Using CAS in Math Education - first experience in Germany with the newest ClassPad technology CP400

Preface

The use of worksheets in mathematics instruction has a long tradition. The use of a worksheet should guide the students to a structured work. Instead of an oral instruction, which requires a synchronous work of all students, the worksheet individually and with their own timing can be processed next. The sequence of the work orders in the worksheet helps to recognize the logical structure of a problem; the work procedures help to penetrate the question.

The disadvantage of a sheet of paper with work orders is seen, that the tools which can be used must be made available about. The students do not know always, how to carry out its solution steps in detail. Moreover, in a classic worksheet are missing the self check of the results, a feedback of the partial steps and also the visualization of the results.

The new developed eActivity in the ClassPad represents an extremely rich extension of the worksheet. The eActivity combines the written representation of the setting of tasks of a worksheet with the tool level of the ClassPad.
These tools are the individual menus or modules, which the ClassPad offers: Computer algebra system, dynamic geometry software, computer statistics, curve plotter, and much more.

Thus, all tasks can be worked on with the possibilities of the computer. At the same time, the documentation of the work can be entered directly. The found results can to be visualized immediately or the results may be in a hidden file, can be viewed. It is an interactive work of the students, between setting of tasks and the results and control of the results themselves.
These new possibilities are to be used by many, when they use a ClassPad. Therefore, an uniform structure of an eActivity appears more than reasonable. The kind of the worksheet should be visible in the name of the file.

It is suggested using a single initial letter, which addresses to this request.

Category	D	D-power	Discovery of a term, a rule, the power, ...
Category	V	V-Heron	reconstructing (Verifying) of a procedure, ...
Category	ES	ES-LSE	Exercises with Solution (linear system of equations)
Category	ET	ET-LSE	Exercises with a Test character
Category	L	L-LSE	Learning units
Category	M	M-rectangle	Modelling a rectangle
Category	P	P-LSE	Protocol on a Linear System of Equations

1. Introduction

In this article the possibilities of the eActivity function are discussed. These electronic worksheets however not only have to be described, but also adapted among themselves. They are intended to supplement the textbook, but still be nevertheless so extensive that can be accessed as detailed as possible and straightforward to subject areas. Therefore, there are categories to create fixed and uniform expressions needed to ensure the users with better visibility and easy handling.

First, however, the applications of the eActivity have to be worked out. In this paper there are two parts, the first part concerns itself with the production and use of an eActivity. Different categories are presented, which are illustrated by examples. In the second part it concerns the technical questions. Which equipment is needed, how to begin, how to create an eActivity, and how to exchanges it?

2. Use of an eActivity

2.1 Repetition

2.1.1 Systematically

A well-stocked database provides an excellent way to quickly and easily repeat subject areas systematically. It is easy for the students to download on the internet the matching themes download and edit. Even with the tuition, the hassle of searching for material to fall away entirely. It can chosen selectively a specific theme, practiced a certain type of task and explain a difficult situation.

2.1.2 Close knowledge gaps

Another form of repetition can be found in the knowledge gaps. This could be not only specifically addressed, but also efficiently eliminated. It is the teacher made it easier to use already past learning units and sample tasks to be solved. This causes the productive repetitions produced a better detection and understanding.

2.2 Start of the lesson

The new medium ClassPad can also serve as an additional motivation, thus facilitating the start of the lesson. Thus, a lesson can begin with a new learning unit or with a protocol of the last hour. Also, homework can be prepared so better and can be compared illustrative.

2.3 Backup of results

A further application type could be the backup of the results, because students can write eActivities. This represents a good exercise and a necessary argumentation. Writing an eActivity presupposes not only a large portion of independence and self-employment, but also an experienced operating authority. Furthermore, the author must prepare on the one hand materially and on the other hand methodically. All of these given requirements can be seen as learning goals and serve the backing up of acquired knowledge.

3. Categories

The eActivities are divided into certain categories. Thus, the user can choose whether he wants to work out the material or a procedure would like to reconstruct only and understand. There is the possibility of working on tasks with or without solution method. But you can itself in addition, only topic-referred a new learning unit to acquire or under „modelling" to materially new tasks of text place themselves. Also it is possible to log lessons for themselves or to monitor for others.

A total of 7 categories are distinguished. Each has its own symbol.

We distinguish:

Category D: Discovery of a term, a rule, ...
Category V: Reconstructing of a procedure, ...
Category ES: Exercises with solution
Category ET: Exercises with a test character
Category L: Learning units
Category M: Modelling
Category P: Protocol

The structure of the directories should be arranged well, and teaching subjects (learning material) according to category (fig. 1) and content (fig. 2) be arranged in the directory.
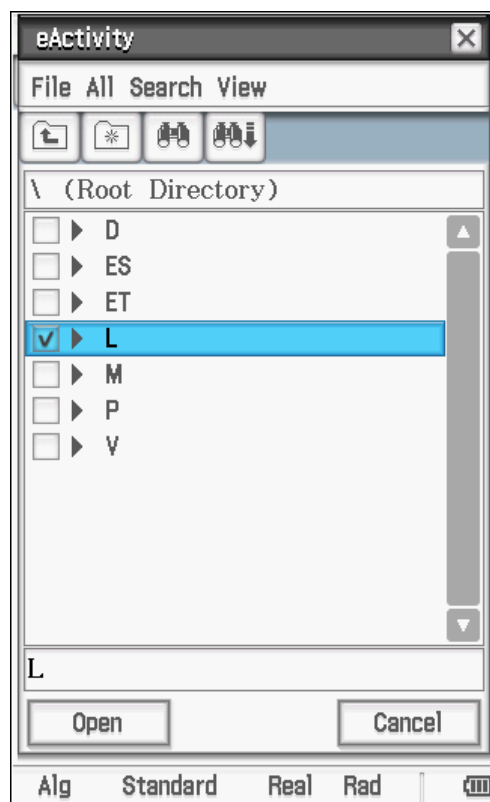

(fig. 1)
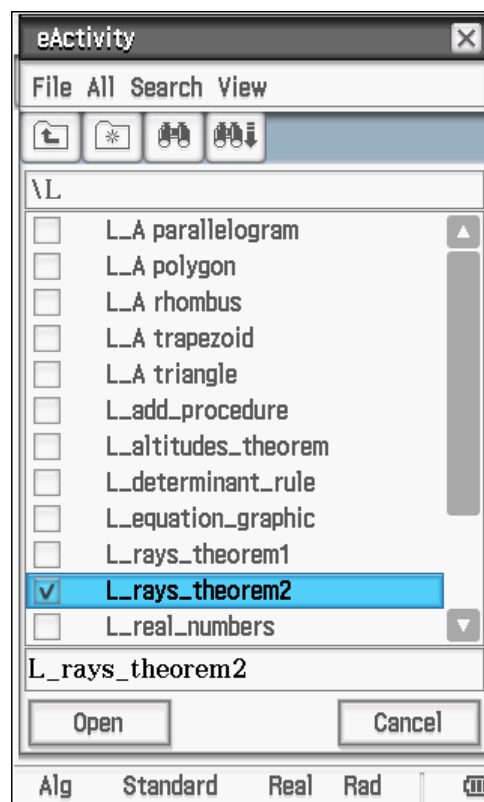
(fig. 2)

3.1 Category D: Discovery of a term, a rule, ...

It's not about to approach a task type on calculators explain, but prepare suitable suggestions by the way to discovery. In this category you proceed e.g. from a well-known concept (fig. 3) and it leads to properties and associated calculation rules (fig. 4).
Try again from the given keywords the logical and visible structure as possible (see also chap. 4) also here to produce.
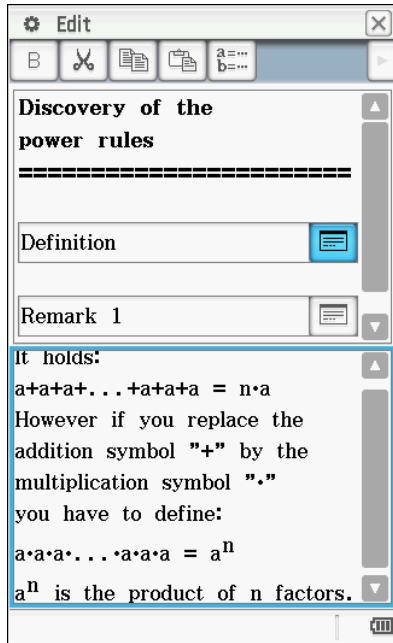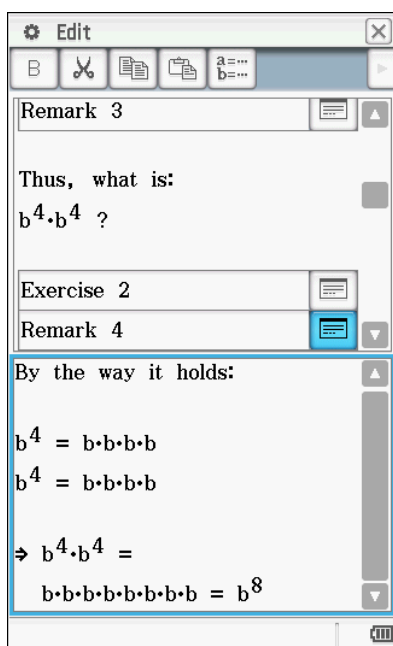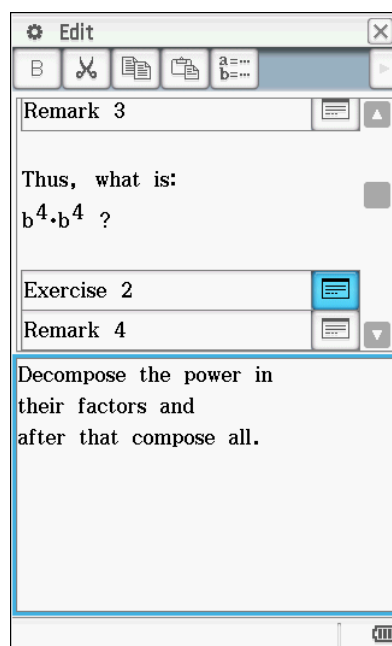
 (fig. 3)
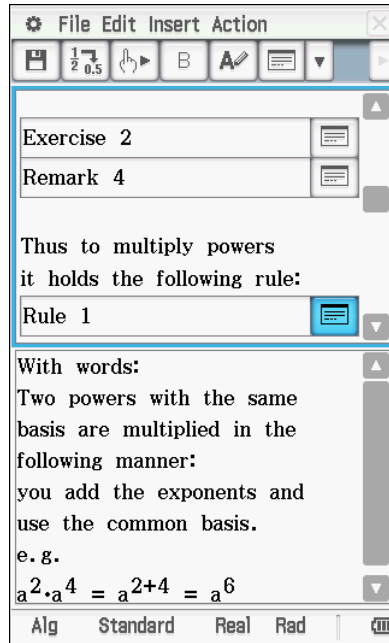
 (fig. 4)

Our example deals with the laws of powers:
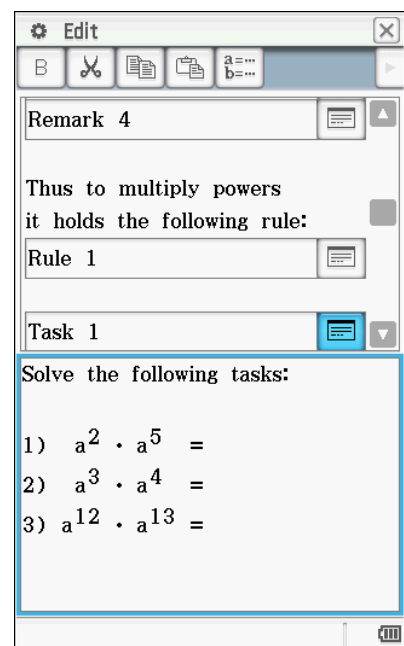It begins with the general definition (fig. 3) and by numerical observations to the rules (fig. 4). It is to be paid attention to neglect none of the accomplished steps. Everything should be formulated as detailed as possible. Nevertheless you should, in order to ensure a good overview, to keep the texts in the individual windows as short as possible. (fig. 5)

 (fig.5)

 (fig.6)

 (fig.7)

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Inserted questions help to advance the development. Under the keyword "exercise" you find after a short guidance the possibility of working on this process. This method to insert tasks in different places should the students to stimulate the self activity (fig.6).

According to a formulated rule it is also advisable to state a small setting of tasks in order to test what they have learned.

These tasks are however so easy to arrange that a sample solution appears invalid and the student is encouraged in his work.

Complete example of this type (D):

☼ Edit

B ✂ 📋 📋 a=⋯ b=⋯

Decompose the power in
their factors and
after that compose all.

☼ Edit

B ✂ 📋 📋 a=⋯ b=⋯

By the way it holds:

$a^2 = a \cdot a$
$a^3 = a \cdot a \cdot a$

$\Rightarrow a^2 \cdot a^3 = a \cdot a \cdot a \cdot a \cdot a = a^5$

☼ File Edit Insert Action

💾 ½ ⅐ ✋▶ B A✎ ▤ ▼

Thus what is:
$a^2 \cdot a^3$?

| Exercise 1 | ▤ |
| Remark 3 | ▤ |

Thus, what is:
$b^4 \cdot b^4$?

| Exercise 2 | ▤ |
| Remark 4 | ▤ |

Thus to multiply powers
it holds the following rule:

Alg   Standard   Real   Rad

☼ Edit

B ✂ 📋 📋 a=⋯ b=⋯

Decompose the power in
their factors and
after that compose all.

☼ Edit

B ✂ 📋 📋 a=⋯ b=⋯

By the way it holds:

$b^4 = b \cdot b \cdot b \cdot b$
$b^4 = b \cdot b \cdot b \cdot b$

$\Rightarrow b^4 \cdot b^4 =$
   $b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b = b^8$

☼ Edit

With words:

Two powers with the same basis are multiplied in the following manner:

you add the exponents and use the common basis.

e.g.

$$a^2 \cdot a^4 = a^{2+4} = a^6$$

☼ Edit

Solve the following tasks:

1) $a^2 \cdot a^5 =$

2) $a^3 \cdot a^4 =$

3) $a^{12} \cdot a^{13} =$

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it holds the following rule:

Rule 1

Task 1

Our next question is:
How to proof the equation
$$a^2 \cdot b^2 = (a \cdot b)^2 \ ?$$

Exercise 3

Remark 5

Rule 2

Alg   Standard   Real   Rad

☼ Edit

Tip:

Try it with skillful transforming!

☼ Edit

It holds:

$$a \cdot a \cdot b \cdot b$$
$$= a \cdot b \cdot a \cdot b$$
$$= (a \cdot b) \cdot (a \cdot b)$$
$$= (a \cdot b)^2$$

⚙ Edit   ☒

| B | ✂ | 🗐 | 📋 | a=⋯ b=⋯ | ▷ |

With words:

Powers with the same exponent are multiplied in the following manner:

you multiply all the bases and than use the common exponent.

e.g.

$$a^4 \cdot b^4 \cdot c^4 = (a \cdot b \cdot c)^4$$

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| 💾 | $\frac{1}{2}\frac{7}{0.5}$ | ✋▶ | B | A✐ | ▤ | ▼ |

Rule 2   ▤ ▲

Task 2   ▤

The last question is:
How to proof the rule
$$\left(a^s\right)^r = a^{(s \cdot r)} = a^{s \cdot r} \quad ?$$

Remark 6   ▤

Rule 3   ▤

Task 3   ▤
                                ▼

Alg   Standard   Real   Rad   ▥

⚙ Edit   ☒

| B | ✂ | 🗐 | 📋 | a=⋯ b=⋯ | ▷ |

Solve the following tasks:

1) $a^2 \cdot b^2 \cdot c^2 =$

2) $f^{13} \cdot g^{13} =$

⚙ Edit   ☒

| B | ✂ | 🗐 | 📋 | a=⋯ b=⋯ | ▷ |

For the term $\left(a^s\right)^r$ we have the following interpretation.

The term $\left(a^s\right)^r$ means:

It exist r factora of the kind a^s.

Thus it follows:

It exist s·r factors of the kind a.

Edit

With words:

We get a power of a power, say

a^s to the r-th power $\left(a^s\right)^r$,

by multiplying the exponents and using the given base.

e.g.

$$\left(a^2\right)^3 = a^{(2\cdot3)} = a^6$$

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Remark 6

Rule 3

Task 3

By the help of the rules 1 and 2 you get the rule for the quotients:

$$\frac{a^r}{a^s} = a^{(r-s)} = a^{r-s}$$

and

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

Alg   Standard   Real   Rad

Edit

Solve the following tasks:

1) $\left(a^2\right)^7 =$

2) $\left(a^5\right)^3 =$

3.2 Category V: Reconstructing of a procedure

The object of the category V is unique. It serves the better understanding of facts or general procedures.
In our example, we approach the solution procedure of square roots by the Heron's iteration. Under the keyword „use" (see also chap. 4) it is widely discussed, where the Heron's procedure finds its use (fig. 8).
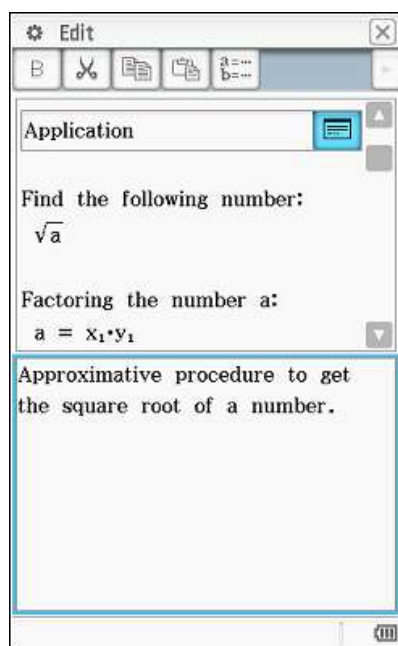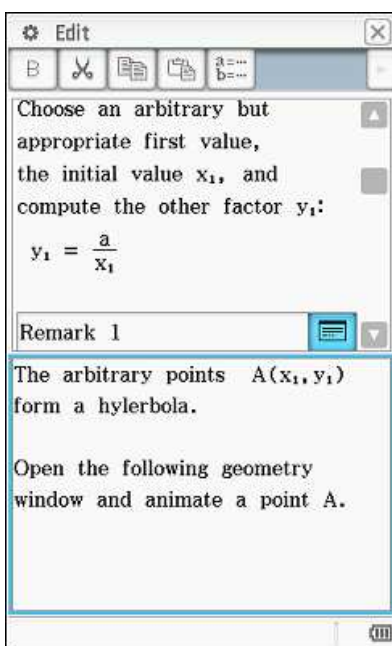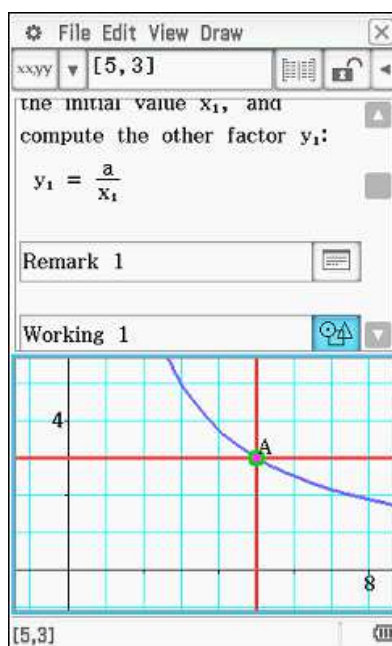
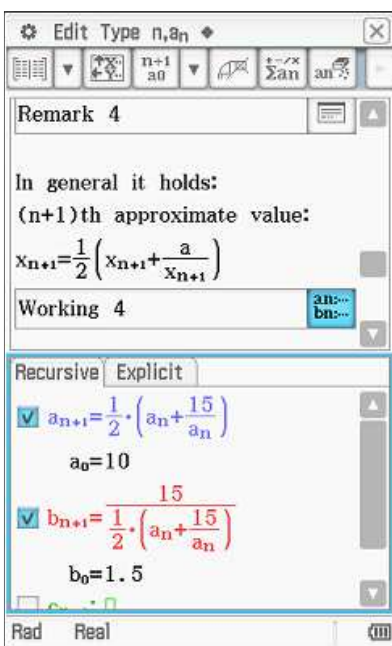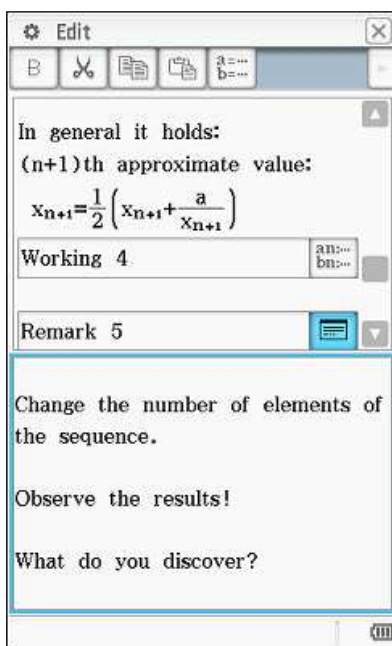| (fig. 8) | (fig. 9) | (fig. 10) |

In the application the further procedure is described. It is to be made certain here that if possible all steps are documented and illustrated (fig. 9-11).
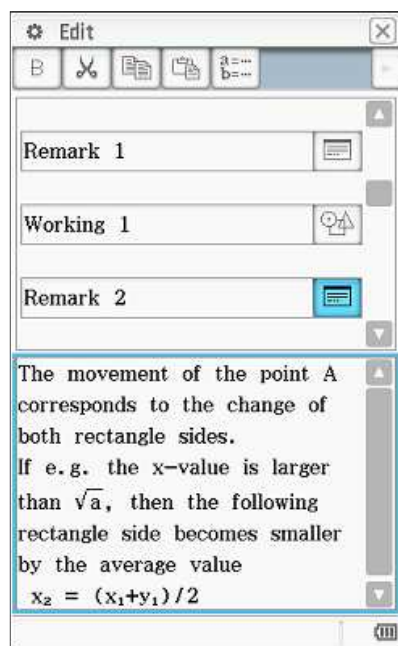

| (fig. 11) | (fig. 12) | (fig. 13) |

The most important attention is the application possibility of the procedure. It therefore is of advantage, if a task of sample is computed. This should happen however not as outstanding in the exercise categories step for step, but should be considered only as additional illustration (fig. 12).

In our example one is to extend and afterwards interpret the result of the task (fig. 13). This helps to understand the calculation steps and ensures again a deeper mental processing. Finally this category with a noticing set or a definition should be locked (fig. 14-15).
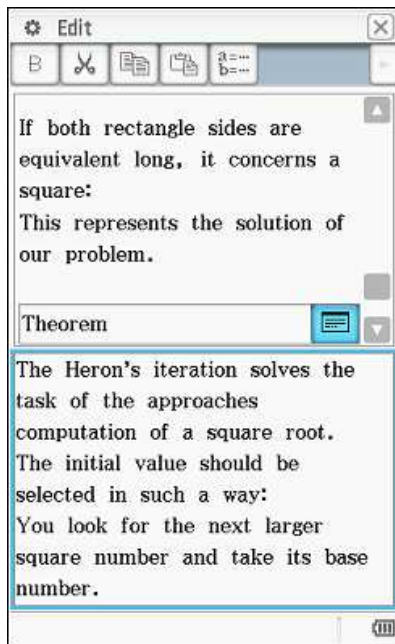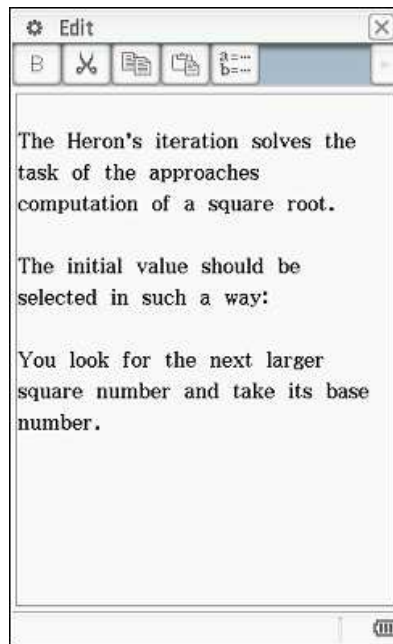
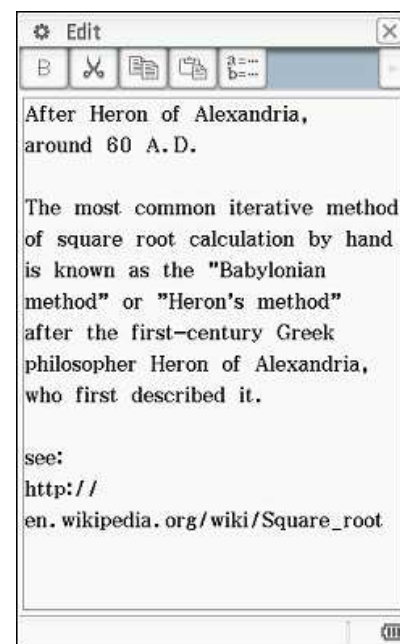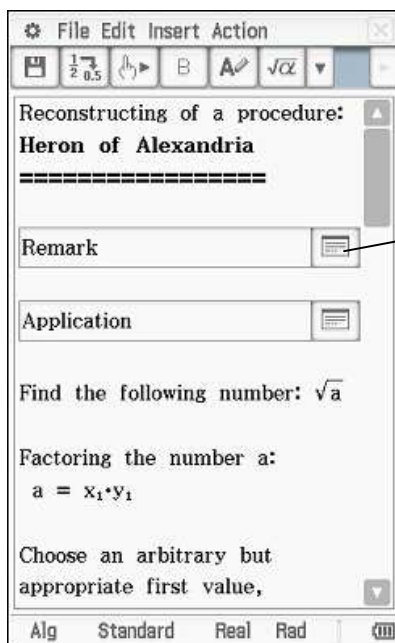(fig. 14)   (fig. 15)

Naturally it can be placed still another control task, in order to give to the learning student an assistance for examining its acquired knowledge.

Complete example of this type (V):

⚙ Edit   ✕

B   ✂   📋   📋   a=⋯
                  b=⋯

Approximative procedure to get
the square root of a number.

⚙ File Edit Insert Action   ✕

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Application   ☰ ▲

Find the following number: $\sqrt{a}$

Factoring the number a:
 $a = x_1 \cdot y_1$

Choose an arbitrary but
appropriate first value,
the initial value x_1, and
compute the other factor y_1:

$y_1 = \dfrac{a}{x_1}$

Remark 1   ☰ ▼

Alg   Standard   Real Rad   ▥

⚙ Edit   ✕

B   ✂   📋   📋   a=⋯
                  b=⋯

The arbitrary points $A(x_1, y_1)$
form a hylerbola.

Open the following geometry
window and animate a point A.

⚙ File Edit View Draw   ✕

x+y= ▼ y=15/x   📖 🔓 ◄

8

The solution of the equation
$x^2 = a$
corresponds to the intersection of
the hyperbola with the bisector.

⚙ Edit   ✕

B   ✂   📋   📋   a=⋯
                  b=⋯

The movement of the point A
corresponds to the change of both
rectangle sides.

If e.g. the x−value is larger than
$\sqrt{a}$, then the following rectangle
side becomes smaller by the
average value $x_2 = (x_1+y_1)/2$

−4

8

A

⚙ File Edit Insert Action   ✕

💾 ½ ⁷₀.₅ ✍► B   A✎ √α ▼

Remark 1   ☰ ▲

Working 1   ⊕△

Remark 2   ☰

Remark 3   ☰

Working 2   ⊕△

Formation of the average value:
$x_2 = \dfrac{x_1+y_1}{2} = \dfrac{1}{2}(x_1+y_1)$

Replace y_1 by $\dfrac{a}{x_1}$ ▼

Alg   Standard   Real Rad   ▥

☼ File Edit View Draw

20

Eq: $y=15/x$

Eq: $y=x$

15

Eq: $y=-x$

-5

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B | 1⃗ 2 0.5 | 👆▶ | B | A✎ | √α | ▼

Working 2

Formation of the average value:
$$x_2 = \frac{x_1+y_1}{2} = \frac{1}{2}(x_1+y_1)$$

Replace y_1 by $\frac{a}{x_1}$

(2nd approximate value):
$$x_2 = \frac{1}{2}\left(x_1+\frac{a}{x_1}\right)$$

It must apply:
$$y_2 \cdot x_2 = a$$

Computation of y_2:

Working 3   √α

Alg   Standard   Real   Rad   ▥

☼ Edit Action Interactive

0.5 1⃗ 2 | 👆▶ | ∫dx⃗ ∫dx◀ | Simp | ∫dx✎ | ▼ | ⊌ | ▼

$$\frac{1}{2}\left(x_1+\frac{a}{x_1}\right) \Rightarrow x_2$$

$$\frac{x_1+\frac{a}{x_1}}{2}$$

$$y_2 \cdot x_2 = a$$

$$\frac{y_2 \cdot \left(x_1+\frac{a}{x_1}\right)}{2} = a$$

$$\text{solve}\left(\frac{y_2 \cdot \left(x_1+\frac{a}{x_1}\right)}{2} = a, y_2\right)$$

$$\left\{y_2 = \frac{2 \cdot a \cdot x_1}{x_1{}^2 + a}\right\}$$

Real   Rad   ▥

☼ Edit

B | ✂ | 🗐 | 🗐 | a:·· b:··

The definition of the rectangle sides is generalized here:

We interpret all rectangle sides as elements of sequences and say

x_n becomes a_n and
y_n becomes b_n

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B | 1⃗ 2 0.5 | 👆▶ | B | A✎ | √α | ▼

Computation of y_2:

Working 3   √α

Remark 4   ▦

In general it holds:
(n+1)th approximate value:
$$x_{n+1} = \frac{1}{2}\left(x_{n+1}+\frac{a}{x_{n+1}}\right)$$

Working 4   a:·· b:··

Remark 5   ▦

Alg   Standard   Real   Rad   ▥

☼ Edit Graph ◆

⊌ | ⊹ | ⊡ | a:·· b:·· | ⟷ | ⟷ | +-/× Σan

Recursive | Explicit

☑ $a_{n+1} = \frac{1}{2}\cdot\left(a_n+\frac{15}{a_n}\right)$

$a_0 = 10$

☑ $b_{n+1} = \frac{1}{2}\cdot\left(a_n+\frac{15}{a_n}\right)$

$b_0 = 1.5$

$a_{n+1} = 1/2 \cdot (a_n+15/a_n)$

n	a_n	b_n
1	5.75	2.6087
2	4.1793	3.5891
3	3.8842	3.8618
4	3.8730	3.8730
5	3.8730	3.8730

5.75   🗐 🗐

Rad   Real   ▥

⚙ Edit   ✕

B ✂ ▤ ▥ a=···
                  b=···

Change the number of elements of
the sequence.

Observe the results!

What do you discover?

⚙ Edit   ✕

B ✂ ▤ ▥ a=···
                  b=···

The Heron's iteration solves the
task of the approaches
computation of a square root.

The initial value should be
selected in such a way:

You look for the next larger
square number and take its base
number.

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💾 ½⁷₀.₅ 👆▸ B A✎ √α ▾

Remark 5   ▤

If both rectangle sides are
equivalent long, it concerns a
square:
This represents the solution of
our problem.

Theorem   ▤

Task 1   ▤

Working 5   an···
                                 bn···

Alg   Standard   Real   Rad   ▥

⚙ Edit   ✕

B ✂ ▤ ▥ a=···
                  b=···

Compute by the help of Heron's
procedure the solution of the
following equation:

$$x^2 = 17$$

⚙ Edit Type n,aₙ ◆   ✕

▤▤ ▾ ▦ n+1 ▾ ⌀ Σan an
                a0

Recursive | Explicit

□ a_{n+1}: □
      $a_0 = 0$
□ b_{n+1}: □
      $b_0 = 0$
□ c_{n+1}: □
      $c_0 = 0$

      n a_n
[Null   Null]

Rad   Real   ▥

## 3.3 Category ES: Exercises with solution

Two further categories represent the exercises.

First the exercises with solution method are mentioned. They lead the student step for step by a certain type of task and can be regarded as a sample solution. In the further process of an application the student should start to solve tasks independent without other help and then with the hidden solution control itself.

It is to be made certain that the examples are not too heavily selected at first, since otherwise understanding the task structure is made more difficult. Only in the process of the worksheet the degree of difficulty can increase. Each working step must be described and accomplished obviously, as with the addition process in fig. 16.
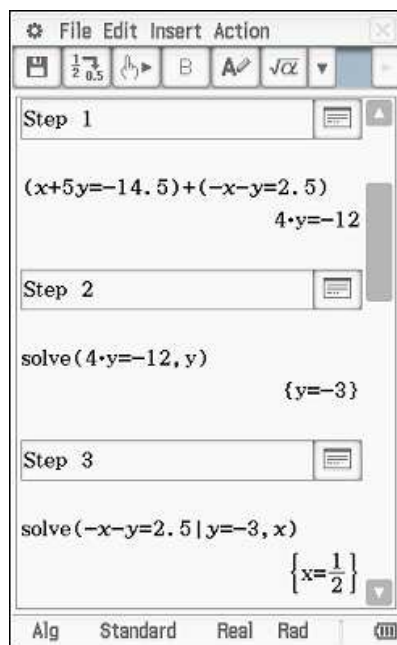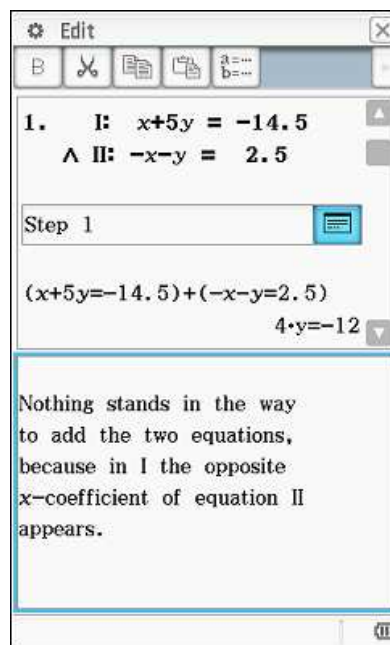
(fig. 16)   (fig. 17)

As you see, it follows after a task clearly placed a gradually accomplished way to the solution. First it is described by clicking the individual windows, what is afterwards accomplished (fig. 17). In order to promote the operating competence of the student, the way to the necessary instruction should be described likewise (fig. 18). If this instruction occurs several times in a worksheet, naturally an unique explanation is sufficient.

The instructions in the inserted windows should be as simple and clearly posed as possible. One should strive to keep the text short so that it can be seized with a view.
During the description it should be fallen back to firm of keywords. Only solution-relevant informations should be mentioned.

For the self check of the students it should be placed far, as previously mentioned, also tasks with independent window of working (fig. 19). These can be compared after the treatment with the sample solutions inserted at the end (fig. 19). Here becomes fast clear, where still pent-up demand exists or which steps are already controlled.
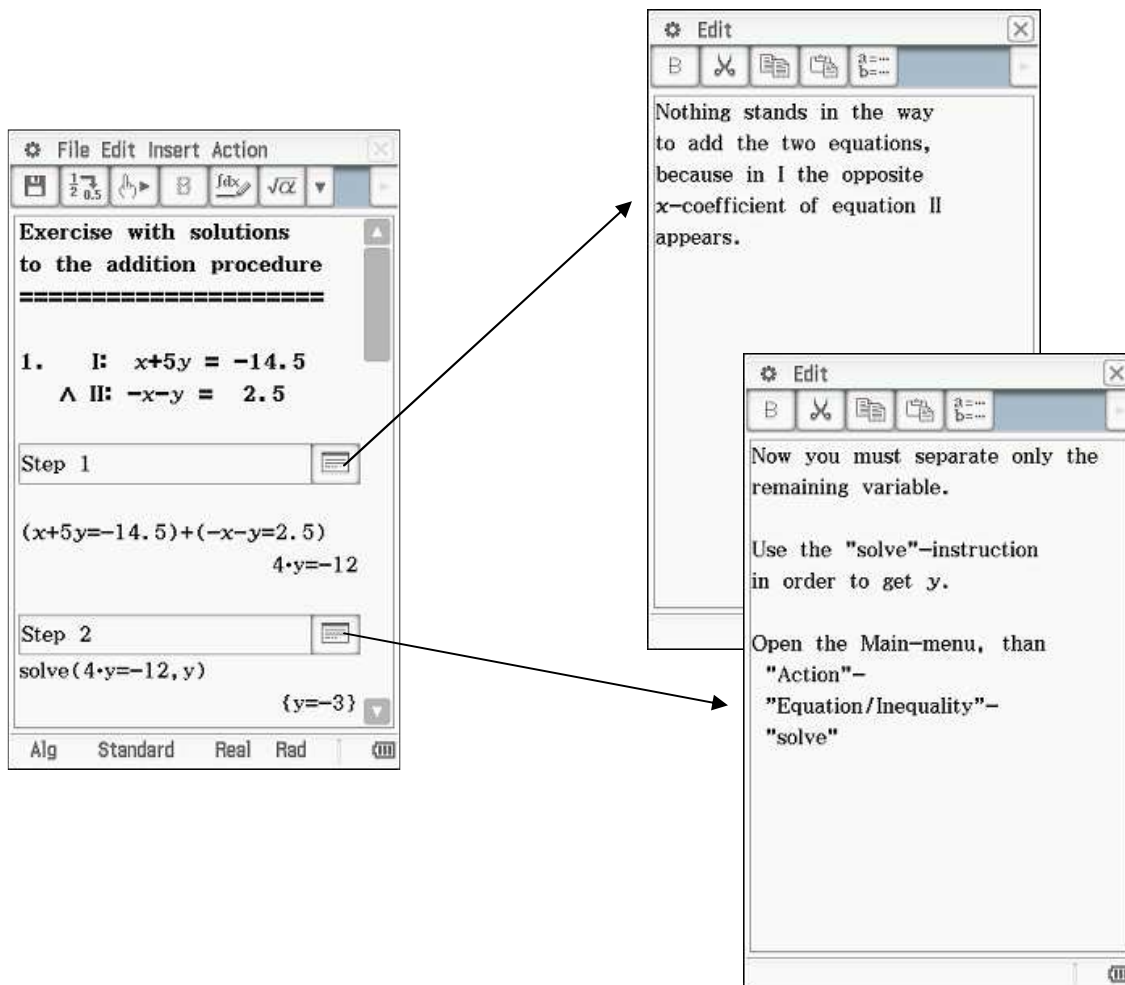
Edit

B ✂ 📋 📋 a:=b:=

$(x+5y=-14.5)+(-x-y=2.5)$
$$4 \cdot y = -12$$

Step 2

solve$(4 \cdot y = -12, y)$
$$\{y = -3\}$$

Now you must separate only the remaining variable.
Use the "solve"-instruction in order to get y.
Open the Main-menu, than "Action"- "Equation/Inequality"- "solve"

(fig. 18)

File Edit Insert Action

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3.   I: $x - 5y = 18.2$
   ∧ II: $x - 2y = 6.2$

Working with 3.   √α

4.   I: $2x - 3y = -39$
   ∧ II: $-2x + 4y = 51$

Working with 4.   √α
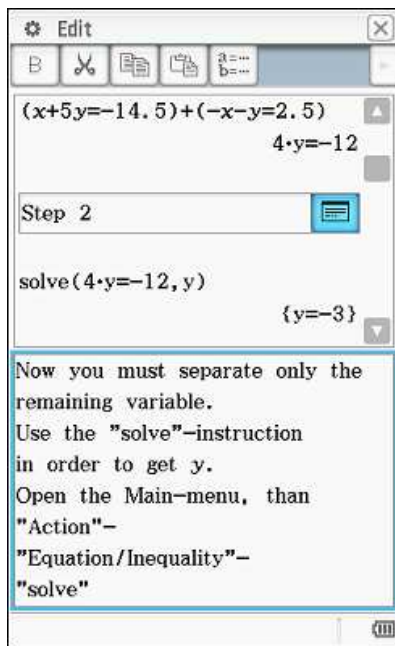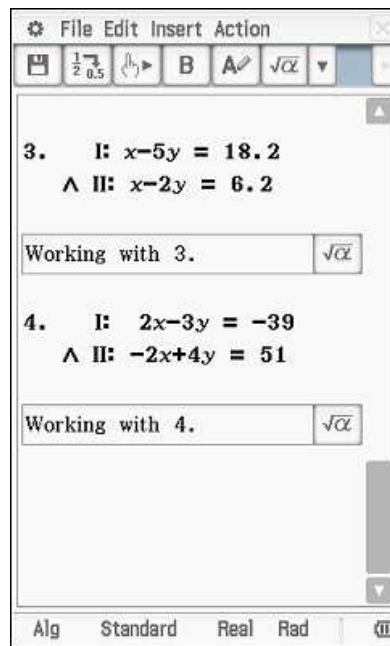
Alg   Standard   Real   Rad   (fig. 19)

Complete example of this type (ES):

File Edit Insert Action

💾 ½ ✋▶ B ∫dx √α ▼

Exercise with solutions
to the addition procedure
======================

1.   I: $x + 5y = -14.5$
   ∧ II: $-x - y = 2.5$

Step 1   ▤

$(x+5y=-14.5)+(-x-y=2.5)$
$$4 \cdot y = -12$$

Step 2   ▤

solve$(4 \cdot y = -12, y)$
$$\{y = -3\}$$

Alg   Standard   Real   Rad

Edit

B ✂ 📋 📋 a:=b:=

Nothing stands in the way to add the two equations, because in I the opposite x-coefficient of equation II appears.

Edit

B ✂ 📋 📋 a:=b:=

Now you must separate only the remaining variable.

Use the "solve"-instruction in order to get y.

Open the Main-menu, than "Action"- "Equation/Inequality"- "solve"

Edit

Now the received y-value
use in equation II,
in order to calculate the
fitting x-value.

File Edit Insert Action

Step 3

solve($-x-y=2.5|y=-3,x$)

$$\left\{x=\frac{1}{2}\right\}$$

solution set: $L = \left\{\left(\frac{1}{2}|-3\right)\right\}$

2.   I: $2x+y = 10.3$
   $\wedge$ II: $3x-y = 1.2$

Step 1

($2x+y=10.3$)+($3x-y=1.2$)

$$5 \cdot x = \frac{23}{2}$$

Alg   Standard   Real   Rad

Edit

Nothing stands in the way to add
the two equations, because in I
the opposite y-coefficient of
equation II appears.

Edit

By the "solve"-instruction
the x separate.

File Edit Insert Action

Step 2

solve($5 \cdot x=\frac{23}{2},x$)

$$\left\{x=\frac{23}{10}\right\}$$

Step 3

solve($2x+y=10.3|x=\frac{23}{10},y$)

$$\left\{y=\frac{57}{10}\right\}$$

solution set: $L= \{(2.3|5.7)\}$

3.   I: $x-5y = 18.2$
    $\wedge$ II: $x-2y = 6.2$

Alg   Standard   Real   Rad

Edit

Use the received x-value
in equation I.
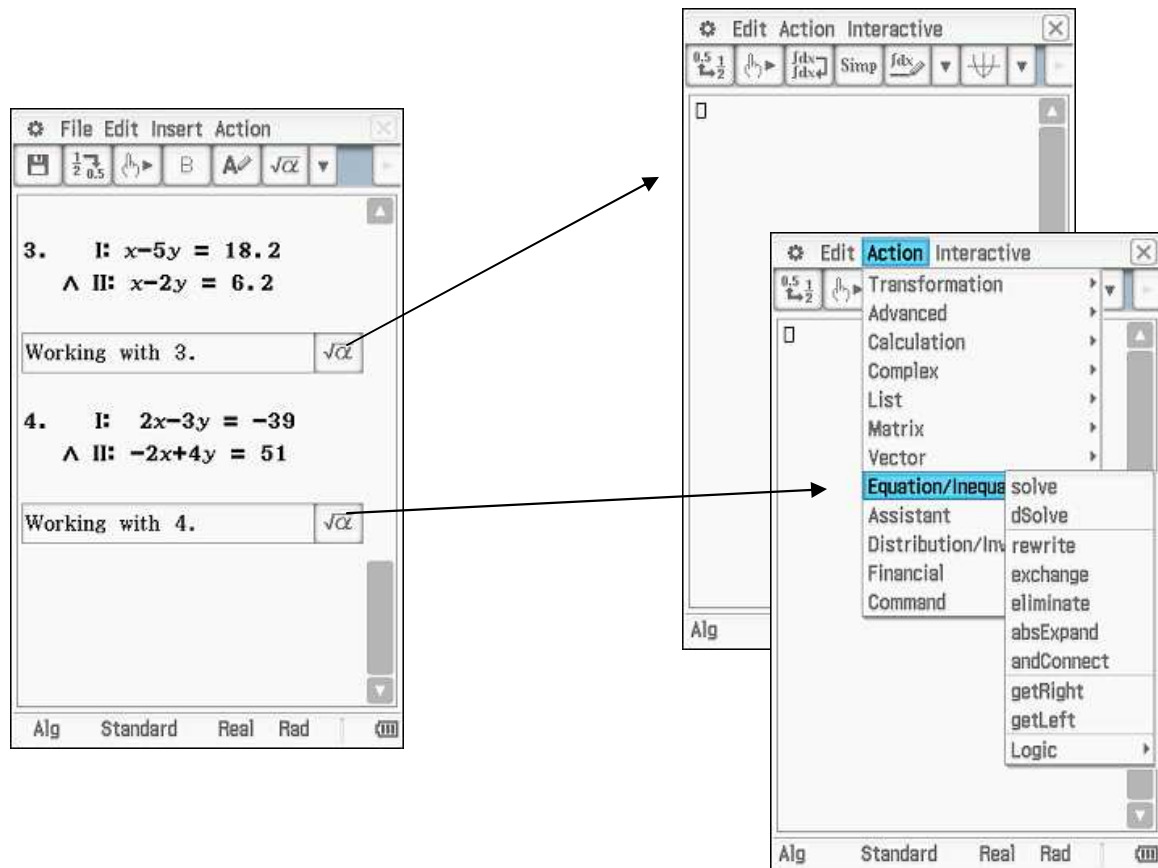
3.4 Category ET: Exercises with a test character

The exercises with a test character are just as developed as the tasks with solution (fig. 20).
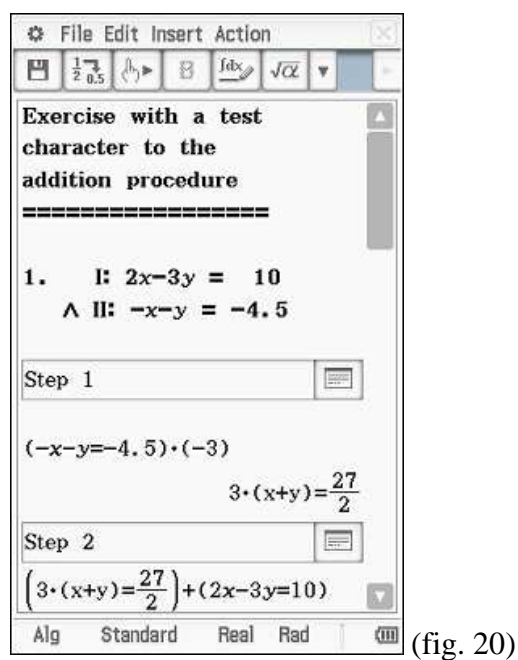
(fig. 20)

A first conception of the type of task is recommended. Misunderstandings over the requirement and the solution method are avoided in this way to a large extent. After this introductory task then as many as desired tasks with rising degree of difficulty follow. This category can be used for the fast examination of learning, for the exercise or for a purposeful test. Here the automaticness is to be considered. Each student can solve as many tasks for itself as he would like. The speed can he here determine. So this work form can be used ideally in practice hours. The teacher has time, itself possible to care questions or problems. The computer places the tasks.

## Complete example of this type (ET):
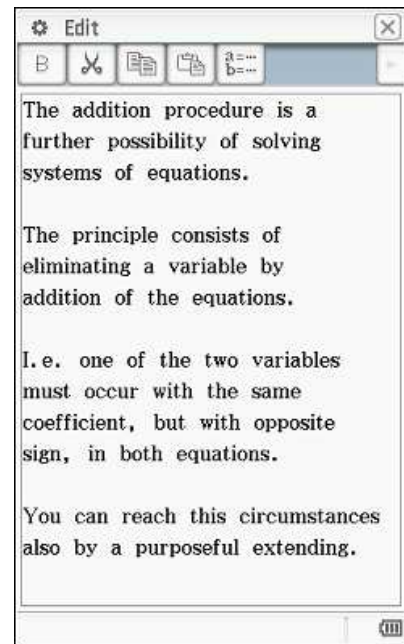
3.5 Category L: Learning units

The category C is generally occupied with learning contents. The material of new contents should be introduced and described here. This is suitable with the entrance into a new topic. With providing should be paid attention to ensure as large an overview as possible. Different goals should have to be called in different windows (fig. 21). So the remark should be separated also here to the general understanding (fig. 22) of intended purpose and proceeding (fig. 23). An example with the accompanying type of task can serve the further illustration.
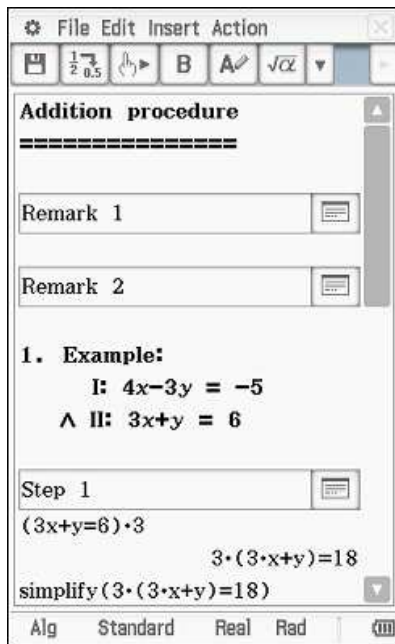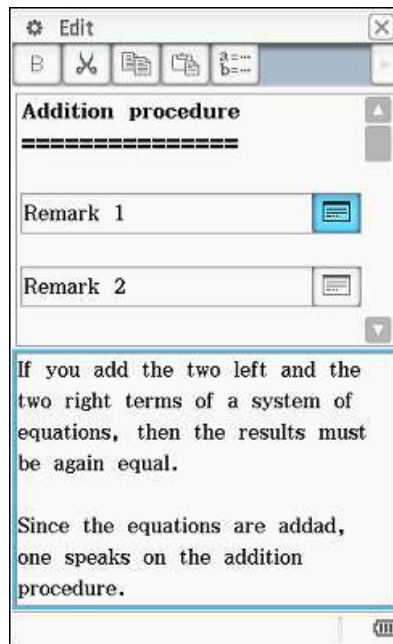
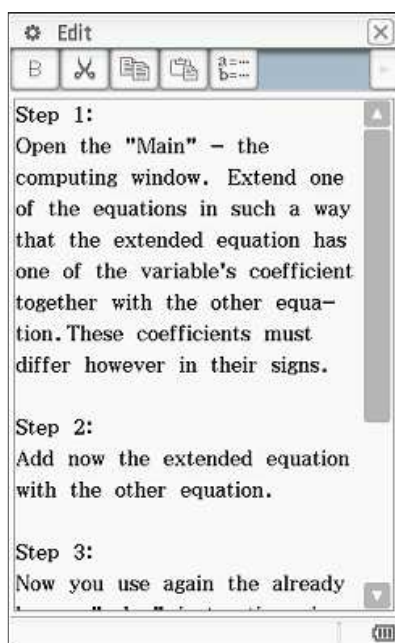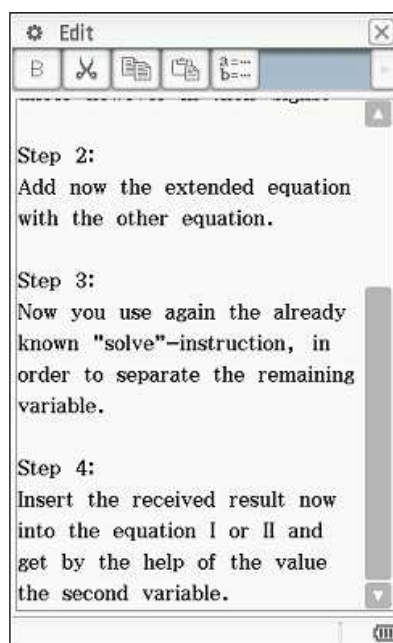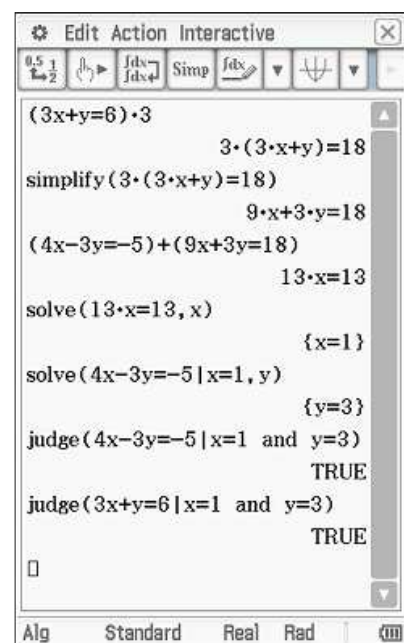(fig. 21)   (fig. 22)   (fig. 23)

It is to be made certain as with the exercises that small solution steps are not ignored. At the conclusion of the category learning unit a verbal (fig. 24-25) and computational (fig. 26) solution of a task should to be represented.

(fig. 24)   (fig. 25)   (fig. 26)

Complete example of this type (L):

Edit

B ✂ 📋 📋 a=...b=...

If you add the two left and the two right terms of a system of equations, then the results must be again equal.

Since the equations are addad, one speaks on the addition procedure.

File Edit Insert Action

💾 ½⅗ ✋▶ B A✐ √α ▼

Addition procedure
================

Remark 1   ▤

Remark 2   ▤

1. **Example:**
    I: $4x-3y = -5$
  ∧ II: $3x+y = 6$

Step 1   ▤

$(3x+y=6)\cdot3$
                $3\cdot(3\cdot x+y)=18$
$simplify(3\cdot(3\cdot x+y)=18)$ ▼

Alg   Standard   Real   Rad   ▥

Edit

B ✂ 📋 📋 a=...b=...

The addition procedure is a further possibility of solving systems of equations.

The principle consists of eliminating a variable by addition of the equations.

I. e. one of the two variables must occur with the same coefficient, but with opposite sign, in both equations.

You can reach this circumstances also by a purposeful extending.

Edit

B ✂ 📋 📋 a=...b=...

If the equation II is extended by the factor 3, then one receives in II the term "3y".

This term is void when adding the two equations by the "−3y" from equation I.

File Edit Insert Action

💾 ½⅗ ✋▶ B A✐ √α ▼

Step 2   ▤ ▲

$(4x-3y=-5)+(9x+3y=18)$
                $13\cdot x=13$

Step 3   ▤
$solve(13\cdot x=13,x)$
                $\{x=1\}$

Step 4   ▤
$solve(4x-3y=-5\,|\,x=1,y)$
                $\{y=3\}$
⇨ $L = \{(1|3)\}$

Remark 3   ▤ ▼

Alg   Standard   Real   Rad   ▥

Edit

B ✂ 📋 📋 a=...b=...

Now the two equations are added with one another.

There one of the variables in both equations with same coefficients, but with reverse sign, is present, is void this variable in the developing new equation in such a way.

Edit

B ✂ 📋 📋 a=...b=...

Now the remaining variable must be only separated. This goes with the "solve"−instruction.

Open in the "Main"−menu:
"Action"−
"Equation/Inequality"−
"solve"

⚙ Edit   ☒

B   ✂   ▤   ▥   a=···
                b=···

Now you insert the in such a way
obtained value into one of the
original equations I or II.

Now this equation must be solved
only concerning the remaining
unknown quantity to get the
solution set L completely.

⚙ File Edit Insert Action   ☒

💾 ½⁷₀.₅ ✋▶ B A✎ √α ▼

Step 3   ▤ △

solve(13·x=13,x)
                        {x=1}

Step 4   ▤

solve(4x−3y=−5|x=1,y)
                        {y=3}

⇒   L = {(1|3)}

Remark 3   ▤

Working 1   ▤

Working 2   √α

Alg   Standard   Real Rad   ▥

⚙ Edit   ☒

B   ✂   ▤   ▥   a=···
                b=···

In the following windows the whole
calculation method is summarized
and represented again.

The "judge"−instruction examines
the result for its correctness.

⚙ Edit Action Interactive   ☒

⁰·⁵₁ ½ ✋▶ ∫dx⤴ Simp ∫dx✎ ▼ ⟂ ▼

(3x+y=6)·3
                3·(3·x+y)=18
simplify(3·(3·x+y)=18)
                9·x+3·y=18
(4x−3y=−5)+(9x+3y=18)
                13·x=13
solve(13·x=13,x)
                        {x=1}
solve(4x−3y=−5|x=1,y)
                        {y=3}
judge(4x−3y=−5|x=1 and y=3)
                        TRUE
judge(3x+y=6|x=1 and y=3)
                        TRUE
□

Alg   Standard   Real Rad   ▥

⚙ Edit   ☒

B   ✂   ▤   ▥   a=···
                b=···

Step 1:
Open the "Main" − the computing window. Extend one of
the equations in such a way that the extended equation
has one of the variable's coefficient together with the
other equation. These coefficients must differ however in
their signs.

Step 2:
Add now the extended equation with the other equation.

Step 3:
Now you use again the already known "solve"−instruction,
in order to separate the remaining variable.

Step 4:
Insert the received result now into the equation I or II
and get by the help of the value the second variable.

3.6 Category M: Modelling

With the modelling it goes around the ability to embed an everyday problem appropriately into a mathematical model. With it also learning will have to be used. It concerns here tasks of text, which above all the transfer thinking are to strengthen. A task in an extra window is placed (fig. 27-28) and carried out in meaningful learning steps the transition of the everyday life world to a mathematical world.
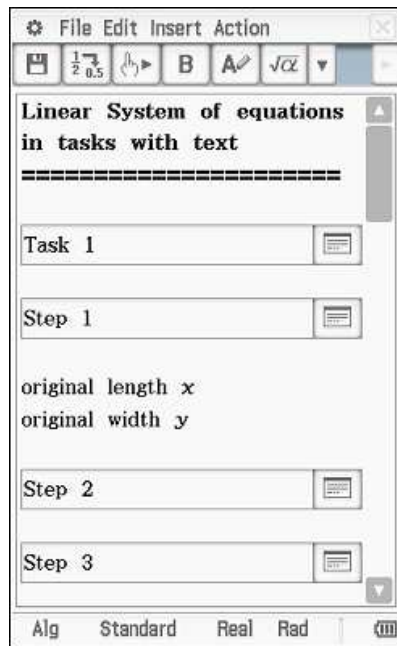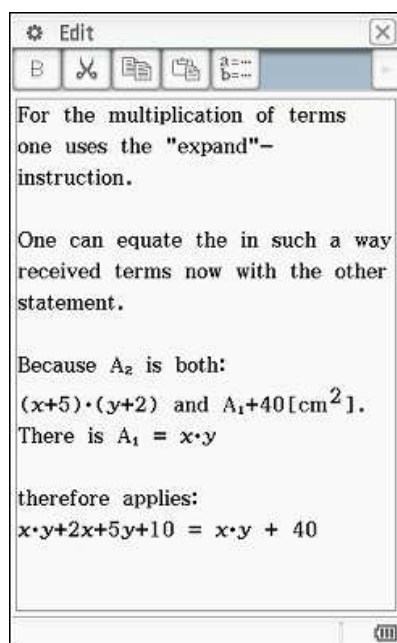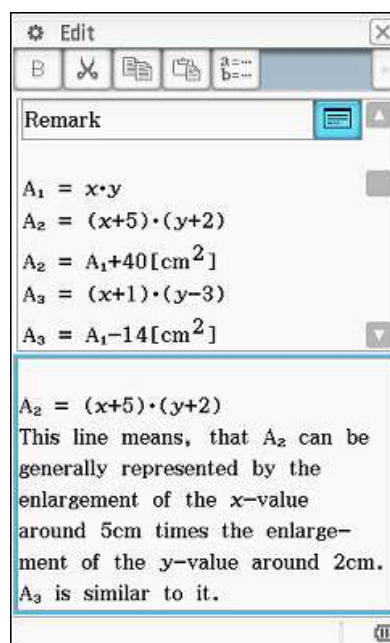

(fig. 27)
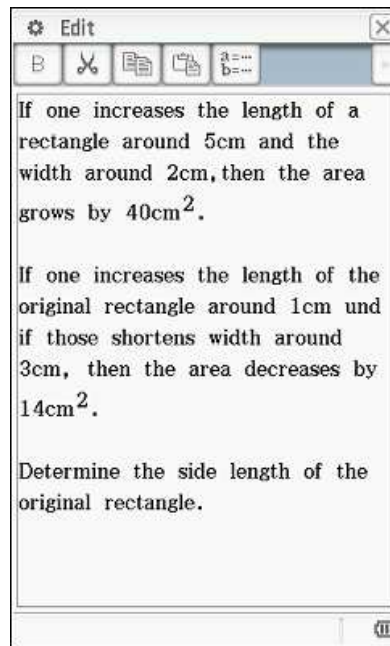
(fig. 28)

The proceeding for the treatment of the task is to be represented clearly. The students are to learn to set up from a text for example an equation. It must therefore be shown, how one selects the variables skillfully and how the individual conditions are used for an equation (fig. 29). Around an excessive demand of the student to avoid, one should interpret and show also the statement of the equation in such a way contentwise, how one can analyse its solution, in order to control itself again (fig. 30).


(fig. 29)

(fig. 30)

Conference "Mathematical Education today and tomorrow"   Moscow, Nov. 28-29, 2013

Prof. Dr. Ludwig Paditz,   Dresden University of Applied Sciences, Germany

Also the investigation of the definition set of the variables should be discussed particularly. This represents an additional examination possibility for the result which can be received and borders the model of the reality (fig. 31).
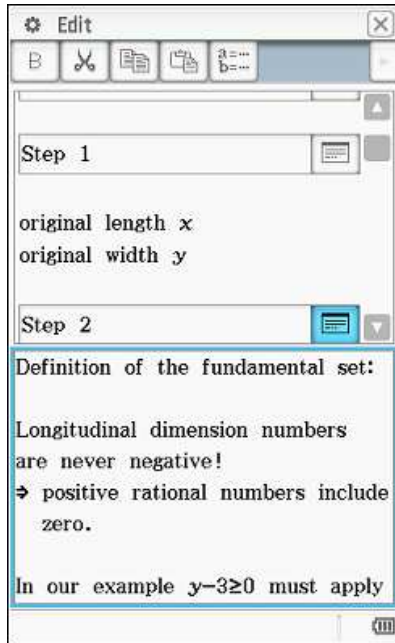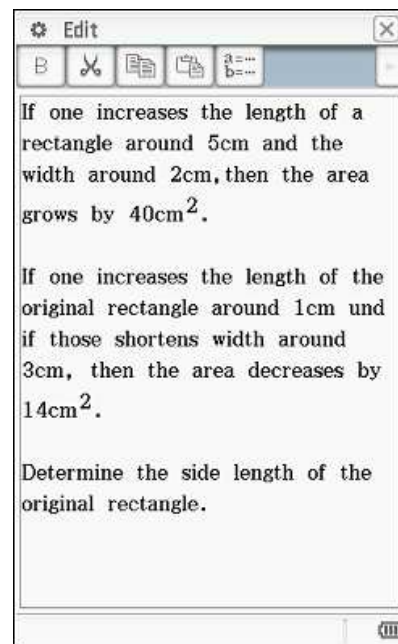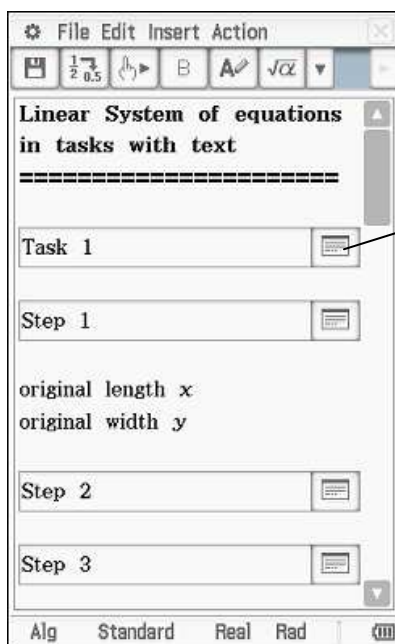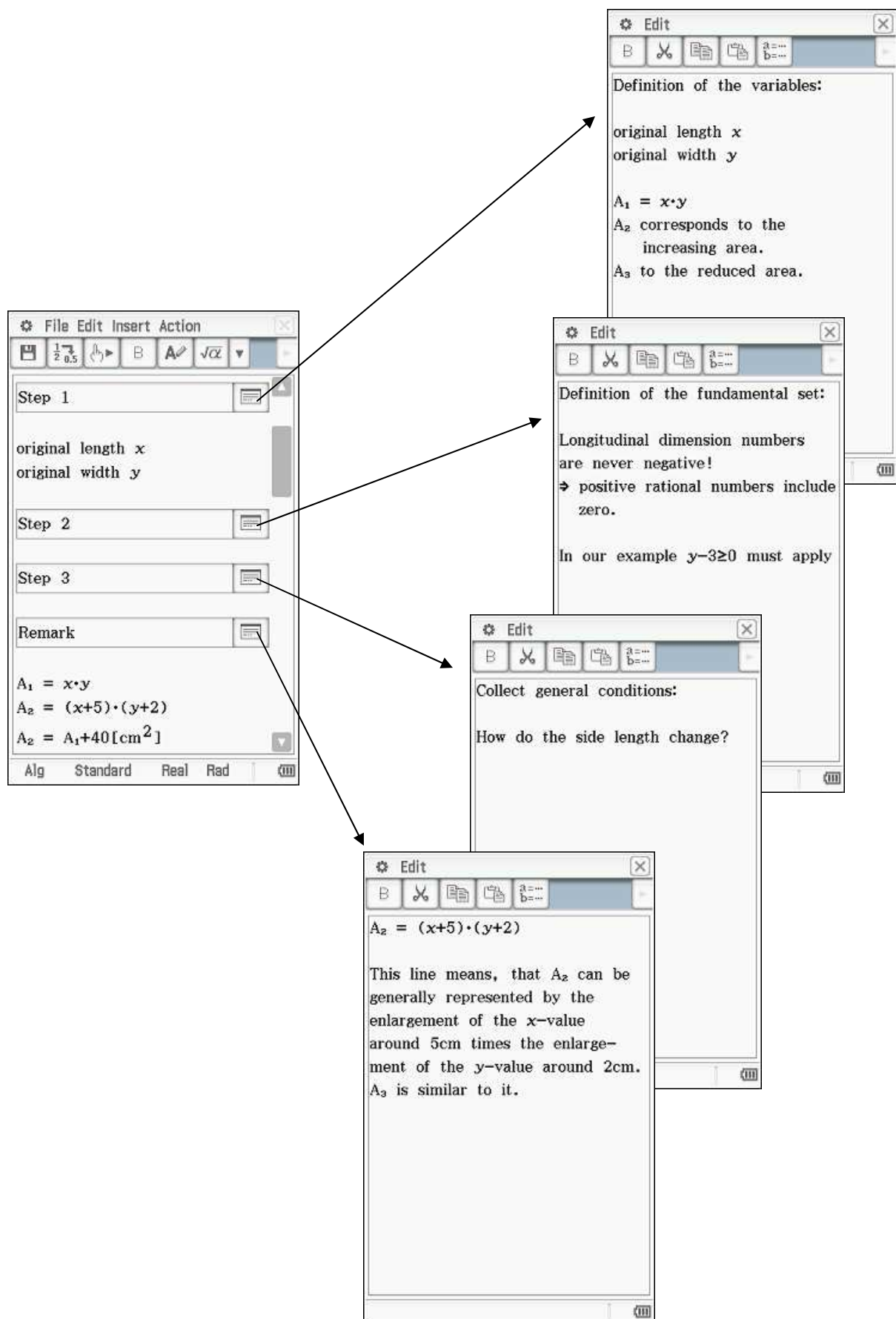

(fig. 31)
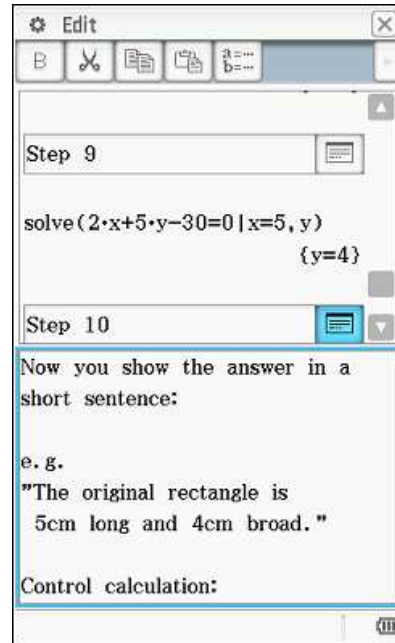
(fig. 32)

The student must learn the fact that a task with text is not terminated with the solution set but finished by an answer in text form (fig. 32). At the end of the detailed discussion a similar task should force the student to even use the learned. Specific practicing on tasks with text improves the text understanding and the transfer service.

Complete example of this type (M):

Definition of the variables:

original length x
original width y

$A_1 = x \cdot y$
A_2 corresponds to the increasing area.
A_3 to the reduced area.

Definition of the fundamental set:

Longitudinal dimension numbers are never negative!
➔ positive rational numbers include zero.

In our example $y-3 \geq 0$ must apply

Collect general conditions:

How do the side length change?

$A_2 = (x+5) \cdot (y+2)$

This line means, that A_2 can be generally represented by the enlargement of the x-value around 5cm times the enlargement of the y-value around 2cm. A_3 is similar to it.

File Edit Insert Action

Step 1

original length x
original width y

Step 2

Step 3

Remark

$A_1 = x \cdot y$
$A_2 = (x+5) \cdot (y+2)$
$A_2 = A_1 + 40 [cm^2]$

Alg   Standard   Real   Rad

For the multiplication of terms one uses the "expand"–instruction.

One can equate the in such a way received terms now with the other statement.

Because A_2 is both:
$(x+5)\cdot(y+2)$ and $A_1+40[cm^2]$.
There is $A_1 = x\cdot y$

therefore applies:
$x\cdot y+2x+5y+10 = x\cdot y + 40$

File Edit Insert Action

$A_1 = x\cdot y$
$A_2 = (x+5)\cdot(y+2)$
$A_2 = A_1+40[cm^2]$
$A_3 = (x+1)\cdot(y-3)$
$A_3 = A_1-14[cm^2]$

Step 4

Working 1

$\quad$ I: $x\cdot y+2\cdot x+5\cdot y+10=x\cdot y+40$
$\wedge$ II: $x\cdot y-3\cdot x+y-3 =x\cdot y-14$

Step 5

Alg   Standard   Real   Rad

Edit Action Interactive

$expand((x+5)\cdot(y+2))$
$\qquad\qquad x\cdot y+2\cdot x+5\cdot y+10$
$expand((x+1)\cdot(y-3))$
$\qquad\qquad x\cdot y-3\cdot x+y-3$

Edit

Summarize and simplify as far as possible.

This transformation fulfils the "rewrite"–instruction.

File Edit Insert Action

Working 2

$\quad$ I: $2\cdot x+5\cdot y-30=0$
$\wedge$ II: $-3\cdot x+y+11=0$

Step 6

$solve(2\cdot x+5\cdot y-30=0, y)$
$\qquad\qquad \left\{y=\dfrac{-2\cdot x}{5}+6\right\}$
$solve(-3\cdot x+y+11=0, y)$
$\qquad\qquad \{y=3\cdot x-11\}$

Step 7

Alg   Standard   Real   Rad

Edit Action Interactive

$rewrite(x\cdot y+2\cdot x+5\cdot y+10=x\cdot y+4$
$\qquad\qquad 2\cdot x+5\cdot y-30=0$
$rewrite(x\cdot y-3\cdot x+y-3 =x\cdot y-14)$
$\qquad\qquad -3\cdot x+y+11=0$

Edit

In order to be able to equate two equations, they must be dissolved first after the same variables.

Alg

⊙ Edit   ⊠

B ✂ ▤ ▥ a=···
                b=···

Now both in the equation I as well
as in II the variable "y" stands.
After this variable can be dis-
solved now.
Also here the rules of the
equivalent transformation apply.

⊙ File Edit Insert Action   ⊠

💾 ¹⁷₂₀₅ ✋▶ B A✐ √α ▼

Step 7   ▤   ▲

   I: $y = \dfrac{-2 \cdot x}{5} + 6$

∧ II: $y = 3 \cdot x - 11$

Step 8   ▤

$solve\left(\dfrac{-2 \cdot x}{5} + 6 = 3 \cdot x - 11, x\right)$

                        {x=5}

Step 9   ▤

$solve(2 \cdot x + 5 \cdot y - 30 = 0 \,|\, x = 5, y)$

                        {y=4} ▼

Alg   Standard   Real Rad   ▥

⊙ Edit   ⊠

B ✂ ▤ ▥ a=···
                b=···

Now the equations can be equated
and the variables be separated.
The received value of 5cm seems
to be meaningful. It means that
the original rectangle length
amounted to 5cm.

⊙ Edit   ⊠

B ✂ ▤ ▥ a=···
                b=···

You insert this value now only
into one of the initial conditions
and so you receive the second
unknown quantity.

Also the statement "y=4" is
realistic. It means that the
original side y is long 4cm.

⊙ Edit   ⊠

B ✂ ▤ ▥ a=···
                b=···

Now you show the answer in a
short sentence:

e.g.
"The original rectangle is
  5cm long and 4cm broad."

Control calculation:

⊙ File Edit Insert Action   ⊠

💾 ¹⁷₂₀₅ ✋▶ B A✐ √α ▼

$solve\left(\dfrac{-2 \cdot x}{5} + 6 = 3 \cdot x - 11, x\right)$ ▲

                        {x=5}

Step 9   ▤

$solve(2 \cdot x + 5 \cdot y - 30 = 0 \,|\, x = 5, y)$

                        {y=4}

Step 10   ▤

Working 3   √α

Task 2   ▤   ▼

Alg   Standard   Real Rad   ▥

⊙ Edit Action Interactive   ⊠

⁰·⁵₁₂ ᑯ▶ ∫dx⁻ᐟ Simp ∫dx✐ ▼ ⊣⊢ ▼

(5+5)(4+2)   ▲

                            60

(5+1)(4-3)

                             6

5·4

                            20

control calculation

                                    ▼

Alg   Standard   Real Rad   ▥

⊙ Edit   ⊠

B ✂ ▤ ▥ a=···
                b=···

A rectangle has the circumference
45cm.

If one extends a side by 3cm and
if one shortens the other side at
the same time around 3cm, then
the area grows by 42cm^2.

Compute the length (width) of
the original rectangle.

                                    ▥

3.7 Category P: Protocol

The protocol has a different application type. Ever more teacher go the way from the closed to open instruction. However it is often connected with time delay, to let collect and speak what in a group compiled. Frequently to the medium „foil for the overhead projector" or „poster" one falls back here. Here the ClassPad could be used.

The material is jointly to prepare, but individually noted. The teacher or the schoolmates have thereafter the possibility, after the lecture of the particular, of seeing the solutions of the group in different representations. The results compiled here can be presented moreover with the ClassPad simply and fast. Week plans or projects could be spoken so medium-supported.

Also concerning the protocols of the students it should be paid attention to the necessary form. The summary of learning contents can contain also elements of an eActivitiy. To self-sufficiency is to be paid attention however nevertheless. So also difficulties or proposals for solution can be stated by the pupil. These results can be worked on then in the class federation or in the single discussion.

Example (fig. 33-34):

(fig. 33)
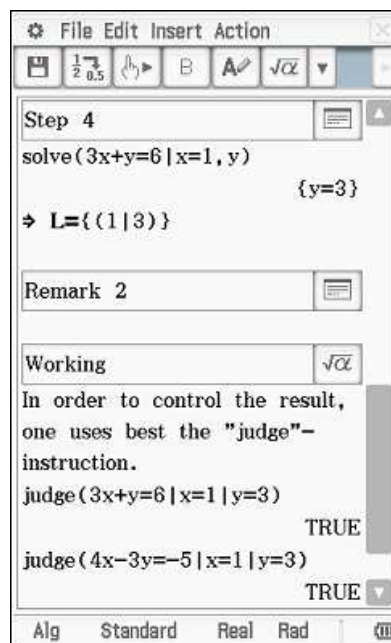
(fig. 34)

Complete example of this type (P):

Protocol by
Markus Mustermann
================

Remark 1

Task

Step 1
$(3x+y=6)\cdot 3$
$$3\cdot(3\cdot x+y)=18$$

Step 2
$(3\cdot(3\cdot x+y)=18)+(4x-3y=-5)$
$$3\cdot(3\cdot x+y)+4\cdot x-3\cdot y=13$$

Alg   Standard   Real   Rad

If you add the two left and the two right terms of a system of equations, then the results must be again equal.

Since the equations are addaed, one speaks on the addition procedure.

This procedure serves the solution a system of equations therefore likewise.

For this type of task the following problem is typical:

I: $4x-3y = -5$
∧ II: $3x+y = 6$

So that one of the two variables is void when adding the equations, it must be first extended.

One wants to reach that in the equation a variable coefficient with opposite sign develops.

Here we extend the equation II.

Step 2
$(3\cdot(3\cdot x+y)=18)+(4x-3y=-5)$
$$3\cdot(3\cdot x+y)+4\cdot x-3\cdot y=13$$
$simplify(3\cdot(3\cdot x+y)+4\cdot x-3\cdot y=1$ ▶
$$13\cdot x=13$$

Step 3

$solve(13\cdot x=13,x)$
$$\{x=1\}$$

Step 4

$solve(3x+y=6\,|\,x=1,y)$
$$\{y=3\}$$
⇒ L = {(1|3)}

Alg   Standard   Real   Rad

Now the two equations are added and the results with "simplify"-instruction is simplified.

The variable y is disappeared by adding with the opposite term.

Now the remaining variable must be only separated. This goes with the "solve"-instruction.

Open in the "Main"-menu:
"Action"-
"Equation/Inequality"-
"solve".

☼ Edit   ✕

| B | ✂ | 📋 | 📄 | a=···b=··· |

Now you insert the in such a way
obtained value into one of the
original equations I or II.

Now this equation must be solved
only concerning the remaining
unknown quantity to get the
solution set L completely.

☼ File Edit Insert Action   ✕

| 💾 | ½ ⁷₀.₅ | 👆▶ | B | A✐ | √α | ▼ |

Step 4   ▤   ▲

solve(3x+y=6|x=1,y)

                              {y=3}

⇒ L = {(1|3)}

Remark 2   ▤

Working   √α

In order to control the result,
one uses best the "judge"–
instruction.

judge(3x+y=6|x=1|y=3)
                              TRUE

judge(4x−3y=−5|x=1|y=3)
                              TRUE ▼

Alg   Standard   Real   Rad   ▥

☼ Edit   ✕

| B | ✂ | 📋 | 📄 | a=···b=··· |

In the following window the whole
calculation method is summarized
and represented again.

The "judge"–instruction examines
the result for its correctness.

☼ Edit Action Interactive   ✕

| ⁰·⁵ ½ | 👆▶ | ∫dx | Simp | ∫dx✐ | ▼ | ⊬ | ▼ |

(3x+y=6)·3   ▲

                     3·(3·x+y)=18

(3·(3·x+y)=18)+(4x−3y=−5)

              3·(3·x+y)+4·x−3·y=13

simplify(3·(3·x+y)+4·x−3·y=1 ▶

                           13·x=13

solve(13·x=13,x)

                             {x=1}

solve(3x+y=6|x=1,y)

                             {y=3}

judge(3x+y=6|x=1|y=3)
                             TRUE

judge(4x−3y=−5|x=1|y=3)
                             TRUE

□   ▼

Alg   Standard   Real   Rad   ▥

4. Keywords

The agreed keywords should not appear as restriction, but an additional internal arrangement they are seen. This makes possible a fast access to looked information. In the following, now the individual keywords on function and contents are described. If individual keywords in an eActivity occur several times, then one becomes number these keywords additionally.

4.1 Setting of tasks

Setting of tasks should appear with their own window. Under the term „setting of tasks" or „task" you can deal on the one hand with the desired solution method, or on the other hand state simply setting of tasks. This window is suitable particularly for tasks with text, it can used also with the category „modelling" (fig. 35).
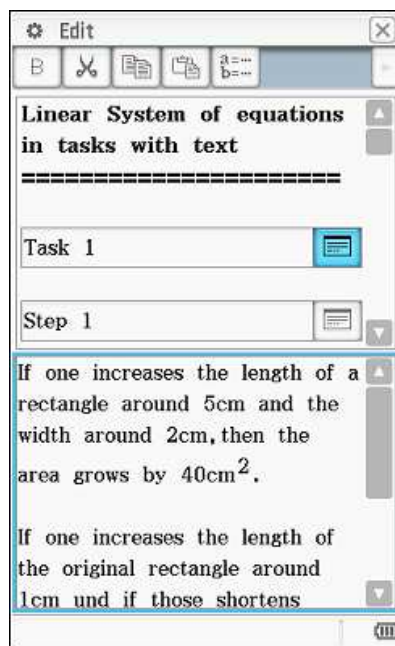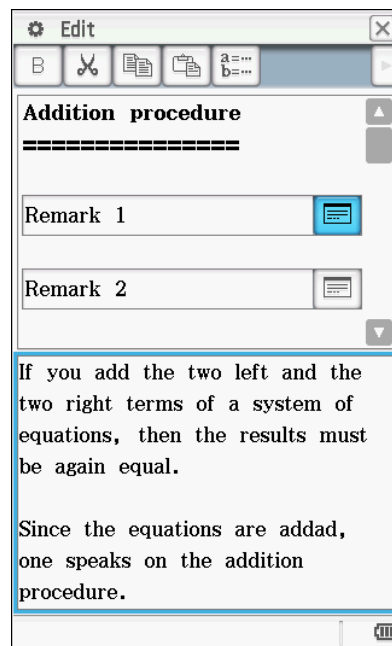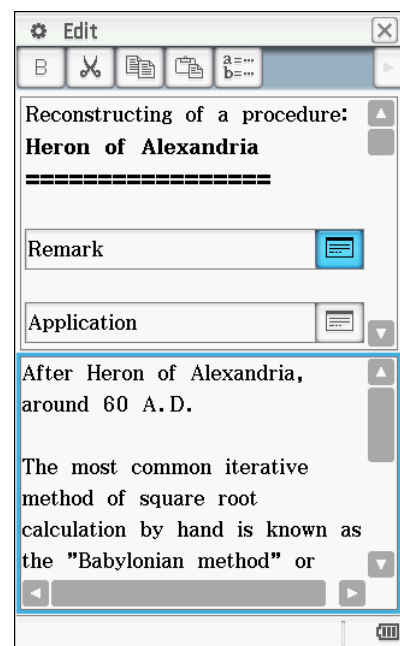
(fig. 35)   (fig. 36)   (fig. 37)

The extensive text can be in this way simply accessed. After reading the text one closes the window and can to the treatment dedicate itself. The e-Activity loses in such a way to extent and the overview is improved.

4.2 Remark

Under remarks solution assistance can be given (fig. 36), or e.g. historical references (fig. 37).

4.3 Exercises

Under „exercise" the student finds the possibility of trying solutions themselves. He is to approach playful the solution, discover rules themselves. These windows find particularly in the category D their employment (fig.38).
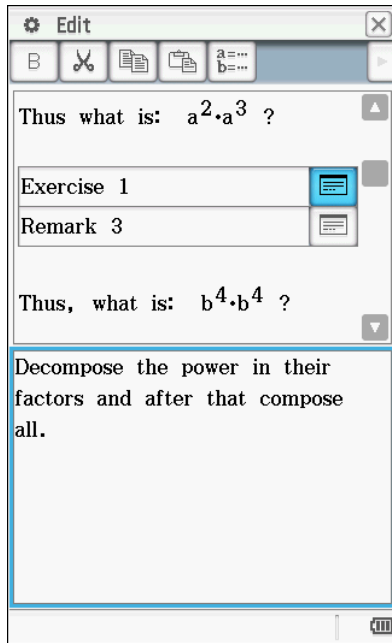
(fig. 38)

4.4 Step

The windows with the designation „step" serve the explanation of the individual steps of computation (fig. 39-41).


(fig. 39)

(fig. 40)

(fig. 41)

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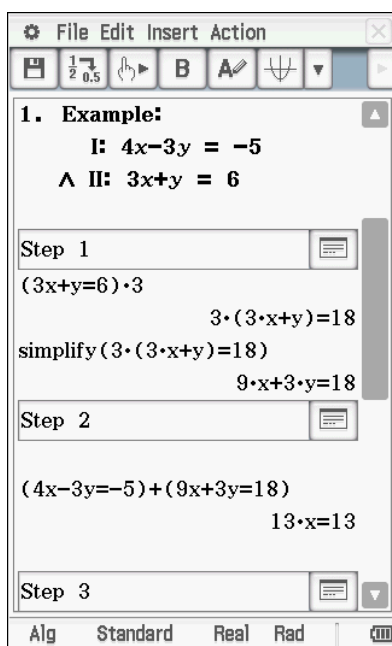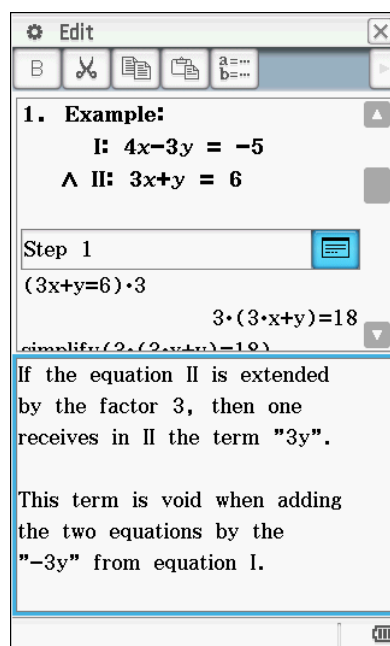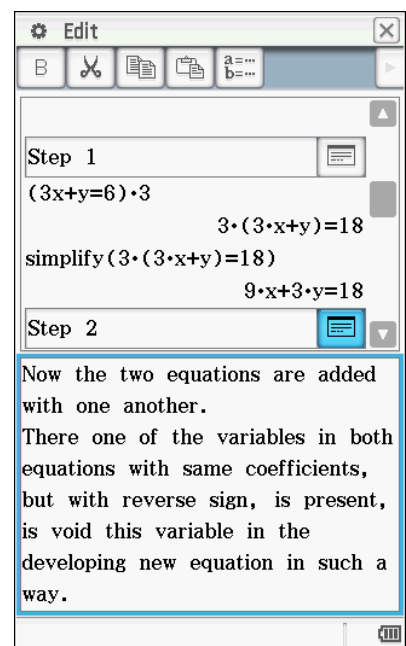
It is recommendable not to give the instructions too in detail. The student should have the possibility of discovering with one view the substantial. The requirements should be set on a suitable level, so that a positive work attitude remains keeping upright. Thus is meant that the student does not to have to consider each step, he can obvious jump over and complicated piece for piece seize.

4.5 Consideration

With the keyword „consideration" it gives to say actually not much. Nevertheless one should define it against the „remarks". The content of a conclusion or of a train of thought can be noted in the „consideration". So it could be written (fig. 42) also as „consideration":
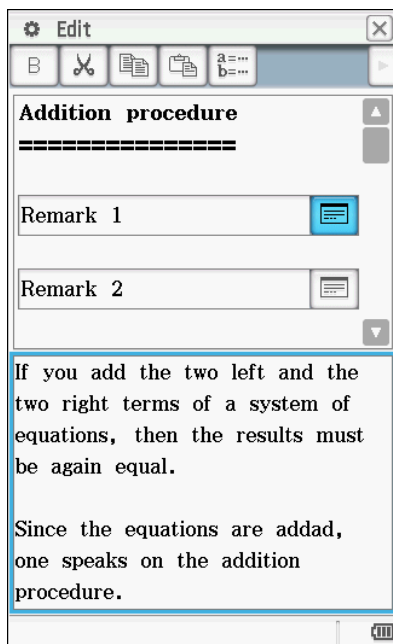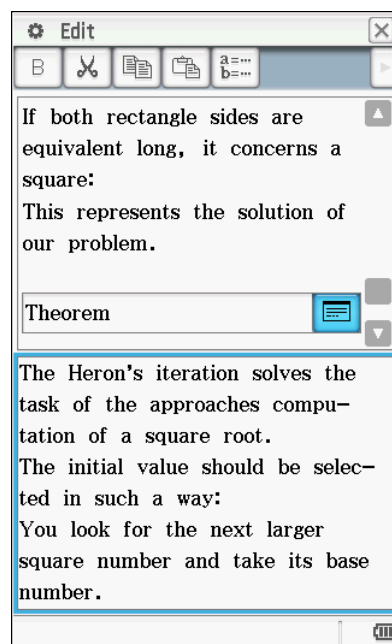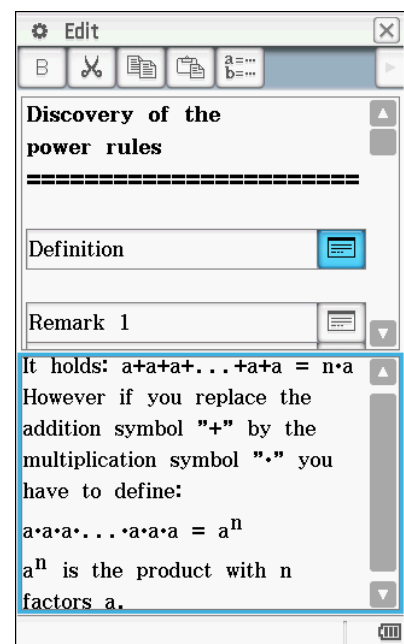
(fig. 42)   (fig. 43)   (fig. 44)

4.6 Theorem (theorem to remember)

The keyword describes circumstances to note a „theorem". It concerns a found result, a theorem a compiled definition, which one is to note. Additionally it can offer however still important hints or noticing assistance (fig. 43).

4.7 Definition

Here it stands generally valid definitions. With these definitions future arithmetic rules can be compiled. They are usually the starting situation of the category D (fig. 44).

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4.8 Rule

With this keyword it concerns the worked out arithmetic rule. The window „rule" closes usually the application and it can be regarded thus also as educational aim (fig. 45).
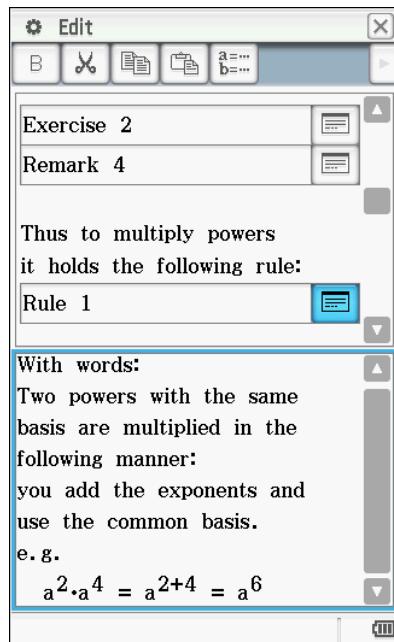
(fig. 45)

5. Writing an eActivity

This document describes the production and the exchange of an eActivity by the help of the ClassPad 400 and the ClassPad Manager II program.

The eActivities are documents, in which you can combine different applications and abilities of the ClassPad with texts. Start your ClassPad or the ClassPad manager (see below).

Change into the main menu, the so called "Menu", and select the icon .
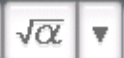
Now you see the eActivity window. The conditions of the last editing are indicated if necessary. In the illustration (fig. 46) you see beside the menu border above to the row of icons completely and under it in addition the drop down menus of the icons. Over the menu and the icons you have access to the eActivity specific elements. The status line (down) is the same as in the „Main" menu.

In order to begin a new eActivity, you select [File], [New] and confirm the enquiry „Clear All" with [OK]. Now the field is empty and you can begin with the working.

The Insert menu to insert an application data strip

Text or Calculation input mode

Switching between **bold** and plain text

(fig. 46)
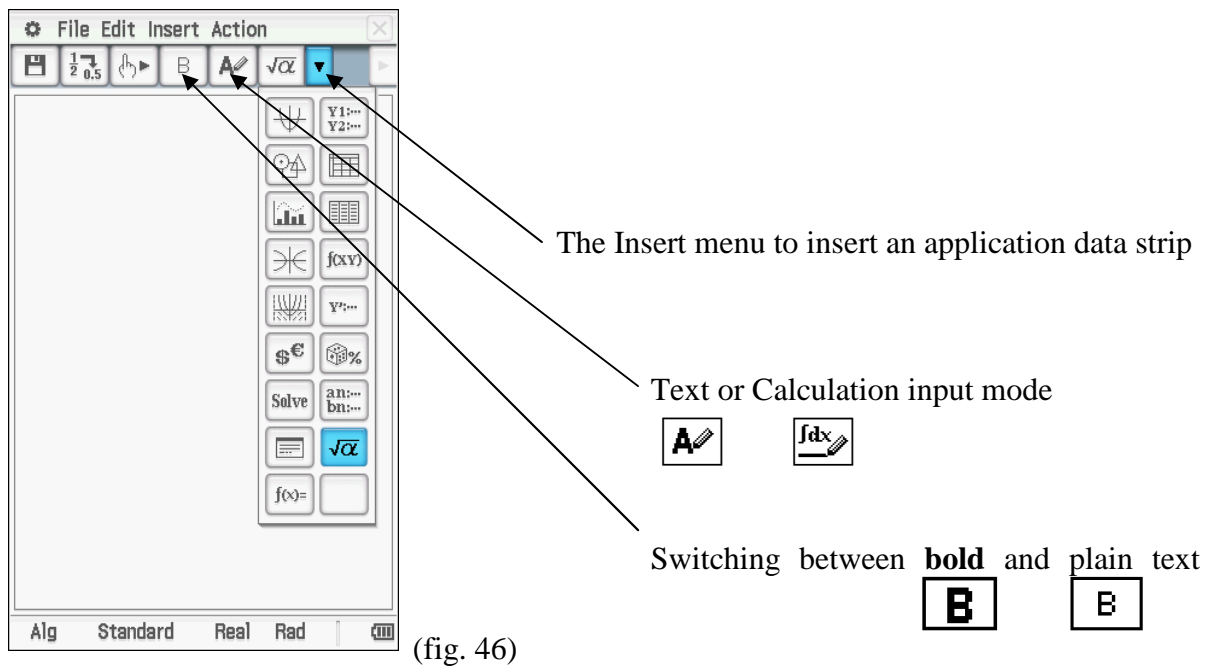
An eActivity consists in individual lines, which are from different nature and which you can combine arbitrarily:

- Text lines you recognize by the fact that the fourth icon of the icon row shows a
  (over this icon you switch also between calculation lines and text lines). Text lines can contain text, formulas etc. Mathematical formulas in text lines are not computed and have thus a purely informative character.
  Over the icon **B** you switch over for the entire line between **bold** and normal writing.
  For longer texts and references there is also an application data field (see below) „Note".

- Computation lines show instead as the fifth icon. Computation lines behave like the lines in the mathematical „Main" menu of the ClassPad.

- Geometry links you recognize by the symbol at the start of line, you can it create over [Insert], [Geometry Link] and geometry objects thereby in dicate and modify (e.g. you can draw a straight line by Drag&Drop this way).

- Application data fields you add over the selection under or over the menu option [Insert]. They become represented by a header, which you can mark, and by an icon of respective application.

The selection of the icon in the application strip opens and closes the respective application. Which field is open straight, recognize by coloured inverting of the icon. They can build data from application with help of „pulling and placing" (Drag&Drop) into your eActivity and in reverse. Changes in an eActivity lead to appropriate changes in application and if necessary in reverse.
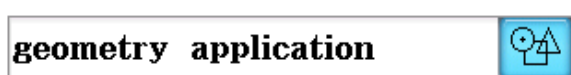
geometry application

fig. 47: a geometry application strip
(with the header line „geometry application")

5.1 A small example of use

For exemplary elucidation:

Change in the eActivity window and provide over [File], [New] a new eActivity. Press the keyboard key to let over indicate the soft keyboard and now you change by choice of the abc rider to the alphabetical keyboard. Give as titles e.g.. „eActivity example".

Click up on [**B**], in order to let the header appear bold.

Press two times [EXE], in order to begin a new line and to insert a further line distance.
Insert over [Insert], [Geometry Link] one geometry link - the start of line shows now the symbol . Enter now e.g. y = 2x+1.

Add over [Insert], [Geometry] or over the icon row a geometry data field - it is opened and the geometry menu border appears automatically.

Now you tap into the eActivity window (above), mark y = 2x+1, in which you pull with the pen over it. Set the pen on the marked expression and pull with the pen into the geometry data field. The straight line is drawn. If you place now two points on the straight line, you can change the straight line in their situation. These changes you see immediately in geometry link.

Close the geometry data field with one click on its icon. Type with the pen into the header line of the geometry data field and enter a title, e.g. „a straight line".
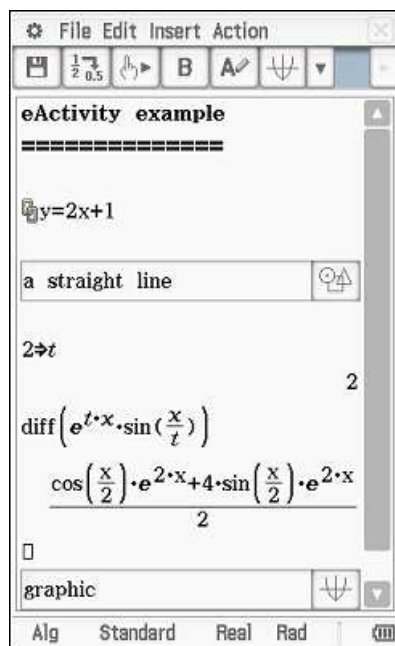
 (fig. 48)
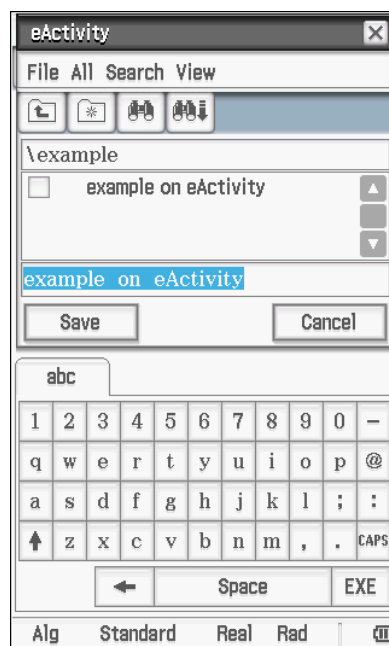
 (fig. 49)

Completely similarly you insert a computation line and e.g. a diagram data field, compute something in the computation line and draw the result into the diagram data field, in order to produce a diagram. In fig. 48 you see the result. The cursor is located in the line over the diagram data field „graphic".

You recognize that it concerns a computation line, by the fact that the empty line ends with a small box and that shows the fifth icon [∫dx] .

Note: Use „Resize" in order to have for the application data fields the whole screen available.

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6. Storage of an eActivity

Store your eActivity over [File], [Save] (store) or the choice of ⊟ , indicate over the soft keyboard a name and press [Save]. By the example: enter as names „example on eActivity" (fig. 49).

7. Opening of an eActivity

An already existing eActivity you open through [File], [Open] or the choice of ⊟ , choice an eActivity and click [Open] (fig. 50).

 (fig. 50)

Note:
If you provide your eActivity with the ClassPad manager, you profit among other things from the possibility of the larger display, to the use of the PC keyboard and you can store your eActivity directly into an *.vcp (virtual ClassPad file) image (to select it in addition in the context menu, which you reach by means of the right mouse button, [File], [Save] or [Save As...]).

The **ClassPad manual** on the ClassPad Utilites CD offers a complete description of all eActivity elements and further examples in chapter 10 „eActivity Application".

8. Data exchange between PC and ClassPad 400

You can connect a ClassPad to a computer (Windows-PC or MAC) or to another ClassPad and exchange data (*.vcp files) with the connected device. Here you need an USB cable or a data communication cable. Further information see [2].

References:

[1] Tschacher, K., Lang, W. and Liebezeit, J.-W.: **eActivity** (in German) 2004,
       Internet: http://www.classpad.de/download/Richtlinien.pdf (Nov. 5, 2013)

[2] **User's Guide ClassPad II** (fx-CP400) (in English)
       Internet: http://support.casio.com/en/manual/004/ClassPadII_UG_EN.pdf (Nov. 5, 2013)

[3] Damköhler, J.: **Der ClassPad in der Schule** – Ideen für den Einsatz im Mathematik-
       Unterricht, Bildungsverlag EINS 2005, ISBN 978-3-427-33400-2