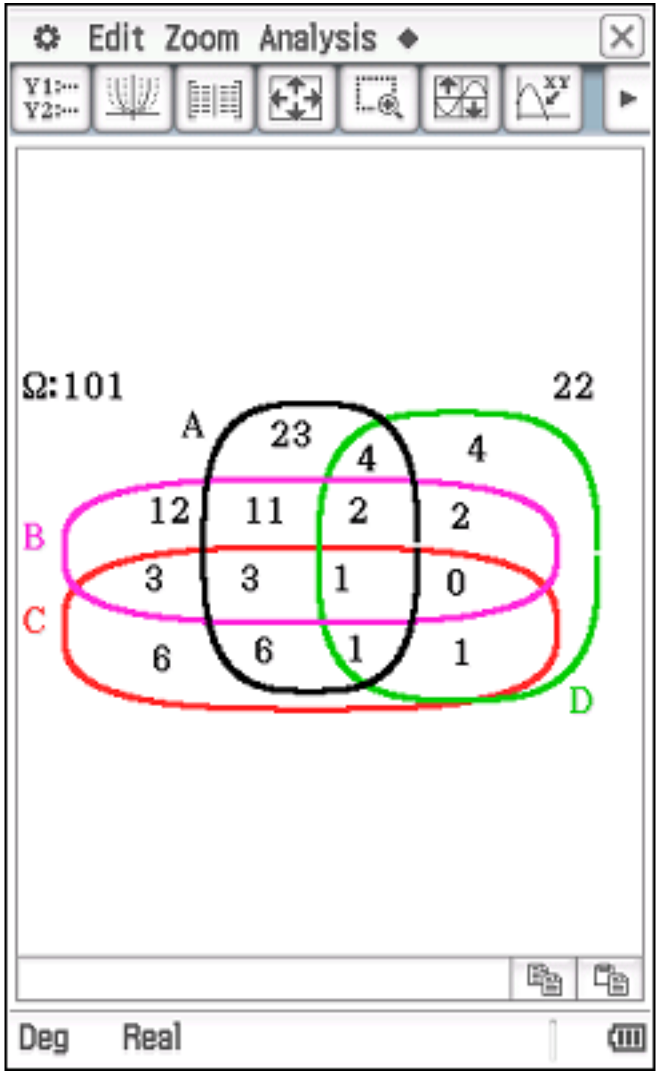
(A∩B)\(C∪D) ?

(17-6)/101=11/101

What is the conditionel probability **P((A∩B)|(C∪D))** with the condition (C∪D) ?

P((A∩B)∩(C∪D)) / P(C∪D) = 6/33 = 2/11

By the help of the Venn-diagram it is easy to get the solution.



Wallisch, F. (2015):

mp4-video of a student:

“Testdurchlauf-vierMengen”

http://www.informatik.htw-dresden.de/~paditz/Testdurchlauf-vierMengen.mp4

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