Multilevel methods for problems with a non-smooth solution

by

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In practical applications we often have to solve boundary value problems with non-smooth input data, e.g. polygonally bounded domains with re-entrant corners, discontinuous coefficients or points where the boundary conditions change their type. In these cases the solution u belongs to a Sobolev-Slobodetskiy space $H^{1+\alpha}(\Omega),\quad 0 < \alpha \leq 1$, only (see, e.g., [1]).

If we apply a usual finite element scheme to solve a boundary value problem with non-smooth input data, we will in general not obtain the same convergence order as in the case of smooth data. But utilizing the knowledge of the asymptotic solution behaviour near the singular points one can construct finite element schemes which guarantee the same convergence order as the usual finite element schemes for problems with smooth input data (see, e.g., [2], [3]).

Based on a sequence of finite element discretizations we study the asymptotic behaviour of the solution and generate locally (in dependence on the behaviour of the solution) refined meshes in a neighbourhood of the singular points. We prove that the corresponding finite element schemes with piecewise linear test functions have got the usual convergence order $O(h)$ in the H^1-norm, where the number of nodes remains in the order $O(h^{-2})$ for two-dimensional problems. Further, we discuss various multilevel algorithms for solving the systems of the algebraic finite element equations. Finally, we show the efficiency of the proposed algorithms by model problems and hard practical problems.

References

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